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NLO EVOLUTION OF COLOR DIPOLE*

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The small-x deep inelastic scattering in the saturation region is governed by the non-linear evolution of Wilson-line operators. In the leading logarithmic approximation it is given by the BK equation for the evolution of color dipoles. In the next-to-leading order the BK equation gets contributions from quark and gluon loops as well as from the tree gluon diagrams with quadratic and cubic nonlinearities.

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A general feature of high-energy scattering is that a fast particle moves along its straight-line classical trajectory and the only quantum effect is the eikonal phase factor acquired along this propagation path. In QCD, for the fast quark or gluon scattering off some target, this eikonal phase factor is a Wilson line — the infinite gauge link ordered along the straight line collinear to particle's velocity n^μ:

$$U^\eta(x_\perp) = \mathrm{P}\exp\left\{ ig \int\limits_{-\infty}^{\infty} du\ n_\mu A^\mu(un + x_\perp) \right\} . \tag{1}$$

Here A_μ is the gluon field of the target, $x_\perp$ is the transverse position of the particle which remains unchanged throughout the collision, and the index η labels the rapidity of the particle (for a review see Ref. [1]).

Let us consider the deep inelastic scattering from a hadron at small $x_\mathrm{B} = Q^2/(2p\cdot q)$. The virtual photon decomposes into a pair of fast quarks moving along straight lines separated by some transverse distance. The

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propagation of this quark–antiquark pair reduces to the "propagator of the color dipole" $U(x_\perp)U^\dagger(y_\perp)$ — two Wilson lines ordered along the direction collinear to quarks' velocity. The structure function of a hadron is proportional to a matrix element of this color dipole operator

$$\hat{\mathcal{U}}^\eta(x_\perp,y_\perp) = 1 - \frac{1}{N_c}\mathrm{Tr}\left\{\hat{U}^\eta(x_\perp)\hat{U}^{\dagger\eta}(y_\perp)\right\} \tag{2}$$

switched between the target's states ($N_c = 3$ for QCD).

The small-x behavior of the structure functions is governed by the rapidity evolution of color dipoles. At relatively high energies and for sufficiently small dipoles we can use the leading logarithmic approximation (LLA) where $\alpha_\mathrm{s} \ll 1$, $\alpha_\mathrm{s}\ln x_\mathrm{B} \sim 1$ and get the non-linear BK evolution equation for the color dipoles [2, 3]:

$$\begin{aligned}\frac{d}{d\eta}\,\hat{\mathcal{U}}(x,y) \;&=\; \frac{\alpha_\mathrm{s}N_c}{2\pi^2}\int d^2z\,\frac{(x-y)^2}{(x-z)^2(z-y)^2}\\ &\quad\left[\hat{\mathcal{U}}(x,z)+\hat{\mathcal{U}}(y,z)-\hat{\mathcal{U}}(x,y)-\hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\right]\,. \end{aligned}\tag{3}$$

The first three terms correspond to the linear BFKL evolution [4] and describe the parton emission while the last term is responsible for the parton annihilation. For sufficiently high x_B the parton emission balances the parton annihilation so the partons reach the state of saturation [5] with the characteristic transverse momentum Q_s growing with energy $1/x_\mathrm{B}$.

As usual, to get the region of application of the leading-order evolution equation one needs to find the next-to-leading order (NLO) corrections. In the case of the small-x evolution equation (3) there is another reason why NLO corrections are important. Unlike the DGLAP evolution, the argument of the coupling constant in Eq. (3) is left undetermined in the LLA, and usually it is set by hand to be Q_s. The precise form of the argument of α_s should come from the solution of the BK equation with the running coupling constant, and the starting point of the analysis of the argument of α_s in Eq. (3) is the calculation of the NLO evolution.

Let us present our result for the NLO evolution of the color dipole [6] (the quark part was calculated in [7, 8]). Hereafter we use notations $X \equiv x - z$, $X' \equiv x - z'$, $Y \equiv y - z$, and $Y' \equiv y - z'$.

$$\begin{aligned}\frac{d}{d\eta}\mathrm{Tr}\left\{\hat{U}_x\hat{U}^\dagger_y\right\} &= \frac{\alpha_\mathrm{s}}{2\pi^2}\int d^2z\,\frac{(x-y)^2}{X^2Y^2}\Bigg\{1+\frac{\alpha_\mathrm{s}}{4\pi}\Bigg[b\ln(x-y)^2\mu^2 - b\\ &\times\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2}+\left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c-\frac{10}{9}n_f-2N_c\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Bigg]\Bigg\}\end{aligned}$$

$$\times\left[\mathrm{Tr}\left\{\hat{U}_x\hat{U}_z^\dagger\right\}\mathrm{Tr}\left\{\hat{U}_z\hat{U}_y^\dagger\right\}-N_c\mathrm{Tr}\left\{\hat{U}_x\hat{U}_y^\dagger\right\}\right]+\frac{\alpha_s^2}{8\pi^4}\int d^2zd^2z'$$
$$\times\left[\left(-\frac{2}{(z-z')^4}+\left\{\frac{X^2Y'^2+X'^2Y^2-4(x-y)^2(z-z')^2}{(z-z')^4\left[X^2Y'^2-X'^2Y^2\right]}\right.\right.\right.$$
$$\left.\left.+\frac{(x-y)^4}{\left[X^2Y'^2-X'^2Y^2\right]X^2Y'^2}+\frac{(x-y)^2}{(z-z')^2}\frac{1}{X^2Y'^2}\right\}\ln\frac{X^2Y'^2}{X'^2Y^2}\right)$$
$$\times\left[\mathrm{Tr}\left\{\hat{U}_x\hat{U}_z^\dagger\right\}\mathrm{Tr}\left\{\hat{U}_z\hat{U}_{z'}^\dagger\right\}\mathrm{Tr}\left\{\hat{U}_{z'}\hat{U}_y^\dagger\right\}-\mathrm{Tr}\left\{\hat{U}_x\hat{U}_z^\dagger\hat{U}_{z'}U_y^\dagger\hat{U}_z\hat{U}_{z'}^\dagger\right\}-(z'\to z)\right]$$
$$+4n_f\left\{\frac{2}{(z-z')^4}-\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2-X'^2Y^2)}\ln\frac{X^2Y'^2}{X'^2Y^2}\right\}$$
$$\left.\times\mathrm{Tr}\left\{t^a\hat{U}_xt^b\hat{U}_y^\dagger\right\}\left[\mathrm{Tr}\left\{t^a\hat{U}_zt^b\hat{U}_{z'}^\dagger\right\}-(z'\to z)\right]\right].$$

Here μ is the normalization point in the $\overline{MS}$ scheme and $b=\frac{11}{3}N_c-\frac{2}{3}n_f$ is the first coefficient of the β-function. The NLO kernel is a sum of the running-coupling part (proportional to b), the non-conformal double-log term $\sim\ln\frac{(x-y)^2}{(x-z)^2}\ln\frac{(x-y)^2}{(x-z)^2}$ and the three conformal terms which depend on the two four-point conformal ratios $\frac{X^2Y'^2}{X'^2Y^2}$ and $\frac{(x-y)^2(z-z')^2}{X^2Y'^2}$. The (almost) conformal kernel (4) was obtained with the "rigid" rapidity cutoff

$$\begin{aligned}U_x^\eta &= \mathrm{P}\exp\left[ig\int\limits_{-\infty}^{\infty}du\ p_1^\mu A_\mu^\eta(up_1+x_\perp)\right],\\ A_\mu^\eta(x) &= \int d^4k\theta(e^\eta-|\alpha_k|)e^{-ik\cdot x}A_\mu(k)\,.\end{aligned}\tag{4}$$

The result for the NLO kernel with the "smooth" cutoff (1) is more complicated and non-conformal so it appears that the color dipoles should be regularized as in Eq. (4).

It should be emphasized that the NLO result itself does not lead automatically to the argument of coupling constant α_s in Eq. (3). In order to get this argument one can use the renormalon-based approach: first get the quark part of the running coupling constant coming from the bubble chain of quark loops and then make a conjecture that the gluon part of the β-function will follow that pattern. The Eq. (4) proves this conjecture in the first nontrivial order: the quark part of the β-function $\frac{2}{3}n_f$ calculated earlier gets promoted to full b. The analysis of the argument of the coupling constant was performed in Ref. [7–9] and the result is

$$\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\left\{\hat{U}_x\hat{U}^\dagger_y\right\} &\simeq \frac{\alpha_{\mathrm{s}}\left((x-y)^2\right)}{2\pi^2} \\
&\times \int d^2z\left[\mathrm{Tr}\left\{\hat{U}_x\hat{U}^\dagger_z\right\}\mathrm{Tr}\left\{\hat{U}_z\hat{U}^\dagger_y\right\} - N_c\mathrm{Tr}\left\{\hat{U}_x\hat{U}^\dagger_y\right\}\right] \\
&\times \left[\frac{(x-y)^2}{X^2Y^2} + \frac{1}{X^2}\left(\frac{\alpha_{\mathrm{s}}\left(X^2\right)}{\alpha_{\mathrm{s}}\left(Y^2\right)} - 1\right) + \frac{1}{Y^2}\left(\frac{\alpha_{\mathrm{s}}\left(Y^2\right)}{\alpha_{\mathrm{s}}\left(X^2\right)} - 1\right)\right] + \ldots . \quad (5)
\end{aligned}$$

It is easy to see that the argument of α_{s} is determined by the size of the smallest dipole $\min(|x-y|,|x-z|,|y-z|)$.

Our result (4) agrees with the forward NLO BFKL kernel [10] up to a term proportional $\alpha_{\mathrm{s}}^2\zeta(3)$ times the original dipole. We think that the difference could be due to different definitions of the cutoff in the longitudinal momenta. There is no any obvious preferred definition of the cutoff in the longitudinal momenta so it can be chosen in any way convenient for practical calculations of higher orders. It is worth noting that all cutoffs should give the same α_{s} correction to the intercept of the BFKL pomeron determined by the rightmost singularity in the complex ω plane. Our goal was to study the dipole amplitudes with the cutoff closely related to the the small-x asymptotics of the anomalous dimensions of twist-2 gluon operator and therefore we imposed the cutoff (4). It would be instructive to get the $j \to 1$ asymptotics of the anomalous dimensions of gluon operators directly from Eq. (4), without a Fourier transformation of our result to the momentum space and comparing to NLO BFKL as it is done in Ref. [6]. The study is in progress.

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