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PHENOMENOLOGICAL FORMULA FOR α-DECAY HALF-LIVES OF HEAVIEST NUCLEI

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A simple phenomenological formula for description of α-decay half-lives T_α of heavy (above ^{208}Pb) and superheavy nuclei is proposed. The formula, expressing T_α as a function of the decay energy Q_α, has five adjustable parameters: three to describe even–even nuclei and two for accounting for effects of an odd proton and an odd neutron. The formula allows one to describe measured values of T_α of 61 even–even nuclei roughly within a factor of 1.3, 45 odd–even nuclei within a factor of 2.1, 55 even–odd nuclei within a factor of 3.2 and 40 odd–odd nuclei within a factor of 4.0, on the average, when measured values of Q_α are taken. Results of its use are compared with those of other formulae. Effects of various changes in the formula are discussed.

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1. Introduction

A rather large number of phenomenological formulae have been used for description of α-decay half-lives T_α in a long history of studying this process (*e.g.* [1–10]). Such formulae are of a practical value, for an easy prediction of not yet measured values of T_α or to see a systematics of this quantity in a large region of nuclei. They certainly cannot replace a deep theoretical studies of α-decay, which aim at a microscopic description of the mechanism of this process. The latter studies are being done parallely (see *e.g.* the book [11] and the review [12]). There are also used models based on various simplifying assumptions (*e.g.* [13]).

The interest in the description of α-decay is continuing and even increasing, as a number of discovered and studied α emitters is continuously increasing, especially among exotic nuclei, in particular the heaviest ones (*e.g.* [14–20]).

In all phenomenological formulae used up to now, the dependence of T_α on the observed kinetic energy of α particle (or decay energy Q_α directly connected with it), as deduced from the Geiger–Nuttall rule, is taken into account. They differ, however, in the assumed dependence of T_α on the proton, Z, and mass, A, numbers of a decaying nucleus. Most of the formulae are rather simple. Some of them, however, are more complex (*e.g.* [7]), introducing, for example, a direct dependence of T_α on the shell structure of a nucleus, besides indirect one that comes via Q_α.

The objective of the present paper is to see how good accuracy can be reached in description of the present-day data for T_α of heaviest nuclei by using simple phenomenological formulae existing presently. It is also aimed to check, if the formulae cannot be still simplified without loosing their accuracy. The intention is to use as few adjustable parameters as possible.

In our analysis, we concentrate on heaviest nuclei with proton number $Z = 84$–111 and neutron number $N = 128$–161. All kinds of nuclei: even–even (e–e), odd–even (o–e), even–odd (e–o) and odd–odd (o–o) are considered, where *e.g.* (o–e) means (odd-Z, even-N) nuclei.

2. Check of the quality of existing formulae

We take the following two formulae for such a check. One is the rather old formula of Viola and Seaborg [4], which has been often used up to the present day (see *e.g.* [21–28]), and the other is a recent formula by Royer [10].

The check consists in testing how well are the measured half-lives $T_\alpha^{\rm exp}$ reproduced by the values $T_\alpha^{\rm ph}(Q_\alpha^{\rm exp})$ obtained by a phenomenological formula, when measured decay energy $Q_\alpha^{\rm exp}$ is taken.

2.1. The Viola–Seaborg formula

This 7-parameter formula reads [4]

$$\log_{10} T_\alpha^{\rm VS}(Z,N) = (aZ+b)Q_\alpha^{-1/2} + (cZ+d) + h_i \,, \tag{1}$$

where Z is proton number, N is neutron number and $Q_\alpha(Z,N)$ is the α-decay energy of a parent nucleus. The quantities a, b, c, d and h_i are adjustable parameters; h_i (originally denoted by $\langle \log F_i \rangle$), $i = p, n, pn$, are the average hindrance factors for o–e (*i.e.* odd-proton), e–o (*i.e.* odd-neutron) and o–o (*i.e.* odd-proton and odd-neutron) nuclei, respectively. For e–e nuclei, $h_i = 0$.

Adjustment of the parameters to experimental values of the half-lives T_α [29] (taken in seconds) with the use of experimental values of $Q_\alpha^{\rm exp}$ [30] (taken in MeV) for 61 even–even nuclei with $Z = 84$–110 and $N = 128$–160, for which both these data exist, leads to

$$a = 1.3892\,,\ b = 13.862\,,\ c = -0.1086\,,\ d = -41.458\,. \tag{1a}$$

With these four values of a, b, c, d kept fixed, adjustment of the three parameters h_p, h_n, h_{pn} to $T_\alpha^{\rm exp}$ [29] with the use of $Q_\alpha^{\rm exp}$ [30] for 45 o–e (with $Z = 85$–107, $N = 128$–160), 55 e–o (with $Z = 84$–110, $N = 129$–161) and 40 o–o (with $Z = 85$–111, $N = 129$–161) nuclei, respectively, leads to

$$h_p = 0.437\,, \quad h_n = 0.641\,, \quad h_{pn} = 1.024\ . \tag{1b}$$

The results are summarized in Table I. The first column specifies the class of nuclei, the second gives the number of nuclei N (with measured both $T_\alpha^{\rm exp}$ and $Q_\alpha^{\rm exp}$, which have been used for the adjustment of the parameters) in each class (group), $\bar{\delta}$ is the average of absolute values of discrepancies,

$$\bar{\delta} = \frac{1}{N}\sum_{k=1}^{N}\left|\log_{10}\left(T_{\alpha k}^{\rm ph}/T_{\alpha k}^{\rm exp}\right)\right|, \tag{2}$$

rms is the root-mean-square value of these discrepancies, $\bar{f} = 10^{\bar{\delta}}$, h is the value of h_i in each group and n_p is the number of adjustable parameters for each group. The difference between $\bar{\delta}$ and rms gives some information about inhomogeneity of the distribution of the discrepancies.

TABLE I

Results obtained with the Viola–Seaborg formula (see text).

Nuclei	N	$\bar{\delta}$	rms	$\bar{f}$	h	n_p
e–e	61	0.129	0.161	1.35	0	4
o–e	45	0.352	0.447	2.25	0.437	1
e–o	55	0.564	0.666	3.66	0.641	1
o–o	40	0.675	0.808	4.73	1.024	1

One can see in Table I that $T_\alpha^{\rm exp}$ are reproduced by the Viola–Seaborg formula roughly within a factor of 1.4 for e–e, 2.3 for o–e, 3.7 for e–o and 4.7 for o–o nuclei, respectively, on the average.

It should be added that in the fitting procedure, nuclei with neutron number close to shell closures at $N = 152$ and 162 have been omitted, as we would not like to include direct shell effects (not contained in Q_α) to the smooth dependence of T_α on Q_α described by a phenomenological formula. These are 3 e–o nuclei with $N = 151$ ($Z = 96, 98, 100$), one with $N = 153$ ($Z = 98$) and one with $N = 161$ ($Z = 110$). Also 3 o–e nuclei: ($Z = 97$, $N = 146$, 148) and ($Z = 101$, $N = 154$), with $T_\alpha^{\rm exp}$ especially much deviating from the average behavior, have been omitted.

2.2. The Royer formula

This 12-parameter formula reads [10]

$$\log_{10} T_\alpha^{\mathrm{R}}(Z,N) = aZQ_\alpha^{-1/2} + bZ^{1/2}A^{1/6} + c\,, \tag{3}$$

where Z and A are proton and mass numbers of a parent nucleus, respectively, and a, b, c are adjustable parameters. All three parameters a, b, c are adjusted separately for each class of nuclei: e–e, o–e, e–o and o–o. Thus, this gives altogether 12 parameters.

Adjustment of the parameters to experimental values $T_\alpha^{\exp}$ with the use of experimental values $Q_\alpha^{\exp}$, both taken the same as in adjustment of the Viola–Seaborg parameters, leads to

$$a = 1.5519,\quad b = -0.9761,\quad c = -28.688 \quad \text{for e–e nuclei}\,, \tag{3a}$$
$$a = 1.6070,\quad b = -0.9467,\quad c = -30.912 \quad \text{for o–e nuclei}\,, \tag{3b}$$
$$a = 1.6327,\quad b = -1.1249,\quad c = -27.287 \quad \text{for e–o nuclei}\,, \tag{3c}$$
$$a = 1.6789,\quad b = -1.0409,\quad c = -30.509 \quad \text{for o–o nuclei}\,. \tag{3d}$$

The results are summarized in Table II. One can see in this table that $T_\alpha^{\exp}$ are better (except e–e nuclei) reproduced by T_α^{R}, than by T_α^{VS} (Table I). This is not unexpected because of the larger number of adjustable parameters in the case of T_α^{R}.

TABLE II

Results obtained with the Royer formula.

Nuclei	N	$\bar{\delta}$	rms	$\bar{f}$	n_p
e–e	61	0.133	0.169	1.36	3
o–e	45	0.321	0.398	2.09	3
e–o	55	0.497	0.602	3.14	3
o–o	40	0.557	0.714	3.61	3

One should remark that the values of parameters of Eqs. (3a)–(3d) differ from those given in [10] (see Eqs. (3e)–(3h) in Sec. 5). This is because the difference in the range of nuclei, the data of which have been taken in the adjusting procedure. If one takes the values of the parameters given in [10] to describe $T_\alpha^{\exp}$ considered in the present paper, the discrepancies are larger, as discussed in Sec. 5.

3. Search for a new formula

Looking at the values of the Viola–Seaborg parameters, Eq. (1a), one can see that the value of the parameter b is small in comparison to values of aZ ($Z = 84$–110). This suggests that a 3-parameter formula

$$\log_{10} T_\alpha^{\rm ph}(Z,N) = aZQ_\alpha^{-1/2} + bZ + c\,, \tag{4}$$

may be not much worse (for e–e nuclei) than the 4-parameter one of Eq. (1).

Really, adjustment of a, b, c to $T_\alpha^{\rm exp}$ with the use of $Q_\alpha^{\rm exp}$ for 61 e–e nuclei (the same as used to obtain the values of Eq. (1a)) leads to

$$a = 1.5372\,, \quad b = -0.1607\,, \quad c = -36.573\,, \tag{4a}$$

with the average discrepancies $\bar{\delta} = 0.128$ and rms$\,= 0.165$, *i.e.* with only rms very little larger than the respective value obtained with the 4-parameter formula of Eq. (1) (see Table I). Due to this, we will be using this 3-parameter formula of Eq. (4) for e–e nuclei.

In the case of odd-A and o–o nuclei, structure of the ground states (g.s.) of a parent and the daughter nuclei are, in general, different. This causes a hindrance of the transition between these states. A parent nucleus prefers to decay from its g.s. to such an excited state of its daughter which has the same (or similar) structure. When we do not know the excitation energy of such a state, it is natural to treat it as an adjustable parameter. Thus, the formula (4), generalized to describe also odd-A and o–o nuclei, takes the form

$$\log_{10} T_\alpha^{\rm ph}(Z,N) = aZ(Q_\alpha - \bar{E}_i)^{-1/2} + bZ + c\,, \tag{5}$$

where $\bar{E}_i = 0$ for e–e nuclei, $\bar{E}_i = \bar{E}_p$ (average excitation energy of proton one-quasiparticle state to which α decay goes) for o–e nuclei, $\bar{E}_i = \bar{E}_n$ (average excitation energy of neutron one-quasiparticle state to which α decay goes) for e–o nuclei and $\bar{E}_i = \bar{E}_{pn}$ (average excitation energy of one-proton and one-neutron quasiparticle state) for o–o nuclei. To minimize the number of adjustable parameters, we put the average excitation energy $\bar{E}_{pn}$ of o–o nuclei as equal to the sum of the average energies of o–e ($\bar{E}_p$) and e–o ($\bar{E}_n$) nuclei, *i.e.*

$$\bar{E}_{pn} = \bar{E}_p + \bar{E}_n\,. \tag{5a}$$

This way, we get only 5 adjustable parameters to describe all four classes of nuclei by the formula (5).

With the 3-parameters a, b, c (fitted to data for e–e nuclei) kept fixed, adjustment of the two parameters $\bar{E}_{\rm p}$ and $\bar{E}_n$ to $T_\alpha^{\rm exp}$ [29] with the use of $Q_\alpha^{\rm exp}$ [30] for 45 o–e and 55 e–o nuclei, respectively, leads to $\bar{E}_p = 0.113\,$MeV and $\bar{E}_n = 0.171$ MeV.

Thus, the new phenomenological formula (5), with the 5 parameters

$$a = 1.5372\,, \quad b = -0.1607\,, \quad c = -36.573\,,$$
$$\bar{E}_p = 0.113 \text{ MeV}\,, \quad \bar{E}_n = 0.171 \text{ MeV}\,, \qquad (5b)$$

and with the value of $\bar{E}_{pn}$ calculated by Eq. (5a) for o–o nuclei, is used to describe T_α of all four classes of heavy and superheavy nuclei.

To test the assumption of Eq. (5a), we treated $\bar{E}_{pn}$ as the additional (6th) adjustable parameter, fitted to data of 40 o–o nuclei of the investigated region. We have obtained $\bar{E}_{pn} = 0.277$ MeV, *i.e.* very close to the value 0.284 MeV obtained from Eq. (5a) with $\bar{E}_p$ and $\bar{E}_n$ taken from Eq. (5b). This is a pleasant result, showing that the assumption of Eq. (5a) is reasonable.

4. Results

Table III shows the results obtained with the new formula of Eq. (5). By comparing it with Table I, one can see that although a smaller number of adjustable parameters (five) used by the new formula, it better describes measured half-lives $T_\alpha^{\rm exp}$ than the Viola–Seaborg formula using 7 adjustable parameters. Comparing Table III with Table II, one can observe that the quality of description of $T_\alpha^{\rm exp}$ by the new formula is quite similar to that of the Royer formula which uses 12 fitted parameters.

TABLE III

Results obtained with the formula of the present paper (Eq. (5)).

Nuclei	N	$\bar{\delta}$	rms	$\bar{f}$	n_p	$\bar{E}$ [MeV]
e–e	61	0.128	0.165	1.34	3	0
o–e	45	0.318	0.407	2.08	1	0.113
e–o	55	0.508	0.602	3.22	1	0.171
o–o	40	0.603	0.724	4.01	0	0.284

Let us illustrate now the quality of the results in a more detailed way than by the average values of discrepancies shown in Table III. Fig. 1 gives logarithm of the ratio of the phenomenological half-live $T_\alpha^{\rm ph}$, calculated according to the new formula of Eq. (5), to experimental one $T_\alpha^{\rm exp}$ for e–e nuclei with $Z = 84$–116. One can see that generally the values are within the range of about ± 0.25 (which corresponds to the values of the ratio $T_\alpha^{\rm ph}/T_\alpha^{\rm exp}$ within the range of about 0.56–1.78). Only for the nuclei ^{212}Po and ^{264}Hs, they are visibly outside this range.

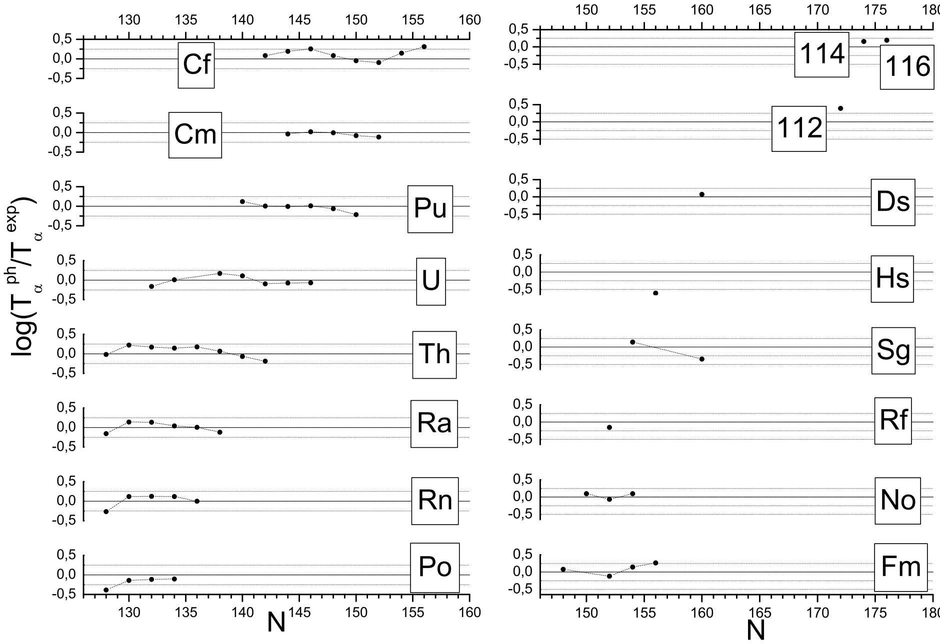

Fig. 1. Logarithm of the ratio $T_\alpha^{\rm ph}/T_\alpha^{\rm exp}$ calculated as a function of neutron number N for even–even nuclei with proton number $Z = 84$–116.

Fig. 2 illustrates the same quantity as in Fig. 1 for o–e nuclei. One can see that here the values of discrepancy are generally within the range of about ± 0.50 (which corresponds to the values of the ratio $T_\alpha^{\rm ph}/T_\alpha^{\rm exp}$ within the range of about 0.32–3.16). Only for the nuclei ^{257}Md and ^{261}Bh, they appear significantly outside this range.

Fig. 3 shows the discrepancies for e–o nuclei. One can see that they are larger than those for o–e nuclei. For more nuclei, the discrepancies appear outside the range of ± 0.50. Especially large discrepancy is obtained for the nucleus ^{237}Pu.

Finally, Fig. 4 presents the discrepancies for o–o nuclei. They are largest among all four classes of nuclei, but not much larger than the discrepancies obtained for e–o nuclei. As for nuclei, for which no adjustable parameters are used, the description of their half-lives is relatively good. The worst case is for ^{244}Bk, where $\log(\mathrm{T}_\alpha^{\rm ph}/\mathrm{T}_\alpha^{\rm exp}) \approx -2$.

Concerning the results presented in Figs. 1–4, one should add that the parameters of the formula of Eq. (5) for $T_\alpha^{\rm ph}$ have been fitted only to the data for nuclei with $Z = 84$–111. Thus, the data for nuclei with $Z = 112$–116, obtained more recently, may be treated as a test of a predictive power of the formula. One can see in the figures that this test appears to be positive. The values $T_\alpha^{\rm ph}$ reproduce $T_\alpha^{\rm exp}$ rather well for these nuclei.

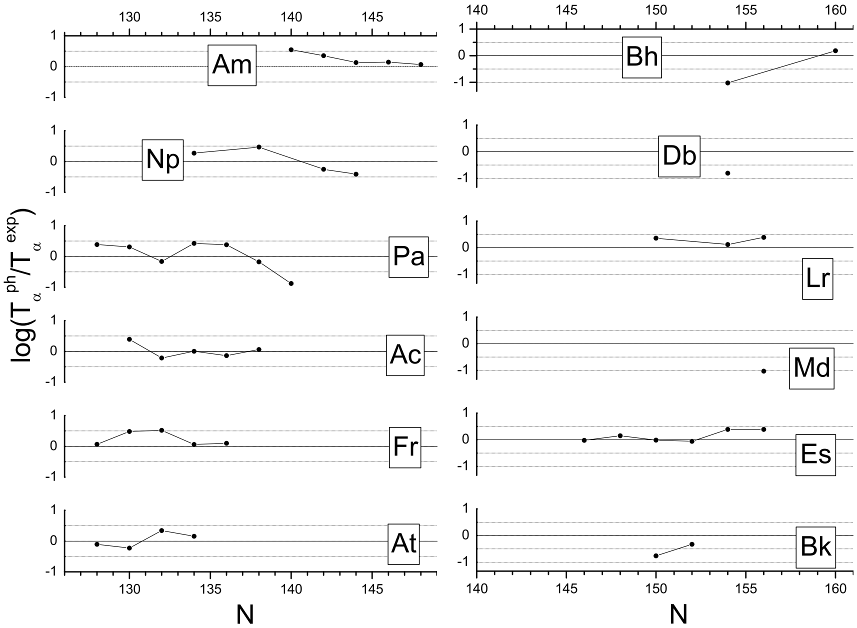

Fig. 2. Same as in Fig. 1, but for odd–even nuclei with $Z = 85$–107.

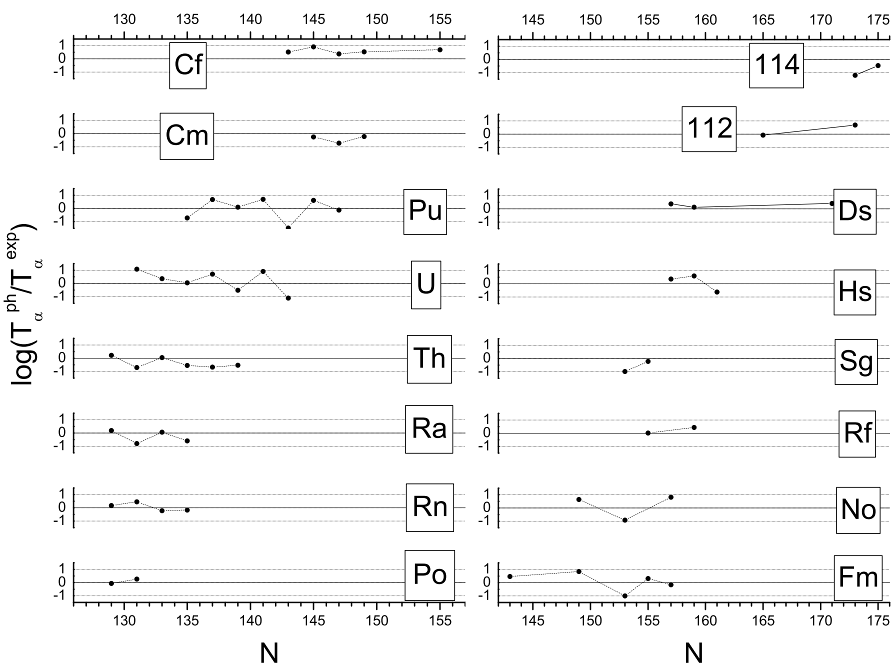

Fig. 3. Same as in Fig. 1, but for even–odd nuclei with $Z = 84$–114.

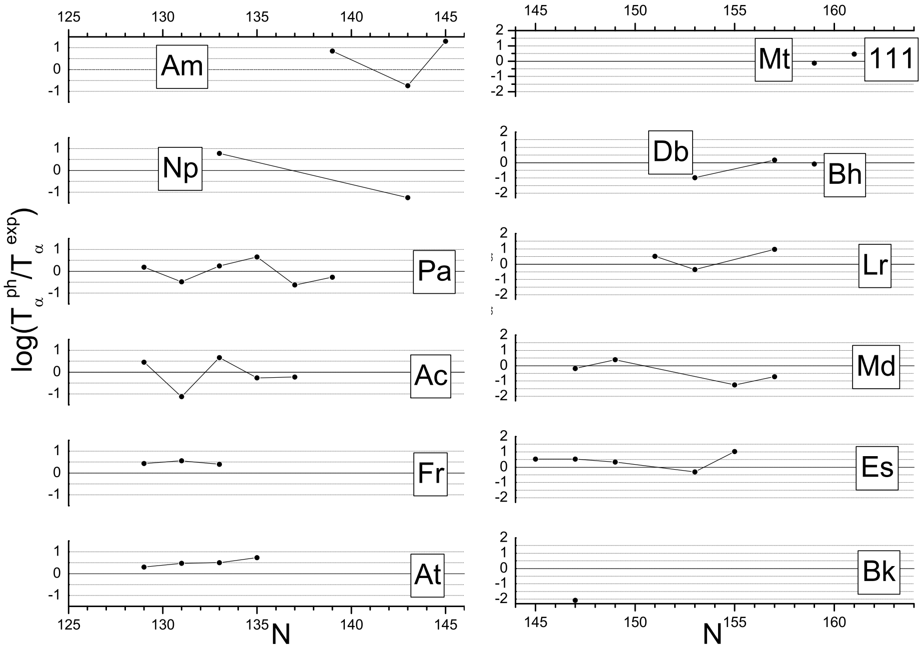

Fig. 4. Same as in Fig. 1, but for odd–odd nuclei with $Z = 85$–111.

5. Discussion of various effects

5.1. Effect of electron screening

This effect consists in a smaller kinetic energy of α particle outside an atom (which is measured) than its energy when it penetrates the Coulomb barrier, due to the orbital electron screening. The screening energy is [31]

$$E_{\rm scr} = \left(65.3Z^{7/5} - 80Z^{2/5}\right)\,{\rm eV}\,. \tag{6}$$

The energy is rather small. For nuclei from ^{212}Po to ^{292}116, considered in Fig. 1, it changes (smoothly) from 32 keV to 50 keV.

Due to this, its effect on T_α is also not large, but still significant. The ratio $T_\alpha^{\rm ph}(Q_\alpha^{\rm eff})/T_\alpha^{\rm ph}(Q_\alpha^{\rm exp})$, where

$$Q_\alpha^{\rm eff} = Q_\alpha^{\rm exp} + E_{\rm scr}\,, \tag{7}$$

and $T_\alpha^{\rm ph}$ is calculated according to Eq. (5), changes within the range from 0.51 to 0.84 for 64 nuclei considered in Fig. 1. Thus, the half-life $T_\alpha^{\rm ph}$ is reduced by this effect from 16% to 49 % for these nuclei. The magnitude of this relative reduction is correlated with the value of $Q_\alpha^{\rm exp}$. It is lowest for the nucleus ^{216}Ra with largest $Q_\alpha^{\rm exp}$ (9.53 MeV) and is highest for the nucleus ^{232}Th with smallest $Q_\alpha^{\rm exp}$ (4.08 MeV).

It is interesting to see how are the values of the parameters of the new formula, Eq. (5b), modified by the inclusion of this effect and if the quality of description of T_α is improved by it.

The results for the parameters are

$$a = 1.5394\,, \quad b = -0.1610\,, \quad c = -36.596\,,$$
$$\bar{E}_{\rm p} = 0.112 \text{ MeV}\,, \qquad \bar{E}_{\rm n} = 0.171 \text{ MeV}\,. \tag{5c}$$

To get them, we put $Q_\alpha^{\rm eff}$ instead of $Q_\alpha^{\rm exp}$ in the fitting procedure, *i.e.* we minimized sum of squares of the differences

$$\log_{10} T_\alpha^{\rm exp}(Z,N) - \left\{ aZ\, \left[Q_\alpha^{\rm eff}(Z,N) - \bar{E}_i \right]^{-1/2} + bZ + c \right\}\,. \tag{8}$$

One can see that the obtained values of the parameters, Eq. (5c), are almost the same as in the case when the effect is not taken into account, Eq. (5b). The results for the quality of description of T_α are shown in Table IV.

Comparison of Table IV with Table III shows that inclusion of the screening effect to the formula of Eq. (5) does not improve description of T_α of considered nuclei.

TABLE IV

Results obtained with the formula of Eq. (5) in the case, when the screening effect is taken into account.

Nuclei	N	$\bar{\delta}$	rms	$\bar{f}$	n_p	$\bar{E}$[MeV]
e–e	61	0.128	0.165	1.34	3	0
o–e	45	0.318	0.408	2.08	1	0.112
e–o	55	0.507	0.602	3.21	1	0.171
o–o	40	0.603	0.724	4.01	0	0.283

5.2. *Effect of using $\bar{E}_i$ instead of $\bar{h}_i$ as adjustable parameters*

To see this effect, we look at the results obtained with the formula

$$\log_{10} T_\alpha^{\rm ph}(Z,N) = \left(aZQ_\alpha^{-1/2} + bZ + c \right) + \bar{h}_i\,, \tag{9}$$

similar to that of Viola and Seaborg, Eq. (1), and compare them with the results obtained with the formula of Eq. (5). Naturally, the results will be different only for o–e, e–o and o–o nuclei, *i.e.* for nuclei with one or two odd nucleons. The results are shown in Table V.

TABLE V

Results obtained with the formula of Eq. (9).

Nuclei	N	$\bar{\delta}$	rms	$\bar{f}$	n_p	$\bar{h}_i$
e–e	61	0.128	0.165	1.34	3	0
o–e	45	0.356	0.456	2.27	1	0.433
e–o	55	0.564	0.645	3.66	1	0.643
o–o	40	0.689	0.810	4.89	0	1.076

A comparison between the results of Table III and those of Table V shows that the former are better than the latter ones. This is probably because the assumption of about the same excitation energy $\bar{E}_i$ of a state of a daughter nucleus, which has the same structure as the g.s. of the parent nucleus, is more realistic than the assumption of about the same hindrance factor $\bar{h}_i$. This may be argued in the following way. The state with the same characteristics as the g.s. of a parent nucleus should not be far in energy from the g.s. of the daughter nucleus, independently where the nucleus is located in the studied region, especially if the region is not too large. Thus, the assumption of constant $\bar{E}_i$ inside the region seems to be realistic. But, effect of this constant $\bar{E}_i$ on T_α may be quite different for nuclei with different Q_α, resulting in different hindrances h_i. Due to this, the assumption of constant h_i for a large region of nuclei seems to be less realistic.

5.3. Effect of level density

As density of single-particle levels increases with increasing mass number A of a nucleus, one might think about modifying, in Eq. (5), the expression for the excitation energy of the state to which α decay goes. As the density increases (within a simple model of harmonic oscillator) proportionally to $A^{1/3}$, one could propose the formula

$$\log_{10} T_\alpha^{\mathrm{ph}}(Z,N) = aZ\left(Q_\alpha - \bar{E}_i A^{-1/3}\right)^{-1/2} + bZ + c\,. \tag{10}$$

A direct check shows, however, that this does not improve the description of T_α of heaviest nuclei considered in the present paper.

5.4. Dependence on the range of described nuclei

Quality of description and values of parameters obviously depend on the range of nuclei, to data of which the parameters are adjusted. Let us directly illustrate this on the example of the Royer formula. The parameters of it, fitted to data of 373 nuclei which include also light (below ^{208}Pb) α emitters, are [10]

$$a = 1.5864, \quad b = -1.1629, \quad c = -25.31 \quad \text{for e–e nuclei}\,, \tag{3e}$$
$$a = 1.592, \quad b = -1.1423, \quad c = -25.68 \quad \text{for o–e nuclei}\,, \tag{3f}$$
$$a = 1.5848, \quad b = -1.0859, \quad c = -26.65 \quad \text{for e–o nuclei}\,, \tag{3g}$$
$$a = 1.6971, \quad b = -1.113, \quad c = -29.48 \quad \text{for o–o nuclei}\,. \tag{3h}$$

The use of these values to description of T_α of 201 heaviest nuclei, on the data of which we base in the present paper, leads to the results given in Table VI.

TABLE VI

Results obtained with the Royer formula with the values of parameters taken from [10].

Nuclei	N	$\bar{\delta}$	rms	$\bar{f}$	n_p
e–e	61	0.247	0.296	1.77	3
o–e	45	0.340	0.461	2.19	3
e–o	55	0.515	0.632	3.27	3
o–o	40	0.561	0.723	3.64	3

In this table N is the number of nuclei for which $T_\alpha^{\rm ph}$ is calculated (and not to which they are fitted) with the parameters of Eqs. (3e)–(3h). Comparing these results with those of Table II, one can see that they are worse, especially for e–e nuclei. They are even worse than the results of the 5-parameter formula of Eq. (5), for all classes of nuclei, except only the o–o one, as can be seen by comparison of Table VI with Table III. This does not certainly mean that one should adjust the parameters to small regions. The regions should be large enough to ensure formulae a generality and a predictive power.

6. Conclusion

A new, simple phenomenological formula for description of α-decay half-lives T_α of heaviest e–e, o–e, e–o and o–o nuclei is proposed. It uses only 5 adjustable parameters: 3 to describe e–e nuclei and 2 for description of nuclei with odd proton and odd neutron, one for each. (As the role of an odd nucleon in T_α is important, we consider such a separation of the roles of adjustable parameters as also significant.) The formula allows one to describe $T_\alpha^{\rm exp}$ of 61 e–e nuclei roughly within a factor of 1.3, 45 o–e nuclei within a factor of 2.1, 55 e–o nuclei within a factor of 3.2 and 40 o–o nuclei within a factor of 4.0, on the average, when $Q_\alpha^{\rm exp}$ is taken. These are the nuclei with $Z = 84$–111, for which both values of Q_α and T_α have been measured. A few still heaviest nuclei with $Z = 112$–116, discovered recently, for which both these values are also known, have not been included into the fitting procedure. The values of $T_\alpha^{\rm exp}$ for them are used as a test of the predictive power of the formula. The test appears to be positive, as illustrated in Figs. 1–4.

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