CHIRAL PHASE TRANSITION SCENARIOS FROM THE VECTOR MESON EXTENDED POLYAKOV QUARK MESON MODEL* **

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(Received June 3, 2015)

Chiral phase transition is investigated in an $\mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}}$ symmetric vector meson extended linear sigma model with additional constituent quarks and Polyakov loops (extended Polyakov quark meson model). The parameterization of the Lagrangian is done at zero temperature in a hybrid approach, where the mesons are treated at tree-level, while the constituent quarks at 1-loop level. The temperature and baryochemical potential dependence of the two assumed scalar condensates are calculated from the hybrid 1-loop level equations of states. The order of the phase transition along the $T=0$ and $\mu_{\mathrm{B}}=0$ axes is determined for various parameterization scenarios. We find that for a first order phase transition at $T=0$ as a function of μ_{B}, a light isoscalar particle is needed.

DOI:10.5506/APhysPolBSupp.8.335
PACS numbers: 12.39.Fe, 12.40.Yx, 14.40.Be, 14.40.Df

1. Introduction

In [1], it was shown through a zero temperature analysis that $q\bar{q}$ scalar states, such as the a_0, $K_0^\star$, and the two f_0s are preferred to have masses above 1 GeV. Similar results were obtained with $q\bar{q}$ states in [2], while in [2–4] it was shown, by using tetraquarks instead of $q\bar{q}$ states, that the (tetraquark) scalar masses are in the range of 0.6–1.0 GeV. These results suggest that the physical states $a_0(1450)$, $K_0^\star(1430)$, $f_0(1370)$, and $f_0(1710)$ (or $f_0(1500)$) are predominantly $\bar{q}q$ states, while $a_0(980)$, $K_0^\star(800)$, $f_0(500)$, and $f_0(980)$ are predominantly tetraquark states. However, states with the same quantum numbers do mix, thus the physical scalar particles are mixtures of $q\bar{q}$ and tetraquarks states. In the current case of the extended linear σ model

* Presented at "Excited QCD 2015", Tatranská Lomnica, Slovakia, March 8–14, 2015.
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(ELσM), we have only one scalar nonet, thus we can describe one a_0, one $K_0^\star$ and two f_0 (which will be denoted by $f_0^{L/H}$) particles. Consequently, one of the most interesting question is in which physical states our fields predominantly are if we investigate the finite temperature/density behavior of our model additionally to the zero temperature properties.

The lightest isoscalar ($J^{PC} = 0^{++}$) $q\bar{q}$ state, f_0 (also called σ) is strongly related to the non-strange condensate. Since the larger is the sigma mass compared to the mass of its chiral partner (the pion) the larger is the temperature at which m_{f_0} approaches m_π in the chiral symmetry restoration, one would expect that a large m_{f_0} mass results in a large pseudocritical temperature (T_c) at zero baryochemical potentials. On the other hand, it is a common expectation that the chiral phase transition is of first order as a function the baryochemical potential (μ_B) at $T = 0$, and since with increasing m_{f_0} mass the transition weakens, at some point it is possible that the transition becomes crossover [5, 6]. This suggests that for a good thermodynamic description, a small m_{f_0} mass is needed, and indeed, as it turns out, our approach supports this requirement.

The paper is organized as follows. In the next subsection, we introduce the model by giving the Lagrangian. In Sec. 3, the determination of the model parameters is shown, while the description of the approximation used to calculate the grand potential together with the field equations are presented in Sec. 4.

2. The model

The Lagrangian we shall use has the following form:

$$\begin{aligned}
\mathcal{L} = {} & \mathrm{Tr}\left[(D_\mu M)^\dagger (D_\mu M)\right] - m_0^2\,\mathrm{Tr}\left(M^\dagger M\right) - \lambda_1\left[\mathrm{Tr}\left(M^\dagger M\right)\right]^2 \\
& -\lambda_2\,\mathrm{Tr}\left(M^\dagger M\right)^2 + c_1\left(\det M + \det M^\dagger\right) + \mathrm{Tr}\left[H\left(M + M^\dagger\right)\right] \\
& -\frac{1}{4}\,\mathrm{Tr}\left(L_{\mu\nu}^2 + R_{\mu\nu}^2\right) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)\left(L_\mu^2 + R_\mu^2\right)\right] \\
& +i\frac{g_2}{2}\left(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}\right) \\
& +\frac{h_1}{2}\,\mathrm{Tr}\left(M^\dagger M\right)\mathrm{Tr}\left(L_\mu^2 + R_\mu^2\right) + h_2\,\mathrm{Tr}\left[(L_\mu M)^2 + (M R_\mu)^2\right] \\
& +2h_3\,\mathrm{Tr}\left(L_\mu M R^\mu M^\dagger\right) + \bar{\Psi}\left[i\gamma_\mu D^\mu - \mathcal{M}\right]\Psi\,. \qquad (1)
\end{aligned}$$

The covariant derivatives above are given by

$$D^\mu M = \partial^\mu M - ig_1\left(L^\mu M - M R^\mu\right) - ieA_e^\mu\left[T_3, M\right]\,, \qquad D^\mu \Psi = \partial^\mu \Psi - iG^\mu \Psi\,, \qquad (2)$$

where $G^\mu = g_s G_i^\mu T_i$, with $T_i = \lambda_i/2$ ($i = 1, \ldots, 8$) denoting the SU(3) group generators given in terms of the Gell-Mann matrices λ_i. Here, $M \equiv M_\mathrm{S} + M_\mathrm{PS}$ stands for the scalar–pseudoscalar fields, $L^\mu \equiv V^\mu + A^\mu$, $R^\mu \equiv V^\mu - A^\mu$ for the left- and right-handed vector fields (which contain the nonets of vector (V_a^μ) and axial-vector (A_a^μ) meson fields), A_e^μ is the electromagnetic field, while G_i^μ are the gluon fields. The field strength tensors are ($Q \in \{L, R\}$)

$$Q^{\mu\nu} = \partial^\mu Q^\nu - ieA_e^\mu[T_3, Q^\nu] - \{\partial^\nu Q^\mu - ieA_e^\nu[T_3, Q^\mu]\} \, , \tag{3}$$

while the external fields related to the scalar and vector fields are $H = \frac{1}{2}\mathrm{diag}(h_{0\mathrm{N}}, h_{0\mathrm{N}}, \sqrt{2}h_{0\mathrm{S}})$, $\Delta = \mathrm{diag}(\delta_\mathrm{N}, \delta_\mathrm{N}, \delta_\mathrm{S})$ (For more details on the model, see [1]).

3. Setting the Lagrange parameters

There are 15 unknown parameters in Eq. (1), which will be present in the field equations at finite temperature and/or density, namely, m_0, λ_1, λ_2, c_1, m_1, h_1, h_2, h_3, δ_N, δ_S, ϕ_N, ϕ_S, g_F, g_1, g_2. From this set, δ_N can be melted into m_1 thus leaving 14 unknowns. These parameters are determined similarly as in [1], that is we calculate values of various masses and decay widths at tree-level and compare them with the corresponding experimental value taken from the PDG [7] through the χ^2 minimalization process of Ref. [8]. It is important to note that we artificially increased the errors of the PDG to a minimum level of 5%, since we do not expect our model to be more precise. Another important points are that now we use a different anomaly term in Eq. (1) (the term proportional to c_1) as was presented in [1], we fit the total width in the case of $a_0(980)$ instead of the amplitudes, we include the f_0 masses and decay width into the global fit, we take into account the effects of the fermion vacuum fluctuations, in the case of which the expression of the (pseudo)scalar masses are modified. Moreover, since now we also included the constituent quarks in the isospin symmetric limit, we use two additional equations to their tree-level masses with the values $m_{u,d} = 330$ MeV, and $m_s = 500$ MeV.

The scalar meson sector below 2 GeV contains more physical particles than we can place into one $q\bar{q}$ nonet (consisting of a_0, $K_0^\star$, f_0^L, f_0^H), since in nature two a_0, two $K_0^\star$ and five f_0 particles exist in that energy range. These particles are the $a_0(980)$ and $a_0(1450)$ (denoted by $a_0^{1/2}$), the $K_0^\star(800)$ and $K_0^\star(1430)$ (denoted by $K_0^{\star\,1/2}$), the $f_0(500)$ (or σ), $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ (denoted by $f_0^{1\ldots5}$). Accordingly, there are 40 particle assignment possibilities to pair the physical particles to the members of the nonet in the model. We performed a χ^2 fit for all the assignments and ordered them according to their χ^2 values. The results of the 5 best solution along with the particle assignments in two cases (with and without the fermionic vacuum fluctuation) are shown in Table I. By a similar fitting procedure in [1], we argued that the best assignment without fitting the $f_0^{L/H}$ is the $a_0^2 K_0^{\star\,2}$, while we reasoned that $f_0^{L/H}$ should correspond to $f_0^{3/5}$. Now, it seems that the situation changes since in Table I one can see that the two best assignment in the case with and without using the fermion vacuum fluctuation are $a_0^1 K_0^{\star\,2} f_0^2 f_0^3$ and $a_0^1 K_0^{\star\,2} f_0^1 f_0^3$, respectively. This suggests that using only zero temperature quantities in this model is not enough to point out uniquely a single particle assignment. Thus, we investigated the properties of the different particle assignments at finite temperature/densities as well.

TABLE I

χ^2 and $\chi^2_{\rm red} \equiv \chi^2/N_{\rm dof}$ values ($N_{\rm dof} = 15$) for the first five best solutions of the fit together with the particle assignment without (left part) and with (right part) the fermionic vacuum fluctuations.

Particle assignment	χ^2	$\chi^2_{\rm red}$	Particle assignment	χ^2	$\chi^2_{\rm red}$
$a_0^1 K_0^{\star\,2} f_0^1 f_0^3$	45.0	3.0	$a_0^1 K_0^{\star\,2} f_0^2 f_0^3$	46.2	3.1
$a_0^1 K_0^{\star\,2} f_0^1 f_0^2$	51.7	3.4	$a_0^1 K_0^{\star\,1} f_0^1 f_0^3$	52.7	3.5
$a_0^2 K_0^{\star\,2} f_0^2 f_0^3$	51.7	3.4	$a_0^1 K_0^{\star\,1} f_0^1 f_0^2$	54.1	3.6
$a_0^1 K_0^{\star\,2} f_0^2 f_0^5$	60.4	4.0	$a_0^1 K_0^{\star\,2} f_0^1 f_0^2$	60.7	4.0
$a_0^2 K_0^{\star\,2} f_0^3 f_0^5$	61.3	4.1	$a_0^1 K_0^{\star\,2} f_0^1 f_0^3$	61.7	4.1

4. Finite temperature field equations

In our model, there are four order parameters, which are the two chiral condensates $\phi_{\rm N}$ (non-strange) and $\phi_{\rm S}$ (strange), and the two Polyakov loop variables Φ and $\bar{\Phi}$ (for the introduction of the Polyakov loop variables and their potential, see [9]). The field equations, which determine the dependence on T and $\mu_{\rm B} = 3\mu_q$ of the order parameters are given by the minimalization of the grand canonical potential (see *e.g.* [11]),

$$\frac{\partial \Omega_H}{\partial \phi_{\rm N}} = \frac{\partial \Omega_H}{\partial \phi_{\rm S}} = \frac{\partial \Omega_H}{\partial \Phi} = \frac{\partial \Omega_H}{\partial \bar{\Phi}} = 0\,, \tag{4}$$

which will result in four coupled equations. In our approach, we only consider vacuum and thermal fluctuations for the constituent quarks and not for the mesons. The explicit expression for the field equations can be found in [10] (Eqs. (2)–(5)).

5. Results and conclusion

By solving the field equations Eq. (4), we can investigate the behavior of the $\phi_{\rm N}$, $\phi_{\rm S}$ order parameters as a function of T at $\mu_{\rm B}=0$ and as a function $\mu_{\rm B}$ at $T=0$. It was calculated on the lattice [12] that at $\mu_{\rm B}=0$ the value of the pseudocritical temperature $T_{\rm c}$ should be 151 MeV, while it is a common belief that the order of the transition in $\mu_{\rm B}$ at $T=0$ should be of first order. In Fig. 1, the $T_{\rm c}$ and $\mu_{\rm B,c}$ values for all the 40 assignments are shown[1] as a function of $m_{f_0^{\rm L}}$ (the low-lying isoscalar mass) together with the lattice value for the $T_{\rm c}$. It can be seen that in order to be consistent with the lattice $T_{\rm c}$, the $m_{f_0^{\rm L}}$ mass should be below 1 GeV, which can be correspond either to $f_0(500)$ or to $f_0(980)$. However, if we would like to have first order phase transition on the $\mu_{\rm B}$ axis, the $m_{f_0^{\rm L}}$ mass should be even smaller ($\lessapprox$ 400 MeV). Consequently, we investigated the phase boundary for parameterizations with relatively small $m_{f_0^{\rm L}}$ masses.

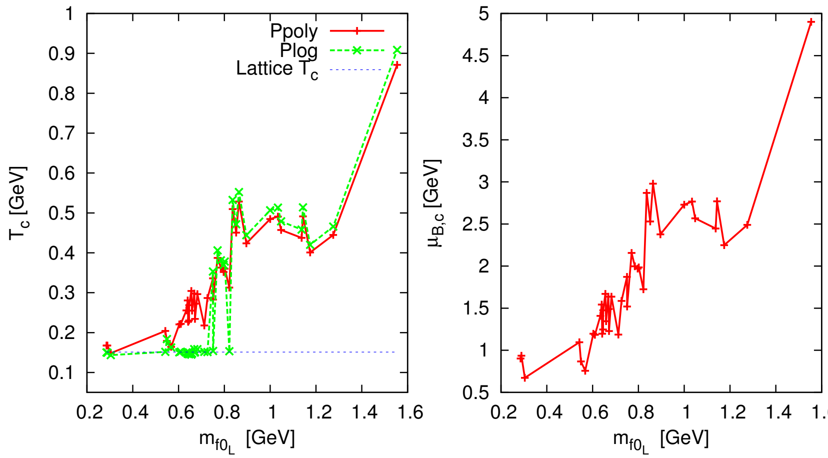

Fig. 1. $T_{\rm c}$ (left) and $\mu_{\rm B,c}$ (right) values *versus* $m_{f_0^{\rm L}}$. On the left, $T_{\rm c}$ is shown for two different Polyakov potential.

In the left panel of Fig. 2, the phase boundary together with the position of the critical endpoint (CEP) is shown, while in the right panel the CEP variation with the $m_{f_0^{\rm L}}$ mass is presented. If we increase the $m_{f_0^{\rm L}}$ mass above $\approx$ 350 MeV, the CEP ceases to exist.

In conclusion, we can say that in order to have a good thermodynamic description within the framework of the current model, we must have the $f_0(500)$ particle in the spectrum.

[1] The lines are only shown to guide the eye.

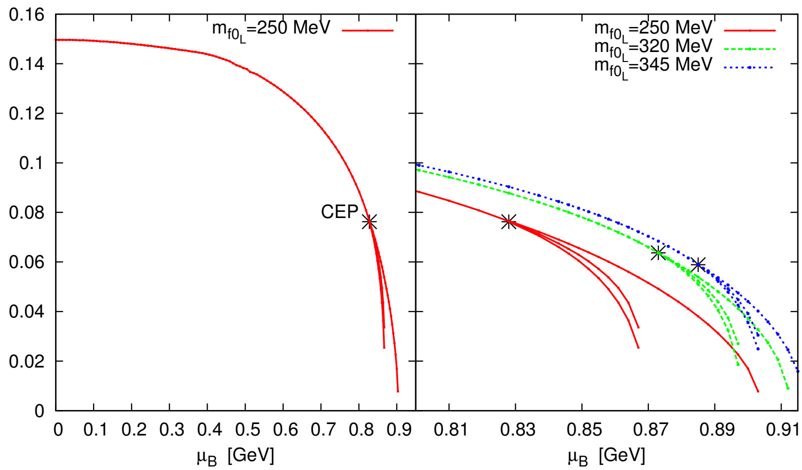

Fig. 2. Phase boundary (left) and CEP variation with $m_{f_0^L}$ mass (right). The CEP position on the left is at $\mu_{B,CEP} = 828$ MeV, $T_{CEP} = 76$ MeV.

The authors were supported by the Hungarian OTKA fund K109462 and by the HIC for FAIR Guest Funds of the Goethe University Frankfurt.

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