

RESONANCES DO NOT EQUILIBRATE*

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We discuss, in qualitative and quantitative fashion, the yields of hadron resonances. We show that these yields, in general, are not in chemical equilibrium. We evaluate the non-equilibrium abundances in a dynamic model implementing the $1 + 2 \leftrightarrow 3$ resonance formation reactions. Due to the strength of these reactions, we show the $\Sigma(1385)$ enhancement, and the $\Lambda(1520)$ suppression explicitly.

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1. Why study strange resonances and quark–gluon plasma?

We study the quark confining vacuum structure, with the objective to liberate quarks and gluons at high temperature. The present day experimental effort involves colliding heavy nuclei and this limits the domain of deconfinement to the size of the atomic nucleus. The ‘free’ quarks and gluons form a thermal gas comprising color charges, the quark–gluon plasma — a hot soup of elementary matter, last seen in the Universe when matter was formed at about $30 \mu\text{s}$ after the big-bang.

The temperature at which we deconfine the quark content of nucleons is $T_c \simeq 160 \text{ MeV}$. However, we reach in heavy ion collisions, $T_{\text{Max}} \simeq 2\text{--}5 T_c$. At this relatively high temperature, we create strange quark and antiquark pairs in an abundance which rivals that of light quarks [1]. While this is going on, the compressed quark matter expands, the expansion consumes the thermal pressure and energy, and ultimately the fireball of hot quarks breaks apart at $T_f < T_c$. Of particular interest is the high abundance of

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antiquarks, including anti-strangeness. The high strange antimatter yield is our evidence that we have recreated the early Universe in a laboratory set experiment.

We are interested in understanding the physics of quark–gluon plasma in the last moments of its existence, and thus the conditions prevailing in the transformation of quarks and gluons into hadrons. The yields of particles produced can be successfully described using statistical physics methods. We use the program SHARE (Statistical HAdronization with REsonances) for this purpose [2, 3]. The remarkable success of this model indicates that the production of hadrons in heavy ion collisions is governed mostly by the accessible phase space. This, in turn, implies that particles are produced in a process resembling vapor evaporation from a hot soup. In this process, hadrons are formed from very ‘sticky’ quarks and this helps to saturate the probability of particle formation.

Aside of stable (under strong interactions) particles, the statistical hadronization of quark–gluon plasma predicts the yields of (anti)baryon resonances. Experimental results available today show that the yields of these states do not always follow the model expectations [4–10]. Given the success of SHARE, we interpret this as a post hadronization evolution of observable yields [11, 12]. Thus, the first objective of this report, see Sec. 2, is to explain why, for stable particles, we can directly apply the statistical hadronization model, while observed resonance yields are subject to post-hadronization dynamics.

After that, we develop the kinetic model tools in Sec. 3, define the model and the initial conditions in Sec. 4, and present, in the following Sec. 5, a detailed numerical study demonstrating that $\Sigma(1385)$ can be greatly enhanced in the observed abundance compared to statistical equilibrium, while other more stable resonances, such as $\Lambda(1520)$ are suppressed.

The understanding of this behavior of strange baryons and antibaryons and their resonances sharpens the tools available to us in the study of quark–gluon plasma properties, at the time of phase conversion into hadrons. This work improves the understanding of the physics of hadronization of quark–gluon plasma, that is of the process of freezing of the deconfined vacuum.

2. Why resonances, in general, do not chemically equilibrate?

The observed stable particle yield is controlled solely by freeze-out temperature, and this yield contains the decay of all resonances. However, the resonance abundances can evolve and mix with stable particles without altering the observed final stable particle yield, since there is no information in the stable particle yield about ‘sharing’ of the yield with resonances. Therefore, we cannot assume that resonance yields are governed by same physics

as the final stable particle yields. We consider, as an example, the reaction



and imagine, for purpose of following simple illustration, that the system we study comprises ONLY these three particles.

Reaction Eq. (1) does not change either the pion π or lambda Λ yield, since all Σ^* -resonances ever made will ultimately contribute to these yields upon resonance decay, which naturally happens before the stable particles are observed. On the other hand, we measure the yield of Σ^* by the invariant mass method. This is done by considering all pair combination of two presumed decay products and evaluating from the energy and momentum of both particles the invariant mass distribution dN/dM where:

$$M = \sqrt{(E_\pi + E_\Lambda)^2 - (\vec{p}_\pi + \vec{p}_\Lambda)^2}.$$

This method implies that as long as the decay products do not rescatter after decay, that is before leaving the medium, the yield of resonances is determined by the observed yield of the decay products. On the other hand, we can assume that each elastic scattering deflects the momentum vector, so that the invariant mass method fails to observe the resonance.

Given this consideration, we can evaluate the relative yield of Σ^*/Λ assuming that the system expands very slowly (and we ignore spin and isospin for simplicity). We are given the inelastic reaction rate, $R(T)_{\text{in}}$, at temperature T derived from the cross-section governing the inelastic reaction Eq. (1). Similarly, we have total elastic rate, $R(T)_{\text{el}}$, originating in any elastic scattering (that is of π or Λ) in the medium. The elastic reaction rate is proportional to the probability of not observing a Σ^* , while the inelastic reaction rate, with the same proportionality constant, is describing the probability of seeing a Σ^* . This means that the *invariant mass method observed* relative Σ^* yield is the ratio of inelastic scattering rate to any reaction rate in the medium, that is,

$$\frac{\Sigma^*}{\Lambda} = \frac{R(T_r)_{\text{in}}}{R(T_r)_{\text{in}} + R(T_r)_{\text{el}}}, \quad (2)$$

considering that the nucleon yield already comprises all $\Sigma^* \rightarrow \Lambda$ decays. Here, we denote by T_r the resonance free-streaming temperature which is typically lower than the stable particle freeze-out condition T_f . Note that if the elastic reactions are negligible, in this greatly simplified model all Λ are descendants of Σ^* and the ratio is unity.

We see, in this example, that the observed relative resonance yield is not governed by thermal/statistical properties of the medium, but depends decisively on the strength of the inelastic reaction rate derived from known

resonance cross-section of reactions, such as Eq. (1). In this work, we will be addressing realistic physical system and will focus our interest on the understanding of the role of the inelastic reaction rate of important resonances. We will be following their abundance evolution as function of time t , that is for a given function, $T(t)$, of temperature. We thus will present our results as a function of T and focus interest on the range T_r to T_f , where T_f is the hadron formation temperature in QGP breakup.

3. Time evolution equations

In Fig. 1, we show the scheme of reactions which all have a noticeable effect on $\Lambda(1520)$ yield after the chemical freeze-out kinetic phase. The format of this presentation is inspired by nuclear reactions schemes. On the vertical axis, the energy scale is shown in MeV. There are three classes of particle states, which we denote from left to right as ‘ N ’ ($S = 0$ baryon), ‘ Σ ’ ($S = -1$, $I = 1$ hyperon) and ‘ Λ ’ ($S = -1$, $I = 0$ hyperon).

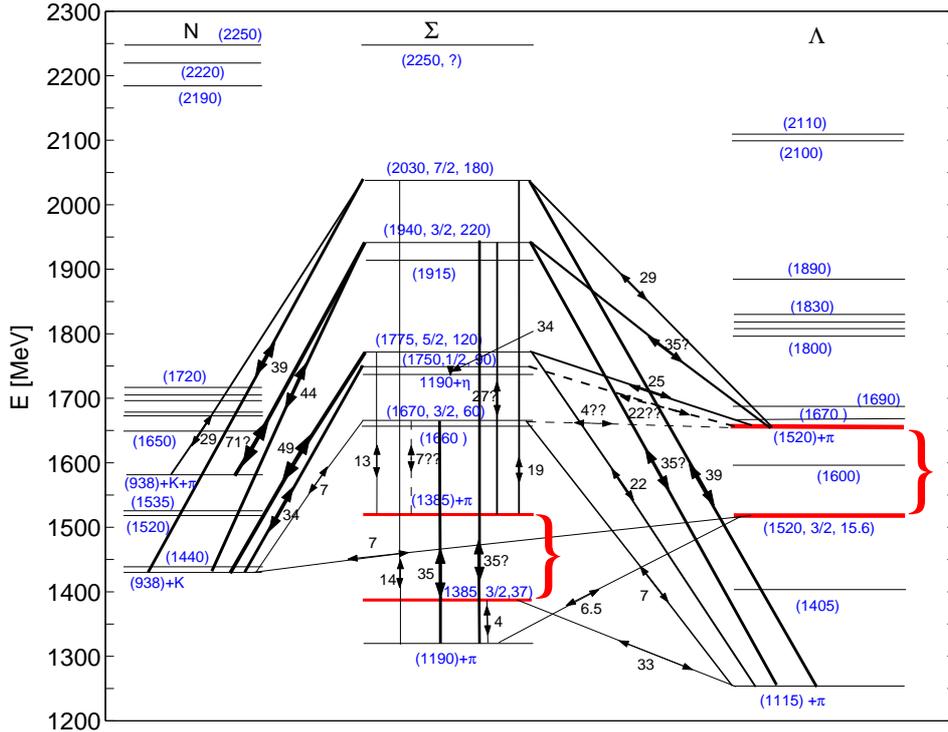


Fig. 1. Reactions scheme for $\Lambda(1520)$ and $\Sigma(1385)$ population evolution.

Near each particle bar, we state (on-line in blue) its mass, and/or angular momentum, and/or total width in MeV. The states $\Lambda(1520)$ and $\Sigma(1385)$ are shown along with the location in energy of $\Lambda(1520) + \pi$ and $\Sigma(1385) + \pi$, respectively, both entries are connected by the curly bracket, and are highlighted (on-line in red). The inclusion of the π -mass is helping to see the kinetic threshold energy of a reaction. The lines connecting the N , Σ and Λ columns are indicating the reactions we consider in the numerical computations. All reactions shown in figure 1 can go in both directions, as shown by the double arrows placed next to the numerical value of the partial decay width Γ_i , in MeV.

The evolution, in time, of the resonance yield is described by the process of resonance formation in scattering, $1 + 2 \rightarrow 3$, less natural decay $3 \rightarrow 1 + 2$:

$$\frac{1}{V} \frac{dN_3}{dt} = \sum_i \frac{dW_{1+2 \rightarrow 3}^i}{dV dt} - \sum_j \frac{dW_{3 \rightarrow 1+2}^j}{dV dt}, \quad (3)$$

where subscripts i, j denote different reactions channels when available. We further allow different sets of subscripts i, j in order to allow more complex dynamical cases in which not all production and/or decay channels are present.

Allowing for Fermi-blocking and Bose enhancement in the final state, where by designation particles 1 and 3 are fermions (heavy baryons) and particle 2 is a boson (often light pion), we have for the two rates:

$$\begin{aligned} \frac{dW_{3 \rightarrow 1+2}^j}{dV dt} &= \int \frac{g_3 d^3 p_3}{2E_3 (2\pi)^3} \int \frac{d^3 p_1}{2E_1 (2\pi)^3} \int \frac{d^3 p_2}{2E_2 (2\pi)^3} (2\pi)^4 \delta_p^4 (1 + 2 - 3) \\ &\quad \times f_3 (1 - f_1) (1 + f_2) \frac{1}{g_3} \sum_{\text{spin}} |\langle p_3 | M^j | p_1 p_2 \rangle|^2, \end{aligned} \quad (4)$$

and in analogy, we have for the 3 back-production rate

$$\begin{aligned} \frac{dW_{1+2 \rightarrow 3}^i}{dV dt} &= \int \frac{g_1 d^3 p_1}{2E_1 (2\pi)^3} \int \frac{g_2 d^3 p_2}{2E_2 (2\pi)^3} \int \frac{d^3 p_3}{2E_3 (2\pi)^3} (2\pi)^4 \delta_p^4 (1 + 2 - 3) \\ &\quad \times f_1 f_2 (1 - f_3) \frac{1}{g_1 g_2} \sum_{\text{spin}} |\langle p_1 p_2 | M^i | p_3 \rangle|^2, \end{aligned} \quad (5)$$

where $\delta_p^4(1 + 2 - 3) \equiv \delta^4(p_1 + p_2 - p_3)$ assures 4-momentum conservation and g_i , $i = 1, 2, 3$ is particle degeneracy. The Bose function for particle 2, and Fermi distribution for particles 1, 3 are:

$$f_2 = \frac{1}{\Upsilon_2^{-1} e^{u \cdot p_2/T} - 1}, \quad f_j = \frac{1}{\Upsilon_j^{-1} e^{u \cdot p_j/T} + 1}, \quad j = 1, 3. \quad (6)$$

Here, Υ_i is particles fugacity, and $u \cdot p_i = E_i$, for $u^\mu = (1, \vec{0})$ in the rest frame of the heat bath where $d^4p\delta_0(p_i^2 - m_i^2) \rightarrow d^3p_i/E_i$ for each particle. Hence, Eq. (4) and Eq. (5) are Lorentz invariant, and thus as presented these rates can be evaluated in any convenient frame of reference. Normally, this is the frame co-moving with the thermal volume element.

Since particles 2, 3 are here heavy baryon (resonances), we can work using the expansion of the relativistic distribution, the first term is the Boltzmann limit:

$$\frac{N_i}{V} = \Upsilon_i \frac{T^3}{2\pi^2} g_i x_i^2 K_2(x_i), \quad (7)$$

where $x_i = m_i/T$, $K_2(x)$ is Bessel function (not to be mixed up with particle 2). However, we use the complete Bose distribution to describe pions.

We introduce in medium lifespan of particle 3:

$$\frac{1}{\tau_3} \equiv \frac{\sum_i R_{123}^i}{V^{-1} dN_3/d\Upsilon_3}, \quad (8)$$

and, similarly, channel lifespan τ_3^i , omitting the sum \sum_i . The rate R_{123} is:

$$R_{123}^i = \iiint \frac{d^3p_1 d^3p_2 d^3p_3}{8E_1 E_2 E_3 (2\pi)^5} \frac{f_1 f_2 f_3 e^{u \cdot p_3/T}}{\Upsilon_1 \Upsilon_2 \Upsilon_3} \delta_p^4(1+2-3) \sum_{\text{spin}} |\langle 12 | M^i | 3 \rangle|^2. \quad (9)$$

R is independent of the fugacity, in the Boltzmann-limit.

The production and decay rates are connected to each other by the detailed balance relation [13, 14]:

$$\Upsilon_1^{-1} \Upsilon_2^{-1} \frac{dW_{1+2 \rightarrow 3}}{dV dt} = \Upsilon_3^{-1} \frac{dW_{3 \rightarrow 1+2}}{dV dt} = R_{123}. \quad (10)$$

Using detailed balance Eq. (10), we obtain for fugacity Υ_3 the evolution equation:

$$\frac{d\Upsilon_3}{d\tau} = \sum_i \Upsilon_1^i \Upsilon_2^i \frac{1}{\tau_3^i} + \Upsilon_3 \left(\frac{1}{\tau_T} + \frac{1}{\tau_S} - \sum_j \frac{1}{\tau_3^j} \right), \quad (11)$$

where we have also introduced characteristic time constants of temperature T and entropy S evolution

$$\frac{1}{\tau_T} = - \frac{d \ln(x_3^2 K_2(x_3))}{dT} \dot{T}, \quad \frac{1}{\tau_S} = - \frac{d \ln(VT^3)}{dT} \dot{T}. \quad (12)$$

The entropy term is negligible, $\tau_S \gg \tau_3$, τ_T since we implement near conservation of entropy during the expansion phase. We implement this in way which would be exact for massless particles taking $VT^3 = \text{const}$. Thus, there is some entropy growth in HG evolution we consider, but it is not signifi-

cant. In order to evaluate the magnitude of τ_T , we use the relation between Bessel functions of order 1 and 2 (not to be mixed up with particles 1, 2) $d(z^2 K_2(z))/dz = -z^2 K_1(z)$. We obtain

$$\frac{1}{\tau_T} = -\frac{K_1(x_3)}{K_2(x_3)} x_3 \frac{\dot{T}}{T}, \quad (13)$$

$\tau_T > 0$. For a static system with $\tau_T \rightarrow 0$, we see that Eq. (11) has transient stable population points whenever

$$\sum_i \mathcal{Y}_1^i \mathcal{Y}_2^i / \tau_3^i - \mathcal{Y}_3 \sum_j 1/\tau_3^j = 0. \quad (14)$$

Finally, we consider the evolution in time of \mathcal{Y}_1 and \mathcal{Y}_2 . In the equation for \mathcal{Y}_1 , we have terms which compensate what is lost/gained in \mathcal{Y}_3 see Eq. (11). Further, we have to allow that particle ‘1’ itself plays the role of particle ‘3’ (for example, this is clearly the case for $\Lambda(1520)$). That is accomplished introducing a chain of populations relations as follows:

$$(1' + 2' \leftrightarrow 1) + 2 \leftrightarrow 3. \quad (15)$$

In the present setting, $\mathcal{Y}_{2=\pi} = \text{const}$. By virtue of entropy conservation the same applies to the case $2' = \pi$. However, if either particle 2 or $2'$ is a kaon, we need to follow the equation for $\mathcal{Y}_{2,2'=K}$ which is analogous to equation for particle 1 or $1'$.

4. Model details and initial conditions

The evolution equations can be integrated once the time dynamics of the fireball and the initial conditions are fixed:

(1) We choose a model of expansion which fixes the behavior $T(t)$; here, we invoke a model of matter expansion where the longitudinal and transverse expansion is considered to be (nearly) independent:

$$\frac{\dot{T}}{T} = -\frac{1}{3} \left(\frac{2(v\tau/R_\perp) + 1}{\tau} \right), \quad (16)$$

where R_\perp is the transverse radius, v is the velocity of expansion in transverse dimension. All flow parameters (or temperature dependence on τ) are the same as in [15, 16].

(2) We determine the *initial* values of particle densities (fugacities) established at hadronization/chemical freeze-out. We determine these for RHIC head-on Au–Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV. We introduce the initial

hadron yields inspired by a picture of a rapid hadronization of QGP in which quarks combine into final state hadrons. For simplicity, we assume that the net baryon yield at central rapidity is negligible. Thus, the baryon-chemical and strangeness potentials vanish. The initial yields of mesons ($q\bar{q}$), ($s\bar{q}$) and baryons (qqq), (qqs) are controlled aside of the ambient temperature T , by the constituent light quark fugacity γ_q and the strange quark fugacity γ_s .

(3) Since high energy pions are moving faster than the bulk of the matter and leave the domain in which the slower baryons are found, we assume that it is impossible to excite reactions with high threshold energy. We thus exclude channels for resonance 3 production with threshold energy $\Delta E > 300$ MeV.

(4) We characterize the hadronization dynamics: we assume that the strangeness pair-yield in QGP is maintained in transition to HG. This fixes the initial value of γ_s . In fact, since we investigate relative chemical equilibrium reactions, our results do not depend significantly on the exact initial value γ_s and/or strangeness content. The entropy conservation at hadronization fixes γ_q . For hadronization temperature $T(t=0) \equiv T_0 = 180$ MeV, $\gamma_q = 1$. However, when $T_0 < 180$ MeV, $\gamma_q > 1$ in order to have entropy conserved at chemical freeze-out. At $T_0 = 140$ MeV, $\gamma_q = 1.6$ that is close to maximum possible value of γ_q , defined by Bose–Einstein condensation condition [17].

(5) The initial particle yields are fixed in terms of fugacities:

$$\mathcal{Y}_{(1=Y)}^0 = \gamma_q^2 \gamma_s, \quad \mathcal{Y}_{(2=\pi)}^0 = \gamma_q^2, \quad (17)$$

or

$$\mathcal{Y}_{(1=N)}^0 = \gamma_q^3, \quad \mathcal{Y}_{(2=K)}^0 = \gamma_q \gamma_s, \quad (18)$$

where $Y \equiv \Sigma$, Λ is a hyperon, the particle 1 is a baryon and particle 2 is a meson. The particle 3 is always a strange baryon:

$$\mathcal{Y}_{(3=Y)}^0 = \gamma_q^2 \gamma_s. \quad (19)$$

Note that for $\gamma_q > 1$, we have always initially

$$\left. \frac{\mathcal{Y}_1 \mathcal{Y}_2}{\mathcal{Y}_3} \right|_{t=0} = \gamma_q^2 \geq 1. \quad (20)$$

As a consequence, initially the pair of particles 1, 2 reacts into 3.

(6) We do not need to follow the evolution in time for the pion yield, which is fixed by conservation of entropy per unit rapidity, as incorporated in Eq. (16). Thus, it is (approximately) a constant of motion. This can be seen recalling that the entropy per pion is nearly 4 within the domain of temperatures considered. The conservation of entropy implies that pion

number is conserved, and with $VT^3 \simeq \text{const.}$ This further implies that, during the expansion,

$$\Upsilon_\pi = \gamma_q^2 = \text{const.},$$

which we keep at the initial value.

5. $\Sigma(1385)$ and $\Lambda(1520)$ yield results

In Fig. 2, we present the fractional yields $\Sigma(1385)/\Lambda_{\text{tot}}$ (left), and $\Lambda(1520)/\Lambda_{\text{tot}}$ (right) as a function of temperature of final kinetic freeze-out T . The results for the hadronization temperatures $T_0 = 140$ (blue lines on-line), $T_0 = 160$ (green lines on-line) and $T_0 = 180$ MeV (red lines on-line) are shown, lines recognized also by their initial value.

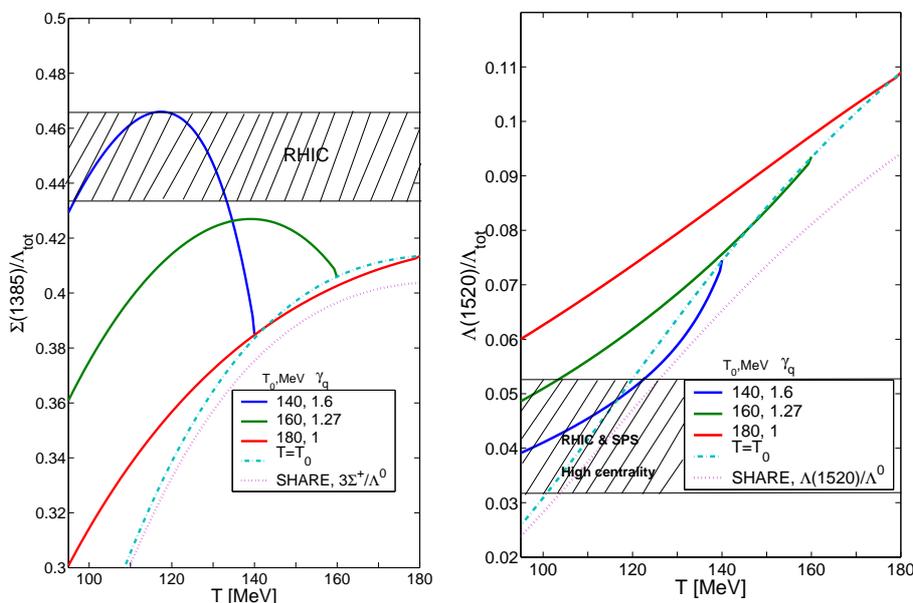


Fig. 2. The ratio $\Sigma(1385)/\Lambda_{\text{tot}}$, on left, and $\Lambda(1520)/\Lambda_{\text{tot}}$, on right, as a functions of temperature $T(t)$ for different initial hadronization temperatures $T_0 = 140, 160$ and 180 MeV (blue, green and red lines, respectively), recognized also by their initial value along the hadronization curve (dot-dashed).

In Fig. 2, the dash-dotted (green)line is the result when the kinetic freeze-out temperature T coincides with the hadronization temperature T_0 . There is no kinetic resonance evolution phase in this case, only resonances decay after hadronization. This result is similar to SHARE result (purple, dotted line). The small difference is mainly due to us taking into account the decays

$$\Sigma(1670, 1750) \rightarrow \Lambda(1520) + \pi, \quad (21)$$

which are expected/predicted in [18]. Similarly, for $\Sigma(1385)$ our results for $T_0 = T$ are different from SHARE results because we include the decay

$$\Sigma(1670) \rightarrow \Sigma(1385) + \pi, \quad (22)$$

which is part of recently updated particle data set [19].

For all initial hadronization temperatures, as the freeze-out temperature decreases, the suppression for $\Lambda(1520)_{\text{ob}}/\Lambda_{\text{tot}}$ ratio is larger than for $\Lambda(1520)/\Lambda(1520)_0$ (at the same temperature T of final kinetic freeze-out). The effect is due in part to the asymmetry to excite $\Sigma(1775)$ and heavier Σ^* directly due to its high mass. The resonance yield suppression effect is approximately of the same magnitude for all hadronization temperatures T_0 . However, the initial hadronization yield of $\Lambda(1520)$ is sensitive to temperature, and decreases rapidly with T . Therefore, only for $T_0 = 140$ MeV, a kinetic freeze-out temperatures ≈ 95 – 105 MeV the ratio $\Lambda_{\text{ob}}(1520)/\Lambda_{\text{tot}}$ reaches the experimental domain $\Lambda_{\text{ob}}(1520)/\Lambda_{\text{tot}} < 0.042 \pm 0.01$ [5, 6] shown, in Fig. 2, by dashed lines. For the same initial conditions, that is for $T_0 = 140$ MeV, we find [15, 16] the ratio $\Sigma(1385)/\Lambda_{\text{tot}} \approx 0.45$ at $T \approx 100$ MeV (and for the entire range 95–135 MeV, in good agreement with experimental data [6, 7]).

6. Conclusions

We find that the resonant hadron states, considering their very large decay and reaction rates, can often interact beyond the chemical and thermal freeze-out of stable particles. Thus, the observed yield of resonances is fixed by the physical conditions prevailing at a later breakup of the fireball matter rather than the production of non-resonantly interacting hadrons. This study quantifies the expectation that, in a dense hadron medium, narrow resonances are ‘quenched’ [11].

Despite a scenario dependent resonance formation or suppression, the stable particle yields used in the study of chemical freeze-out remain almost unchanged, since all resonances ultimately decay into the lowest ‘stable’ hadron. Therefore, after a description, *e.g.*, within a statistical hadronization model of the yields of stable hadrons, the understanding of resonance yields is a second, and separate task which helps to establish the consistency of our physical understanding of the hadron production process.

Our results show that the observable ratio $\Lambda(1520)_{\text{ob}}/\Lambda_{\text{tot}}$ can be suppressed by two effects. First $\Lambda(1520)$ yield is suppressed due to excitation of heavy Σ^* s in the resonance scattering process. Moreover, the final $\Lambda(1520)_{\text{ob}}$ yield is suppressed, because Σ^* s, which decay to $\Lambda(1520)$, are suppressed at the end of the kinetic phase evolution by their (asymmetric) decays to lower mass hadrons.

We resolve resonance puzzle in that we find that some resonances can be enhanced and some suppressed. Specifically $\Sigma(1385)$ is strongly enhanced, since the dense pion gas especially for $\gamma_q > 1$ pushes the Λ into $\Sigma(1385)$. On the other hand, the narrow $\Lambda(1520)$ is depleted by pions pushing it over to high mass resonances, which later can decay without repopulating $\Lambda(1520)$. This effect is particularly strong if we observe that there are fewer high energy particles than Boltzmann distribution predicts in a rapidly expanding and cooling fireball.

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