NEUTRINOS FROM MUON STORAGE RINGS*

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The enthusiasm for building a muon storage ring as an intense source of neutrinos is growing rapidly world-wide. This paper will describe how such a neutrino facility, called a neutrino factory, might be built and outline the physics that could be done with it. The status of related R&D projects in Europe and the US will also be discussed.

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1. Introduction

The probable observation of neutrino oscillations from recent measurements of the flux of neutrinos produced in the atmosphere by the SuperKamiokande experiment [1] is one of the most exciting particle physics results of the last decade. This result will be checked in the near future by long baseline neutrino experiments such as Minos [2], Opera [3] and ICANOE [4], which will use neutrinos produced by the decay of pions at an accelerator (see, for example, a description of the Minos NUMI beam [5]). In the case of Minos, the beam composition will be measured by two functionally similar detectors, one at the accelerator and the second 730 km away. A difference between the two will be a signal for neutrino oscillations. However, the sensitivity that can be achieved by these experiments is limited by uncertainties in the beam composition and energy spectrum and the beam intensities, particularly of $\nu_e$. A systematic investigation of the oscillations and a precise determination of the oscillation parameters requires a dedicated facility, a neutrino factory, which will produce an intense beam of neutrinos of precisely known composition, intensity and energy spectrum. Neutrinos from the decay of muons in a storage ring are ideal for this purpose.

This paper will describe how such a neutrino factory could be built and give more details of the physics it can do, concentrating especially on neutrino oscillations.

2. The neutrino factory

A conceptual design for a neutrino factory [6], illustrating the component parts, is shown in figure 1. These consist of a proton driver, a pion production target and capture system, a complex region for muon capture and cooling, muon accelerators and the muon storage ring. Each of these is discussed in more detail below.

![Diagram of neutrino factory](image)

**Proton Driver**

- **Target**
- **Phase Rotate #1** (42 m rf)
- **Mini Cooling** (3.5 m H₂)
- **Drift** (160 m)
- **Phase Rotate #2** (10 m rf)
- **Cooling** (80 m)
- **Linac** (2 GeV)
- **Recirc. Linac #1** (2-8 GeV)
- **Recirc. Linac #2** (8-50 GeV)
- **Storage Ring** (50GeV 900m circ)

Fig. 1. The main components of a neutrino factory

2.1. Proton driver

There are a number of criteria the proton driver must satisfy. The most important is the neutrino intensity: it was decided at NuFact’99 in Lyon that the baseline for this is \(10^{21}\) muons decays per year in the storage ring [7]. It is estimated that a proton beam power of 4 MW is required to achieve this.
As shown in figure 2, although this is a significant increase over existing machines, it is still comparable with other planned proton accelerators. It should be noted, however, that the neutrino factory lies in the region of this plot occupied by neutron and muon sources, rather than HEP machines, and hence the experience of the designers of such machines is of great value to the neutrino factory project.

![Diagram of beam currents and energies](image)

**Fig. 2.** The beam currents and energies are plotted for various existing and planned accelerators world-wide. Shown dashed are lines of constant power, the lower corresponding to 0.1 MW and the upper to 1 MW. A 4 MW neutrino factory is shown for a range of beam energies.

Although a beam power of 4 MW is not a particularly difficult problem, the resulting power density in the pion production target is! The highest power densities of existing proton machines are achieved in the absorption targets of neutron sources. Currently, these are in the range 300-400 Wcm\(^{-3}\).
(peak) and lead to the failure of the target after 9 months to a year. Neutron
targets tend to be large, while that for a neutrino factory will have to be
much smaller to facilitate pion capture. This results in a power density
(peak) for the latter that could exceed the former by a factor of 250!

The second criterion the proton source has to satisfy is a bunch length
of 1 ns. This is required by the pion and muon capture and the muon accel-
eration systems. This is probably the most difficult criterion to achieve as
it necessitates squeezing very intense bunches into a short space.

The third requirement is frequency: the rf used to capture the pions
has to be normally conducting (see the next section). As a result, it takes
unrealistic power levels to run it at high frequency, leading to a maximum
of about 100 Hz.

![Neutrino factory at RAL](image)

Fig. 3. A schematic layout for a neutrino factory at RAL. For the proton driver
this uses a 5 GeV rapid cycling synchrotron option, running at 50 Hz.

Various designs to satisfy these requirements exist. Most (Brookhaven,
Fermilab, KEK and RAL) use rapid cycling synchrotrons (RCS) with varying
beam energies and powers (see figure 3 for the RAL design). CERN, on the
other hand, has a design using a superconducting linac at 2 GeV [8]. To
achieve a 4 MW beam power, this runs at a frequency much higher than
100 Hz and a circular accumulator is required to reduce the frequency and
a bunch compressor to get close to the required 1 ns bunch length.
2.2. Target and pion capture

A possible layout for the target and pion capture region is shown in figure 4. The target is surrounded by a 20 T solenoid consisting of a normally-conducting inner core and a superconducting outer core. This acts to focus the pions produced through a magnetic channel into rf cavities which capture them. The superconducting magnets must be shielded because most of the beam power ends up as pions and the whole region is bathed in the resulting radiation. One of the R&D activities being performed in Europe is to test that rf cavities will work under these conditions.

As mentioned in the previous section, the major problem for the target is the enormous power density it will be subject to. This will result in the target becoming thermally and radioactively hot. In addition, as the beam will arrive in short bunches at low frequency, the target will be subject to a large mechanical shock. A number of ideas are being considered for targets that will survive this, in particular liquid metal jets and rotating solid bands. Much R&D is required to show these will work. In addition, R&D is being done in Europe and the US to measure the rate of pion production and to test the pion capture system.
2.3. Muon phase rotation and cooling

Following the decay of the captured pions, the resulting muon bunches are short, \( \sim 1-2 \text{ m} \), but have a large spread in momentum, \( \sim 100-300 \text{ MeV} / c \). This momentum spread is a problem for the muon accelerators and must be reduced. The process used to do this is called phase rotation: rf cavities are used to slow down the faster muons and to speed up the slower muons. As this happens over some distance, it results in a long bunch, but with 75\% of the muons in a high density core in energy centred around 100 MeV/\( c^2 \) and with a width of 3\%. In the plane of bunch length versus energy, the bunch has been “rotated” and hence the name.

In addition to phase rotation, the muon beam needs some transverse cooling, i.e. a reduction in the transverse normalised emittance (see [6]). Due to the muon lifetime, any cooling used needs to be fast and this eliminates most of the methods employed to-date. The currently favoured technique is ionisation cooling [9].

In ionisation cooling, the muons are passed through an absorber made from a low Z material, such as liquid hydrogen. In this, they lose both longitudinal and transverse momentum by ionisation energy loss. If the lost longitudinal momentum is restored after the absorber using rf, there will be a net reduction in transverse momentum and hence transverse cooling. A low Z absorber is required to minimise a heating effect coming from multiple scattering. To achieve the required level of cooling, a number of cooling stages are required, as shown in figure 5.

![Fig. 5. Schematic of an ionisation cooling facility](image)

Ionisation cooling is a completely unproven technique and much R&D is required to test it. In particular, in the US the MUCOOL collaboration has proposed a large R&D program to test all the critical components [9]. In Europe, smaller projects are being discussed to check the principles, in particular a muon scattering experiment and a mini-cooling experiment.
2.4. Muon acceleration and storage

The muon acceleration system consists of a linac and two re-circulating
linacs, in which the muons are passed through the same accelerating sections
a number of times, (see figure 1). The latter give the same momentum
increase per pass and allow the final momentum to be varied in steps by
changing the number of passes. The currently proposed final momentum is
50 GeV/c.

Once the muons have been accelerated to the final momentum, they are
injected into a storage ring. As intense neutrino beams are produced in the
directions of the straight sections of this machine, the size of these relative
to the rest of the machine must be maximised and they must be pointed
in the directions of the target experiments. To hit a number of different
experiments, e.g. 2 or 3, the storage ring will need to be some sort of bowtie
or triangular shape.

In Europe, it is felt that the target experiments should be underground
to reduce the background from cosmic muons. Table I shows the distances
between the 5 candidate accelerator laboratories, Brookhaven, CERN, Fermi-
ilab, KEK and RAL, and various underground laboratories. Recent work
suggests two baselines of about 3500 and 7000 km would be the best [10].
The Canary Islands and SLAC are being considered as possible surface ex-
periment locations.

<table>
<thead>
<tr>
<th>Location</th>
<th>BNL</th>
<th>CERN</th>
<th>FNAL</th>
<th>KEK</th>
<th>RAL</th>
</tr>
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<td>Baksan lab, Russia</td>
<td>8050</td>
<td>2950</td>
<td>8550</td>
<td>7300</td>
<td>3400</td>
</tr>
<tr>
<td>Bouby mine, UK</td>
<td>5200</td>
<td>1050</td>
<td>6000</td>
<td>8500</td>
<td>3500</td>
</tr>
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<td>Gran Sasso lab, Italy</td>
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<td>750</td>
<td>7350</td>
<td>8800</td>
<td>1500</td>
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<td>Homestake mine, US</td>
<td>2550</td>
<td>7350</td>
<td>1300</td>
<td>8250</td>
<td>6700</td>
</tr>
<tr>
<td>Kamioka lab, Japan</td>
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<td>9100</td>
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<td>8600</td>
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<td>6900</td>
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<tr>
<td>Sudbury mine, Canada</td>
<td>900</td>
<td>6050</td>
<td>800</td>
<td>8900</td>
<td>5350</td>
</tr>
</tbody>
</table>

Note that for the longer baselines, the neutrino factory will have to
be built at a steep angle pointing into the ground, up to almost 50° from
horizontal!
3. Physics possibilities

There are extensive physics possibilities at a neutrino factory, falling into three main categories: (1) neutrino oscillations, (2) short baseline neutrino physics and (3) "slow" muon physics. It should be noted that the primary justification for building such a facility would be category (1) and most of the emphasis will be on this. However, if a neutrino factory is built, the other categories will also be done and the resulting physics will come almost free!

3.1. Neutrino oscillations

A neutrino factory would be an ideal second generation long baseline neutrino oscillation facility. There are many advantages in producing neutrinos from muon decays, rather than the traditional accelerator source, pion decays. In particular:

1. The muon decay is

\[ \mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e. \]

Thus, for a stored \( \mu^- \) (\( \mu^+ \)) beam, a pure neutrino beam of equal numbers of \( \nu_\mu \) (\( \bar{\nu}_\mu \)) and \( \bar{\nu}_e \) (\( \nu_e \)) will be produced. This is the only source of high energy \( \nu_e \), which are vital for neutrino oscillation investigations, as described below.

2. The muon momentum is well defined in a storage ring, so that the \( \nu_\mu \) and \( \bar{\nu}_e \) energy spectra are perfectly calculable if the beam polarisation is known.

3. As mentioned above, the muon energy is tunable to the experimental requirements.

4. The neutrino fluxes can be precisely calculated from the muon current, which is straightforward to measure.

5. Intense fluxes are possible, for example giving 3 or 4 orders of magnitude more \( \nu_\mu \) and 6 or 7 orders of magnitude more \( \nu_e \) per year than Minos.

6. In principle, all the oscillation channels listed in Table II are available. Note that the appearance of a wrong sign lepton is proof of oscillations, but requires the detector to be able to distinguish the charge of the leptons. This may be difficult for electrons and taus.

The neutrino oscillation terminology used in this paper is the same as that in [11].
Oscillation channels available at a neutrino factory

\[ \mu^- \rightarrow e^- \quad \bar{\nu}_\mu \quad \bar{\nu}_e \]
\[ \bar{\nu}_e \rightarrow \bar{\nu}_e \rightarrow e^+ \quad \text{disappearance} \]
\[ \bar{\nu}_e \rightarrow \bar{\nu}_\mu \rightarrow \mu^+ \quad \text{appearance} \]
\[ \bar{\nu}_e \rightarrow \bar{\nu}_\tau \rightarrow \tau^+ \quad \text{appearance} \]
\[ \nu_\mu \rightarrow \nu_\mu \rightarrow \mu^- \quad \text{disappearance} \]
\[ \nu_\mu \rightarrow \nu_\mu \rightarrow e^- \quad \text{appearance} \]
\[ \nu_\mu \rightarrow \nu_\tau \rightarrow \tau^- \quad \text{appearance} \]

3.1.1. Near-future experiments

As it is very unlikely a neutrino factory will be taking data in less than 10 years, the status of experimental neutrino oscillation measurements will have changed considerably before this happens. In particular, the LSND [12] result will have been checked by (Mini)Boone [13], the atmospheric neutrino oscillations will have been verified by K2K [14], Minos and the CERN to Gran Sasso project and the correct solar neutrino solution identified by SNO [15], Borexino [16] and Kamland [17]. In addition, the atmospheric oscillation parameters, \( \Delta m^2_{23} \) and \( \sin^2 2\theta_{23} \), will have been measured to about 10%.

As a result, the so-called “golden” measurements left for the neutrino factory are:

- \( \theta_{13} \), the link between the solar and atmospheric measurements,
- the sign of \( \Delta m^2_{23} \),
- \( \delta \), the CP-violation angle.

However, the neutrino factory will also be able to improve the measurement of the atmospheric parameters considerably and may have some impact in the solar measurements. These will all be discussed in the following subsections.

3.1.2. \( \Delta m^2_{23} \) and \( \sin^2 \theta_{23} \)

The measurement of these parameters has been extensively studied. In particular, Barger et al. [18] have looked at \( \nu_\mu \rightarrow \nu_\tau \) oscillations using 2-flavour mixing with a 50 GeV muon beam in a storage ring and a baseline of 732 km, i.e. CERN to Gran Sasso or Fermilab to Soudan. They assume that \( 2 \times 10^{20} \) muons will decay in the straight-section pointing at the experiment. Figure 6 shows contours of the number of oscillated events per kT of detector per year. It should be noted that the detector is likely to be > 10 kT and the
Lyon workshop design intensity would give twice as many events per year, so the actual event numbers should be at least 10 times bigger than this. In any case, the SuperKamiokande preferred region is well covered and Barger et al. predict that the atmospheric oscillation parameters will be measured to about 1% within 1 to 2 years.

![Graph showing the number of $\nu_\mu \rightarrow \nu_\tau$ oscillations per kT per year at 732 km from a 50 GeV stored muon beam as a function of $\Delta m^2_{32}$ and $\sin^2 2\theta_{23}$. The allowed regions from Kamiokande and SuperKamiokande are shown, along with expected 90% CL curves from K2K and Minos.](image)

3.1.3. $\theta_{13}$

The most promising channel to measure this in is $\nu_e \rightarrow \nu_\mu$. This is simply because charge identification is easier for muons than for taus or electrons. A study of this channel has been made by Gavela et al. in [11]. They used a 3-flavour analysis with a 10 kT detector at a 732 km baseline and $2 \times 10^{20}$ muon decays per year in straight section pointing at the detector. Their results are shown in figure 7, which indicates that using the muon appearance channel above, a sensitivity of about $10^{-4}$ at 90% CL can be achieved for $\theta_{13}$ in 1 year.

If $\theta_{13}$ is small, in particular zero, then the $\nu_e \rightarrow \nu_\mu$ channel might be sensitive to the solar oscillation parameters, $\theta_{12}$ and $\Delta m^2_{12}$. This has also been studied by Barger et al. [18]. Figure 8 shows the single event contours
Fig. 7. Sensitivity reach in the plane $[\sin^2 \theta_{13}, \Delta m^2_{23}]$ at 90% confidence for a 10 kT detector at 732 km using the muon appearance ($N[\mu^+]$) and disappearance ($N_\mu$) channels. The continuous (dashed) lines correspond to $\theta_{23} = 45^\circ$ (30') and the rectangular domain is approximately the region allowed by the SuperKamiokande data up to 1999.

Fig. 8. The single event per 10 kT per year contours for $\nu_e \rightarrow \nu_\mu$ at a number of muon beam energies. The LSND and the MSW large and small mixing angle solar allowed regions are also shown.
per 10 kT per year for $\nu_e \to \nu_\mu$ for a number of muon beam energies. Note that these contours cut into the large mixing angle MSW solar solution. Barger et al. conclude that at 50 GeV, with a useful muon intensity of $7 \times 10^{20}$ decays per year, a 20 kT detector, a muon identification efficiency of 50\%, negligible background and a few years running, the entire LMA region could be covered.

3.1.4. The sign of $\Delta m^2_{23}$

Of the three neutrino species, only $\nu_e$ and $\bar{\nu}_e$ have charged current elastic scattering amplitudes on electrons. This induces “effective” masses $\mu = \pm 2\sqrt{G_F n_e E_\nu}$ for $\nu_e$ and $\bar{\nu}_e$, respectively, in matter, where $G_F$ is the Fermi constant, $n_e$ the electron number density and $E_\nu$ the neutrino energy.

This matter term modifies the transition probability for channels involving electrons:

$$P(\nu_e \to \nu_\mu) = \sin^2 \theta^\pi_m \sin^2 \left(1.27 \Delta m^2_{23} \frac{\lambda^\pi}{E_\nu} \right), \quad (1a)$$

where

$$\sin^2 \theta^\pi_m = \frac{\sin^2 \theta}{\sin^2 \theta + (x \mp \cos \theta)^2}, \quad (1b)$$

$$\lambda^\pi = L \times \frac{\sin^2 2\theta}{\sin^2 2\theta + (x \mp \cos \theta)^2}, \quad (1c)$$

$$x = \frac{2\sqrt{2} G_F n_e E_\nu}{\Delta m^2}, \quad (1d)$$

A few of notes:

- For a positive $\Delta m^2$, there is a resonance in the transition probability for $\nu_e$ when $x = \cos \theta$ as $\sin^2 \theta^\pi_m$ in (1b) will be 1. Using a constant density of 3.6 g cm$^{-3}$ for the Earth, this resonance will occur at a neutrino energy of $E_\nu^{\text{res}} = 3.7 \times 10^3 \Delta m^2$. For $\Delta m^2$ in the range $10^{-3}$ to $10^{-2}$, this corresponds to a neutrino energy between 3.7 and 37 GeV. Hence, this so-called MSW resonance would be observable in the Earth with a neutrino factory.

- These matter effects are only important for long distances, > 3000 km. This can be seen as follows: for small $\lambda^\pi$, i.e. small $L$, the second sin in (1a) can be expanded. Cancelling out then leaves the standard 2-flavour oscillation probability in the absence of matter effects.

- If $\Delta m^2_{23} < 0$, the resonance occurs for $\bar{\nu}_e$ rather than $\nu_e$ and hence matter effects can be used to determine the sign of $\Delta m^2_{23}$. 

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A study of this has been made by Campanelli et al. [19]. They have performed a 3-flavour analysis using a baseline of 6500 km, the distance from Brookhaven to Gran Sasso. They classify events as (a) electron like (with no charge discrimination), (b) right-sign muon, i.e. the same as the muons in the storage ring, (c) wrong-sign muon, i.e. coming from a $\nu_e$ oscillation, and (d) no leptons. They then determine what they would see in a detector using $10^{21}$ decays of both $\mu^-$, which gives a $\bar{\nu}_e$ beam, and $\mu^+$, which gives a $\nu_e$ beam, with and without oscillations. The results are shown in figure 9 for $\mu^-$ and in figure 10 for $\mu^+$. The important plots are (c), the wrong-sign muons, where there are many more entries for the latter, as one would expect from the MSW resonance. The event numbers are clearly large enough to determine the sign of $\Delta m^2_{23}$ from this.

![Energy distributions](image)

Fig. 9. The energy distributions of the 4 categories of events defined by Campanelli et al, seen in a detector 6500 km away from a stored $\mu^-$ beam. The full line shows what happens if oscillations occur and the dashed, if they do not.
3.1.5. CP-violation

For neutrino oscillations, CP- and T-violation would lead to the following inequalities in the transition probabilities for two neutrino flavours $\alpha$ and $\beta$:

\begin{align}
\text{CP-violation:} & \quad P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta), \\
\text{T-violation:} & \quad P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\nu_\beta \rightarrow \nu_\alpha). \quad (2a) \quad (2b)
\end{align}

Experimentally, as charge discrimination is required, CP-violation is the easier of these to do if $\alpha = e$ and $\beta = \mu$.

To make the measurement the following CP-asymmetry is defined:

\begin{equation}
A_{\alpha\beta}^{\text{CP}} = \frac{P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)}{P(\nu_\alpha \rightarrow \nu_\beta) + P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)}. \quad (3a)
\end{equation}

As the numerator has the same magnitude for all neutrino flavours, the asymmetry $A_{\mu\mu}^{\text{CP}}$ is the largest from current experimental constraints on the transition probabilities. However, for this asymmetry there is a background
from the matter effect as $\nu_e$ and $\bar{\nu}_e$ interact differently with matter, as we have just seen. The true CP-odd term, to leading order in $\Delta m_{12}^2$ is

$$P_{\text{CP}}(\nu_e \to \nu_\mu) = -8c_{12}^2c_{23}^2s_{12}s_{13}s_{23}\sin \delta \left( \frac{\Delta m_{12}^2 L}{4E_\nu} \right) \sin^2 \left( \frac{\Delta m_{23}^2 L}{4E_\nu} \right).$$

(4a)

Note that this depends on all the oscillation parameters and $\delta$, so if any of these are zero, there is no CP-violation. In addition, the magnitude depends directly on $\Delta m_{12}^2$ and it will be difficult to measure if this is small. Finally, the baseline at which the violation is largest depends on $\Delta m_{23}^2$.

CP-violation has been studied by Donini et al. [20]. They have determined the quantity

$$\frac{|A_{\text{CP}}^{\nu\mu}(\pi/2) - A_{\text{CP}}^{\nu\mu}(0)|}{\Delta A_{\text{CP}}}$$

as a function of baseline and neutrino energy, $E_\nu$. This is essentially the number of sigma for the signal, though the error used, $\Delta A_{\text{CP}}$, is only statistical and they have assumed that the matter effect, $A_{\text{CP}}^{\nu\mu}(0)$, is known.

Fig. 11. The CP-asymmetry signal over statistical uncertainty as a function of baseline. The solid (dashed) lines correspond to oscillations in matter (vacuum). In the left-hand plot, the lower and upper lines have $E_\nu = 10$ and $20$ GeV, with $2 \times 10^{20}$ useful muon decays per year. In the right-hand plot, the energies are 20 and 50 GeV and the intensity is $2 \times 10^{21}$ decays per year.

The results of these calculations are shown in figure 11. The parameter values used are:
\[ \Delta m^2_{12} = 10^{-4} \text{ and } \Delta m^2_{23} = 2.8 \times 10^{-3}. \]

- \( \theta_{23} = 45^\circ, \theta_{13} = 13^\circ \text{ and } \theta_{12} = 22.5^\circ. \]

- For the left-hand plot, \( E_\nu = 10 \) and 20 GeV, and there are \( 2 \times 10^{20} \)
  useful muon decays per year.

- For the right-hand plot, \( E_\nu = 20 \) and 50 GeV, and there are \( 2 \times 10^{21} \)
  useful muon decays per year.

Note that at 50 GeV, the signal peaks at a distance of about 5000 km. From Table I, the closest baselines are Brookhaven to Boulby, RAL to Sudbury and RAL to Soudan. However, if the assumption that the matter effect is known is removed, the optimum baseline for measuring CP-violation moves down to 3500 km [10], as matter effects become larger at longer baselines.

### 3.2. Short baseline neutrino physics

As already mentioned, a neutrino factory would not be built to study short baseline neutrino physics. However, having built one to study neutrino oscillations, it would certainly be used for this purpose and the physics done can be considered as a bonus.

The idea is to place a detector as close to the end of a straight section of the neutrino factory storage ring as possible, allowing for radiation, \textit{etc.}

In the study by McFarland [21] (see figure 12) this distance is 29 m. At this distance, with a 50 GeV beam, about 10% of the neutrinos would pass through a detector 10 cm radius. With \( 10^{21} \) muon decays per year, this would give about \( 2.5 \times 10^7 \) charged current neutrino interactions per kg of detector per year! This makes it possible to do high precision physics with neutrinos in a small, fine-grained detector.

There is an extensive amount of physics that can be done with such a neutrino source, in the areas of

- electroweak physics,
- charm physics,
- hadronic structure,
- rare neutrino physics,

- measurements related to long baseline neutrino oscillation studies.

In this report, only a few selected examples of possible measurements will be given.
Fig. 12. Schematic layout of a possible short baseline neutrino physics detector

3.2.1. $\sin^2 \theta_W$

This is measured from the neutral current to charged current event rate in inelastic neutrino–nucleon interactions:

$$R^- = \frac{\sigma(\nu_\mu N \rightarrow \nu_\mu X) - \sigma(\bar{\nu}_\mu N \rightarrow \bar{\nu}_\mu X)}{\sigma(\nu_\mu N \rightarrow \mu^- X) - \sigma(\bar{\nu}_\mu N \rightarrow \mu^+ X)} \approx \frac{1}{2} - \sin^2 \theta_W. \quad (6a)$$

For example, NuTeV has measured [22]

$$\sin^2 \theta_W^{(\text{on-shell})} = 0.2253 \pm 0.0019(\text{stat}) \pm 0.0010(\text{syst}).$$

Using the relation

$$\sin^2 \theta_W \equiv 1 - \frac{M_W^2}{M_Z^2} \quad (7a)$$

this gives a value for the $W$ mass of

$$M_W = 80.26 \pm 0.10 \pm 0.05 \text{ GeV}.$$  

The error on $\sin^2 \theta_W$ from a neutrino factory is expected to be 5 to 20 times better than from NuTeV, leading to an error on the $W$ mass which would be comparable or better than that achieved at LEP or the Tevatron.

3.2.2. Charm production

The charm production at a neutrino factory will be vast: about $10^8$ events per year in a 40 kg detector. Approximately 10% of these will be easy to tag through di-lepton production:

$$\nu_l N \rightarrow l^- \bar{c} \bar{X}; \quad c \rightarrow l^+ \nu_l.$$
With this, it will be possible to measure the CKM matrix element \( V_{cd} \) from the difference in the production rate by \( \nu \) and \( \bar{\nu} \) with an error of about 2\%, comparable to that of \( V_{us} \). This would provide a test of the unitarity of the CKM matrix.

In addition, it will be possible to measure \( D^0 - \bar{D}^0 \) mixing from the ratio of like-sign to unlike-sign di-lepton events. This would be a clean signature for new physics if seen above the 10^{-6} level.

3.2.3. Nucleon structure

The differential cross-section for the process

\[ \nu_{\mu}N \rightarrow \mu X \]

can be written

\[
\frac{d^2\sigma}{dxdy} = \frac{G_F^2 M_N E_{\nu}}{\pi} \left[ xy^2 F_1 + \left( 1 - y - \frac{M_N xy}{2E_{\nu}} \right) F_2 \pm y \left( 1 - \frac{y}{2} \right) x F_3 \right],
\]

where \( F_1, F_2 \) and \( F_3 \) describe the structure of nucleon, \( x \) is the Bjorken scaling variable, \( y \) the elasticity and \(-Q^2\) the square of the 4-momentum transfer. A large number of related measurements can be made, in particular

1. \( \alpha_s \), from the \( Q^2 \) dependence of the parity violating structure function \( x F_3 \) or, better, from the Gross Llewellyn Smith sum rule \([23]\):

\[
\int_0^1 x F_3(x, Q^2) \frac{dx}{x} = 3 \left[ 1 - \frac{\alpha_s}{\pi} - a(n_f) \left( \frac{\alpha_s}{\pi} \right)^2 - b(n_f) \left( \frac{\alpha_s}{\pi} \right)^3 \right]
\]

CCFR have measured the following for this \([24]\):

\[
\int x F_3 \frac{dx}{x} = 2.64 \pm 0.06
\]

leading to value of \( \alpha_s \) of

\[
\alpha_s(M_{\rho}^2) = 0.114^{+0.000}_{-0.012}.
\]

The neutrino factory is expected to measure this integral with an error of about 0.1\%, leading the world best value for \( \alpha_s \).

2. Scattering from polarised targets. No deep inelastic neutrino scattering data from polarised targets exists due to the problems of building a large enough target. For a neutrino factory, large targets are not required and this becomes a possibility. It will allow
the measurement of the unknown spin structure functions $g_1$ and $g_3$ of the proton and neutron and hence will test the “spin crisis” seen with muons;

• the polarisation of all flavours of valence and sea quarks to be measured separately, to a precision of $\leq 1.5\%$.

3.3. Slow muon physics

A neutrino factory will produce vast numbers of “slow” ($<300$ MeV/c) muons, of the order of $10^{15}$ per second. If only a small fraction (e.g. 0.1%) can be captured (e.g. from back-scattered pion decays) the flux will still exceed current sources by 4 orders of magnitude!

The physics possibilities with these muons are extensive. However, as they are beyond the scope of this conference, a few are merely listed here:

• Beyond the standard model: lepton flavour violation; the muon anomalous magnetic moment, $(g_\mu - 2)$; the muon electric dipole moment; search for parity violation in muonic atoms; etc.

• Standard model: the muon lifetime, $\tau_\mu$; the Michel parameters; many tests of QED using muonic atoms; etc.

• Non-particle physics: muon catalysed fusion; muon capture; muSR.

4. Conclusions

Enthusiasm for a neutrino factory is growing rapidly world-wide. In particular, studies of both the machine and physics are taking place in Europe, Japan and the US and it is looking more likely that such a machine will be built on a timescale of 10-15 years. However, most of the parameters of the machine, including the location, are still under investigation. The most likely sites are Brookhaven, CERN, Fermilab, KEK–JAERI and RAL.

The physics possibilities are extensive and fall into three main categories:

1. Neutrino oscillations — depending on the values, measurement of all the oscillation parameters could be possible.

2. Precision neutrino physics — with studies in the areas of electroweak physics, heavy flavour physics, QCD, hadronic structure and new physics.

3. Slow muon physics — with measurements in the areas of electroweak, searches and other branches of science.
REFERENCES

[16] See http://almime.mi.infn.it