KALUZA–KLEIN UNIVERSE FILLED WITH WET DARK FLUID IN $f(R, T)$ THEORY OF GRAVITY*

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(Received December 21, 2016)

Kaluza–Klein metric is considered with wet dark fluid (WDF) source in $f(R, T)$ gravity, where $R$ is the Ricci scalar and $T$ is the trace of the energy-momentum tensor proposed by Harko et al. (2011). The exact solutions of the field equations are derived from a time varying deceleration parameter.

DOI:10.5506/APhysPolBSupp.10.369

1. Introduction

The nature of the dark energy (DE), a component of the Universe [1–3], remains one of the greatest mysteries of cosmology. There are many candidates for DE such as: cosmological constant, quintessence [4], k-essence [5], phantom energy [6] etc. Modified or alternative theories of gravity are the second proposal to justify the current expansion of the Universe. The recently developed $f(R, T)$ theory of gravity is one such example.

In this work, we use WDF as a candidate for DE. This model is in the spirit of generalized Chaplygin gas (GCG), where a physically motivated EOS is offered with the properties relevant for DE problem. The EOS for WDF [7] is

$$p_{\text{WDF}} = \omega(\rho_{\text{WDF}} - \rho^*) .$$

(1)

This EOS is a good approximation for many fluids, including water, in which the internal attraction of molecules makes negative pressure. The parameters $\omega$ and $\rho^*$ are taken to be positive and $0 \leq \omega \leq 1$. If $c_s$ denote the adiabatic sound speed in WDF, then $\omega = c_s^2$ [8]. The energy conservation equation for WDF is

$$\rho_{\text{WDF}}^* + 4H(p_{\text{WDF}} + \rho_{\text{WDF}}) = 0 .$$

(2)

Using $4H = \frac{V'}{V}$ and EOS (1), the above equation can be written as

$$\rho_{\text{WDF}} = \frac{\omega}{1 + \omega} \rho^* + \frac{C}{V(1+\omega)},$$

where $C$ is the constant of integration and $V$ is the volume expansion. Wet dark fluid naturally includes two components: one of them behaves as a cosmological constant as well as a standard fluid with an equation of state $p = \omega \rho$. If we consider $C > 0$, this fluid will not violate the strong energy condition $p + \rho \geq 0$. Hence,

$$p_{\text{WDF}} + \rho_{\text{WDF}} = (1 + \omega) \rho_{\text{WDF}} - \omega \rho^* = (1 + \omega) \frac{c}{V(1+\omega)} \geq 0. \quad (4)$$

The action for $f(R, T)$ gravity is

$$S = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} \, d^4x + \int L_m \sqrt{-g} \, d^4x, \quad (5)$$

where $f(R, T)$ is an arbitrary function of Ricci scalar $R$, $T$ be the trace of stress-energy tensor $T_{ij}$ of the matter. $L_m$ is the matter Lagrangian density. The energy momentum tensor $T_{ij}$ is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} L_m)}{\delta g^{ij}}. \quad (6)$$

By the help of matter Lagrangian $L_m$, the matter energy tensor is given by

$$T_{ij} = (p_{\text{WDF}} + \rho_{\text{WDF}}) u_i u_j - p_{\text{WDF}} g_{ij}, \quad (7)$$

where $u^i = (1, 0, 0, 0, 0)$ is the five velocity in comoving coordinates satisfying the condition $u^i u_i = 1$ and $u^i \nabla_j u_i = 0$. In equation (7), $\rho_{\text{WDF}}$ is the energy density, $p_{\text{WDF}}$ is pressure and the matter Lagrangian can be taken as $L_m = -p_{\text{WDF}}$.

Harko et al. [9] presented three classes of models. In this paper, we consider $f(R, T) = R + 2f(T)$, where $f(T)$ is an arbitrary function of energy tensor. The $f(R, T)$ gravity field equations are derived by varying the action $S$ with respect to metric tensor $g_{ij}$. For a WDF matter source, (7) takes the form of

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2f'(T) T_{ij} + [2p_{\text{WDF}} f'(T) + f(T)] g_{ij}. \quad (8)$$

2. Field equations and solution

The five-dimensional Kaluza–Klein metric is

$$ds^2 = dt^2 - A(t)^2 (dx^2 + dy^2 + dz^2) - B(t)^2 d\psi^2,$$

where the fifth coordinate $\psi$ is space-like.
Here, we consider \( f(T) = \lambda T \), where \( \lambda \) is a constant.

The \( f(R, T) \) gravity field equations (8) for the metric (9) can be written as

\[
2H_x' + 3H_x^2 + 2H_x H_\psi + H'_\psi + H_\psi^2 = (8\pi + 4\lambda)p_{\text{WDF}} - \lambda \rho_{\text{WDF}},
\]

(10)

\[
3 \left( H'_x + 2H_x^2 \right) = (8\pi + 4\lambda)p_{\text{WDF}} - \lambda \rho_{\text{WDF}},
\]

(11)

\[
3 \left( H_x^2 + H_x H_\psi \right) = -(8\pi + 3\lambda)\rho_{\text{WDF}} + \lambda p_{\text{WDF}},
\]

(12)

where prime denotes derivative with respect to time \( t \). \( H_x = \frac{A'}{A} = H_y = H_z \) and \( H_\psi = \frac{B'}{B} \) are the directional Hubble parameters.

Here, we have three equations with four unknowns. To get a deterministic solution, we assume the deceleration parameter as

\[
q = -\frac{a''}{a'^2} = -1 + \frac{\beta}{1 + a^\beta},
\]

(13)

where \( \beta > 0 \) is a constant and \( a \) is the average scale factor.

The mean Hubble parameter \( H = \frac{\dot{a}}{a} \) can be obtained from the above equation as

\[
H = \frac{\dot{a}}{a} = A_1 \left( 1 + a^{-\beta} \right),
\]

(14)

where \( A_1 \) is an integration constant. Again, integrating (14), we get

\[
a = \left( e^{A_1 \beta t} - 1 \right)^\frac{1}{\beta}.
\]

(15)

Using the spatial volume \( V = a^4 = A^3 B \), we obtain the scale factors \( A, B \) as

\[
A = \left( e^{A_1 \beta t} - 1 \right)^\frac{1}{3\beta}, \quad B = \left( e^{A_1 \beta t} - 1 \right)^{\frac{3}{\beta}}.
\]

(16)

The directional Hubble parameters are obtained as

\[
H_x = \frac{A_1}{3 \left( e^{A_1 \beta t} - 1 \right)} = H_y = H_z, \quad H_\psi = \frac{3A_1}{e^{A_1 \beta t} - 1}.
\]

(17)

Using the above values, the energy density \( \rho_{\text{WDF}} \) and the pressure \( p_{\text{WDF}} \) are obtained as

\[
\rho_{\text{WDF}} = \frac{A_1^2}{(8\pi + 3\lambda)(8\pi + 4\lambda) - \lambda^2} \left[ \frac{\beta \lambda e^{A_1 \beta t}}{e^{A_1 \beta t} - 1} \frac{(3\beta \lambda + 80\pi + 38\lambda)e^{2A_1 \beta t}}{3(e^{A_1 \beta t} - 1)^2} \right],
\]

(18)

\[
p_{\text{WDF}} = \frac{A_1^2}{(8\pi + 3\lambda)(8\pi + 4\lambda) - \lambda^2} \times \left[ \frac{(8\pi + 3\lambda)\beta e^{A_1 \beta t}}{e^{A_1 \beta t} - 1} + \frac{16\pi - 4\lambda - 3\beta(8\pi + 3\lambda)}{3(e^{A_1 \beta t} - 1)^2} \right].
\]

(19)
The anisotropy parameter of the expansion is \( \Delta = \frac{1}{4} \sum_{i=1}^{4} \left( \frac{H_i - H}{H} \right)^2 = \frac{4}{3} \).

The scalar expansion (\( \theta = 4H \)) and the shear (\( \sigma^2 = \frac{4}{3} \Delta H^2 \)) are
\[
\theta = 4A_1 e^{A_1 \beta t} \left( e^{A_1 \beta t} - 1 \right)^{-1}, \quad \sigma^2 = \frac{8}{3} A_1^2 e^{2A_1 \beta t} \left( e^{A_1 \beta t} - 1 \right)^{-2} .
\]

(20)

3. Conclusion

The spatial volume and average scale factor of the model is zero at initial time \( t \to 0 \) indicating that the model starts at Big Bang and has a point type singularity too. The anisotropic parameter becomes constant and our model is expanding with time. From Figs. 1 (left) and (right), the energy density of WDF is a decreasing function of time and remains positive throughout the Universe whereas the pressure is the increasing function of time and becomes constant.

![Fig. 1. Left: Plot of \( \rho_{WDF} \) vs. \( t \) with \( A_1 = 1, \beta = 1.5, \lambda = -6 \). Right: Plot of \( p_{WDF} \) vs. \( t \) with \( A_1 = 1, \beta = 1.5, \lambda = -6 \).](image)

REFERENCES