Vol. 47 (2016) ACTA PHYSICA POLONICA B No 2

η' INTERACTIONS WITH NUCLEONS AND NUCLEI*

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(Received October 21, 2015)

We summarise recent progress in theory and experiment towards understanding of η'-meson interactions with nucleons and nuclei. Highlights include the production mechanism of η' mesons in proton–proton collisions close to the threshold, the η' effective mass shift in nuclei and the determination of the η'-nucleon scattering length in free space.

DOI:10.5506/APhysPolB.47.373

1. Introduction

The last 20 years have witnessed a dedicated programme of experimental and theoretical studies of low-energy η' interactions with nucleons, nuclei and other mesons. The η' meson is special in QCD because of its strong affinity to glue. While pions and kaons are would-be Goldstone bosons associated with chiral symmetry, the isosinglet η and η' mesons are too massive by about 300–400 MeV for them to be pure Goldstone states. They receive extra mass from non-perturbative gluon dynamics associated with the QCD axial anomaly. Taking the η–η' mixing angle between -15° and -20°, the η' is predominantly a flavour-singlet state with strong coupling to gluonic intermediate states meaning that its interactions with other hadrons are, in general, characterised by OZI violation, for recent reviews, see [1–3]. The experimental programme has focussed on near-threshold η' production in proton–nucleon collisions using the COSY-11 facility at FZ-Jülich [4], η'-photoproduction experiments at ELSA in Bonn [5] and Jefferson Laboratory [6], studies of the η' in medium and the search for η'-bound states in nuclei at ELSA, GSI and LEPS2 [7] and production of hadronic states with exotic quantum numbers at COMPASS at CERN [8].

Highlights from COSY-11 [4, 9] include studies of the η'- and η-production mechanisms in proton–nucleon collisions close to the threshold through measurements of the total and differential cross sections and varying the isospin of the second nucleon. The η' is observed to be produced primarily in s-wave up to the excess energy of at least $\mathcal{E} = 11$ MeV. A first quantitative value of the η'-nucleon scattering length has been obtained [10] as well as the most accurate measurement of the η' total width in free space [11]. Photoproduction measurements [5, 6] from proton and deuteron targets have recently been extended by the CBELSA/TAPS Collaboration in Bonn to carbon and niobium to make a first (indirect) measurement of the η'-nucleus optical potential [7, 12]. One finds an η' effective mass shift in nuclei of about -37 MeV at nuclear matter density, in excellent agreement with the prediction of the Quark Meson Coupling model, QMC [13], through coupling of the light quarks in the meson to the σ mean field inside the nucleus. The η' experiences an effective mass shift in nuclei which is catalysed by its gluonic component [1, 14]. Although the η'-nucleon interaction in free space is much weaker [10] than the η-nucleon interaction [15], the small width of the η' in medium [16] means that the η' may be a good candidate for possible bound state searches, *e.g.* in experiments at ELSA, GSI and LEPS2 [7]. Searches for η-mesic nuclei are ongoing with data from WASA-at-COSY [17]. The odd L exotic partial waves L^{-+} are strongly enhanced in $\eta'\pi$ relative to $\eta\pi$ exclusive production in collisions of 191 GeV negatively charged pions from hydrogen at COMPASS, consistent with expectations based on the axial U(1) extended chiral Lagrangian [18].

2. QCD symmetries and the η and η'

Spontaneous chiral symmetry breaking in QCD induces an octet of Goldstone bosons associated with SU(3) and also (before extra gluonic effects in the singlet channel) a flavour-singlet Goldstone boson. The squared mass of these Goldstone bosons is proportional to the current mass of their valence quarks. While the pion and kaon fit well in this picture, to understand the isosinglet η and η' masses, one needs 300–400 MeV extra mass in the flavour-singlet channel which is associated with non-perturbative topological gluon configurations [3, 19] related perhaps to confinement [20] or instantons [21]. The gluonic mass contribution $\tilde{m}^2_{\eta_0}$ satisfies the Witten–Veneziano mass formula [22, 23]

$$m^2_\eta + m^2_{\eta'} = 2m^2_K + \tilde{m}^2_{\eta_0} \tag{1}$$

and has a rigorous interpretation in terms of the QCD Yang–Mills topological susceptibility. SU(3) breaking generates mixing between the octet and singlet states which, together with the gluonic mass term, yields the massive η and η' bosons.

Phenomenological studies of various decay processes give a value for the η–η' mixing angle between $-15°$ and $-20°$ [24]. The η' has a large flavour-singlet component with strong affinity to couple to gluonic degrees of freedom, *e.g.* in OZI violating interactions. For the η meson, the singlet component is also important, particularly in understanding of the η in nuclei and potentially accounting for 50% of the η-nucleon scattering length in medium [13]. In the OZI limit of no gluonic mass term, the η would be approximately an isosinglet light-quark state ($\frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle$) with mass $m_\eta \sim m_\pi$ degenerate with the pion and the η' would be a strange-quark state $|\bar{s}s\rangle$ with mass $m_{\eta'} \sim \sqrt{2m_K^2 - m_\pi^2}$, mirroring the isoscalar vector ω and ϕ mesons.

The gluonic mass term is related to the QCD axial anomaly in the divergence of the flavour-singlet axial-vector current. While the non-singlet axial-vector currents are partially conserved (they have just mass terms in the divergence), the singlet current $J_{\mu 5} = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s$ satisfies the anomalous divergence equation

$$\partial^\mu J_{\mu 5} = 6Q + \sum_{k=1}^{3} 2im_k \bar{q}_k \gamma_5 q_k \,, \tag{2}$$

where $Q = \partial^\mu K_\mu = \frac{\alpha_s}{8\pi} G_{\mu\nu}\tilde{G}^{\mu\nu}$ is the topological charge density. The integral over space $\int d^4z\, Q = n$ measures the gluonic winding number [19] which is an integer for (anti-)instantons and which vanishes in perturbative QCD.

The anomalous glue that generates the large η and η' masses also drives OZI violating η and η' production and decay processes [18, 24–26] and enters in the η'-nucleon interaction [27]. In high energy processes, B and charm-quark meson decays involving an η' in the final state are driven in part by the strong coupling to gluonic intermediate states [24, 26]. In low energy QCD, the η' experiences an effective mass shift in nuclei that, within the QMC model, is catalysed by its gluonic component [1]. The η'-nucleon coupling constant is, in principle, sensitive to OZI violation [27]. The QCD axial anomaly also plays an important role in the interpretation of the flavour-singlet Goldberger–Treiman relation [28] and the nucleon's flavour-singlet axial-charge (or "quark spin content") measured in polarised deep inelastic scattering associated with the proton spin puzzle [29, 30]. We refer to Ref. [31] for a discussion of gluonic components in the η' wave function and mixing with pseudoscalar glueball states.

The axial U(1) extended chiral Lagrangian [32] incorporates the chiral and axial U(1) symmetries and allows us to study low-energy QCD processes involving the η'. The gluonic mass term $\tilde{m}_{\eta_0}^2$ is introduced via a flavour-singlet potential involving the topological charge density Q which is constructed such that the Lagrangian also reproduces the axial anomaly [32]. Potential terms involving Q generally describe OZI violation, *e.g.* the term $Q^2 \partial_\mu \pi_a \partial^\mu \pi_a$ with π_a the pseudoscalar Goldstone fields drives the decay $\eta' \to \eta\pi\pi$ [25] and plays an important role in dynamical generation of a light mass exotic with quantum numbers 1^{-+} [18], see below.

3. η' production experiments

η' production has been measured in proton–proton collisions close to the threshold (excess energy $\mathcal{E}$ between 0.76 and $\sim$ 50 MeV) by the COSY-11 Collaboration at FZ-Jülich [33–37] and at $\mathcal{E}$ = 3.7 MeV and 8.3 MeV by SPESIII [38] and 144 MeV by the DISTO Collaboration at SATURNE [39].

For the η', production is s-wave dominated for $\mathcal{E}$ up to at least 11 MeV. The proton–proton and η'-proton invariant mass distributions determined for the $pp \to pp\eta'$ reaction at the excess energy $\mathcal{E}$ = 16.4 MeV show an enhancement which might indicate a non-negligible p-wave contribution from the proton–proton subsystem [37]. Fitting the low $\mathcal{E}$ data to models of the η' final state interaction allowed COSY-11 to extract a first measurement of the η'-proton scattering length [10], see Eq. (4) below.

Comparison of π^0, η and η' production in proton–nucleon collisions close to the threshold was performed at COSY-11. For near-threshold meson production, the production cross section is reduced by initial state interaction between the incident nucleons and enhanced by final state interactions between the outgoing hadrons. For comparing production dynamics, a natural variable is the volume of available phase space which is approximately independent of the meson mass. Making this comparison for the neutral pseudoscalar mesons, it was found that production of the η meson is about six times enhanced compared to the π^0 which is six times further enhanced compared to the η' [35]. One may conclude that the production of the η' and π^0 close to the threshold is non-resonant in contrast to η production which proceeds through strong coupling to $S_{11}(1535)$ [40]. However, it should be noted that as advocated in Ref. [41], η' meson production may also be explained by the relatively weak coupling to a rather not well established set of s-wave and p-wave resonances. Based on the comparison of excitation functions for the $pp \to pp\eta$ and $pp \to pp\eta'$ reactions close to the threshold, it was concluded that the η-proton interaction is much stronger than for η'-proton [35]. In higher energy experiments with proton–proton collisions at 450 GeV, the η and η' seem to have a similar production mechanism which differs from that of the π^0 [42].

Measurements of the isospin dependence of η meson production in proton–nucleon collisions revealed that the total cross section for the quasi-free $pn \to pn\eta$ reaction exceeds the corresponding cross section for $pp \to pp\eta$ by a factor of about three at the threshold and by factor of six at higher excess energies between about 25 and 100 MeV [43, 45]. Combining information about the strong isospin dependence and the isotropic angular distributions of the η meson emission angle in the centre-of-mass frame, it was established that the η meson is predominantly created via excitation of one of the nucleons to the $S_{11}(1535)$ resonance via a strong isovector exchange contribution. The angular dependence of the analysing power slightly indicated that the process proceeds via exchange of the π meson [46].

Measurements of the isospin dependence of η' production further suggest a different production mechanism for this meson [35, 44]. Using the quasi-free proton–neutron interaction [47], COSY-11 placed an upper bound on $\sigma(pn \to pn\eta')$ and the ratio $R_{\eta'} = \sigma(pn \to pn\eta')/\sigma(pp \to pp\eta')$ [44]. For the excess energy between 8–24 MeV, $R_{\eta'}$ was observed to be consistently one standard deviation below the corresponding ratio for η production [43]. In the gendanken limit that η' production proceeded entirely through gluonic excitation in the intermediate state, this ratio would go to one. The data is consistent with both a role for OZI violating η' production [27] and the meson exchange model [48]. The data do not favour a dominant role for the $S_{11}(1535)$ in the η' production mechanism, unlike for η production.

As an extra bonus from these experiments, the total width of the η' was determined from its mass distribution to be $\Gamma = 0.226 \pm 0.017(\text{stat.}) \pm 0.014(\text{syst.})$ MeV [11], an order of magnitude more accurate than previous determinations.

η' (quasi-free) photoproduction from proton and deuteron targets was studied at ELSA [5] and JLab [6]. The production cross section is isospin independent for incident photon energies greater than 2 GeV, where t-channel exchanges are important. At lower energies, particularly between 1.6 and 1.9 GeV, where the proton cross section peaks, the proton and quasi-free neutron cross sections show different behaviour, perhaps associated with resonances or interference terms [5].

4. The η and η' in nuclei

The recent progress in theoretical and experimental studies of the η- and η'- (as well as pion and kaon) nucleus systems promises to yield valuable new information about dynamical chiral and axial U(1) symmetry breaking in low energy QCD [1]. With increasing nuclear density, chiral symmetry is partially restored corresponding to a reduction in the values of the quark condensate and pion decay constant f_π [49, 50]. This, in turn, leads to changes in the properties of hadrons in medium including the masses of the Goldstone bosons. There is presently a vigorous experimental [7, 12, 16, 17, 51–58] and theoretical [1, 13, 59–64] activity aimed at understanding of the η and η' in medium and to search for evidence of possible η and η' bound states in nuclei.

How does the gluonic part of their mass change in nuclei? Medium modifications need to be understood self-consistently within the interplay of confinement, spontaneous chiral symmetry breaking and axial U(1) dynamics.

The η- and η'-nucleon interactions are believed to be attractive corresponding to a reduced effective mass in the nuclear medium and the possibility that these mesons might form strong-interaction bound-states in nuclei. For the η, one finds a sharp rise in the cross section at the threshold for η production in both photoproduction from ^{3}He [57] and in proton–deuteron collisions [58] which may hint at a reduced η effective mass in the nuclear medium. The measurement of the η'-nucleus optical potential by the CBELSA/TAPS Collaboration suggests that the effective η' mass drops by about 40 MeV at nuclear matter density [12]. For the pion and kaon systems, one finds a small pion mass shift of the order of a few MeV in nuclear matter [49], whereas kaons are observed to experience an effective mass drop for the K^- to about 270 MeV at two times nuclear matter density in heavy-ion collisions [65, 66]. The same heavy-ion experiments also suggest the effective mass of anti-protons is reduced by about 100–150 MeV below their mass in free space [65]. Experiments in heavy-ion collisions [67] and η photoproduction from nuclei [68, 69] suggest little modification of the $S_{11}(1535)$ excitation in medium, though some evidence for the broadening of the S_{11} in nuclei was reported in Ref. [69].

Building on η' photoproduction from proton targets, the meson mass shifts in medium can be investigated through studies of excitation functions in photoproduction experiments from nuclear targets and through searches for possible meson bound states in nuclei. In photoproduction experiments, the production cross section is enhanced with the lower effective meson mass in the nuclear medium. When the meson leaves the nucleus, it returns on-shell to its free mass with the energy budget conserved at the expense of the kinetic energy so that excitation functions and momentum distributions can provide essential clues to the meson properties in medium [70]. Using this physics, a first (indirect) estimate of the η' mass shift has recently been deduced by the CBELSA/TAPS Collaboration [12]. The η'-nucleus optical potential $V_{\rm opt} = V_{\rm real} + iW$ deduced from these photoproduction experiments is

$$\begin{aligned} V_{\rm real}(\rho_0) = m^* - m &= -37 \pm 10({\rm stat.}) \pm 10({\rm syst.})\ {\rm MeV}\,, \\ W(\rho_0) &= -10 \pm 2.5\ {\rm MeV} \end{aligned} \tag{3}$$

at nuclear matter density ρ_0. In this experiment, the average momentum of the produced η' was 1.1 GeV and the mass shift was measured in production from a carbon target. This optical potential corresponds to an effective scattering length in medium with the real part of about 0.5 fm assuming we switch off the Ericson–Ericson rescattering denominator [71].

The COSY-11 Collaboration have recently determined the η'-nucleon scattering length in free space to be

$$\begin{aligned} \mathrm{Re}(a_{\eta' p}) &= 0 \pm 0.43 \text{ fm}, \\ \mathrm{Im}(a_{\eta' p}) &= 0.37 \, ^{+0.40}_{-0.16} \text{ fm} \end{aligned} \tag{4}$$

from studies of the final state interaction in η' production in proton–proton collisions close to the threshold [10]. Theoretical models, in general, prefer a positive sign for the real part of $a_{\eta' p}$.

The mass shift, Eq. (3), is very similar to the expectations of the Quark Meson Coupling model (QMC, for a review, see [72]). QCD inspired models of the η- and η'-nucleus systems are constructed with different selections of "good physics input": how they treat confinement, chiral symmetry and axial U(1) dynamics. In the QMC model, medium modifications are calculated at the quark level through coupling of the light quarks in the hadron to the scalar–isoscalar σ (and also ω and ρ) mean fields in the nucleus. In these calculations, the large η and η' masses are used to motivate taking a MIT Bag description for the meson wave functions. Gluonic topological effects are understood to be "frozen in", meaning that they are only present implicitly through the masses and mixing angle in the model. The strange-quark component of the wave function does not couple to the σ field and η–η' mixing is readily built into the model. Possible binding energies and the in-medium masses of the η and η' are sensitive to the flavour-singlet component in the mesons and hence to the non-perturbative glue associated with axial U(1) dynamics [13].

With an η–η' mixing angle of $-20°$, the QMC prediction for the η' mass in medium at nuclear matter density is 921 MeV, that is a mass shift of -37 MeV. This value is in excellent agreement with the mass shift $-37 \pm 10 \pm 10$ MeV deduced from photoproduction data [12]. Mixing increases the octet relative to singlet component in the η', reducing the binding through the increased strange quark component in the η' wave function. Without the gluonic mass contribution, the η' would be a strange-quark state after η–η' mixing. Within the QMC model, there would be no coupling to the σ mean field and no mass shift so that any observed mass shift is induced by glue associated with the QCD axial anomaly that generates part of the η' mass.

Increasing the flavour-singlet component in the η at the expense of the octet component gives more attraction, more binding and a larger value of the η-nucleon scattering length, $a_{\eta N}$. η–η' mixing with the phenomenological mixing angle $-20°$ leads to a factor of two increase in the mass-shift and in the scattering length obtained in the model relative to the prediction for a pure octet η_8. This result may explain why values of $a_{\eta N}$ extracted from phenomenological fits to experimental data where the η–η' mixing angle is unconstrained [15] give larger values (with real part about 0.9 fm) than those predicted in theoretical coupled channels models where the η is treated as a pure octet state [73, 74].

The QMC model also predicts an effective proton mass of about 755 MeV at nuclear matter density [72] and for the S_{11} an excitation energy of ~ 1544 MeV [13], consistent with observations. For the η' in medium, larger mass shifts, downwards by up to 80–150 MeV, were found in recent Nambu–Jona-Lasinio model calculations (without confinement) [62] and in linear sigma model calculations (in a hadronic basis) [63] which also suggest a rising η effective mass at finite density.

New experiments are looking for possible η' bound states in carbon using the (p,d) reaction at GSI [54] and in photoproduction at ELSA [55] and LEPS2 at SPring-8 [56]. The small η' width in nuclei 20 ± 5.0 MeV at nuclear matter density in Eq. (3) was extracted from measurements of the transparency ratio for η' photoproduction from nuclear targets [16] and suggests the possibility of relatively narrow bound η'-nucleus states accessible to experiments. For clean observation of a bound state, one needs the real part of the optical potential to be much bigger than the imaginary part.

COSY experiments are focussed on possible η bound states in ^{3}He and ^{4}He [17, 51, 52]. The search for a signature of a bound state in the excitation functions for the reactions $dd \to {}^3\mathrm{He}\, p\pi^-$ and $dd \to {}^3\mathrm{He}\, n\pi^0$ below the threshold for the reaction $dd \to {}^4\mathrm{He}\,\eta$ gave a negative result and no narrow structure which could correspond to the ^{4}He–η mesic nucleus was found thus far [75]. However, the new high statistics data collected by the WASA-at-COSY Collaboration for the pd reaction in 2014 gives a hope to observe a sharper state for the ^{3}He–η system. This is because the ^{3}He–η interaction is much stronger than the ^{4}He–η interaction, which may be inferred from the much steeper rise of the total cross section at the threshold for the η meson production via the $pd \to {}^3\mathrm{He}\,\eta$ reaction than via $dd \to {}^4\mathrm{He}\,\eta$. It is expected that in the pessimistic case, the new data will permit us to lower the upper bound for the cross section of the production of the $^3\mathrm{He}\,\eta$, *e.g.* via the $pd \to ({}^3\mathrm{He}\,\eta)_{\mathrm{bound}} \to ppp\,\pi^-$ reaction from the present limit of 270 nb [51] by about an order of magnitude. Such a sensitivity should permit us to reach the range of values of the cross section expected for the creation of the η-mesic ^{3}He [60].

5. η'–π interactions and 1^{-+} exotics

Following the discussion in Section 2, the leading contribution to the decay $\eta' \to \eta\pi\pi$ within the QCD effective Lagrangian approach is associated with the OZI violating interaction $\lambda Q^2 \partial_\mu \pi_a \partial^\mu \pi_a$ [25]. When iterated in the Bethe–Salpeter equation for $\eta'\pi$ rescattering, this interaction yields a dynamically generated resonance with quantum numbers $J^{PC} = 1^{-+}$ and the mass of about 1400 MeV. The generation of this state is mediated by the OZI violating coupling of the η' [18]. One finds a possible dynamical interpretation of the light-mass 1^{-+} exotics observed in experiments at BNL and CERN [76]. This OZI violating interaction will also contribute to higher L odd partial waves with quantum numbers L^{-+}. These states are particularly interesting because the quantum numbers $1^{-+}, 3^{-+}, 5^{-+} \ldots$ are inconsistent with a simple quark–antiquark bound state. The COMPASS experiment at CERN has recently measured exclusive production of $\eta'\pi^-$ and $\eta\pi^-$ in 191 GeV π^- collisions on a hydrogen target [8]. They find the interesting result that $\eta'\pi^-$ production is enhanced relative to $\eta\pi^-$ production by a factor of 5–10 in the exotic $L = 1, 3, 5$ partial waves with quantum numbers L^{-+} in the inspected invariant mass range up to 3 GeV. No enhancement was observed in the even L partial waves. We note also recent calculations where the observed light 1^{-+} states have been interpreted within the Dyson–Schwinger–Bethe–Salpeter framework in a quark–gluon basis [77].

6. Conclusions

Dedicated studies of the η' and its interactions with nucleons, nuclei and other mesons have revealed a rich phenomenology characterised by the OZI violation. Gluonic degrees of freedom play a vital role in generating the η' mass, medium modification of the η' properties including the effective mass shift and in-medium scattering length, as well as driving decay processes involving the η' and dynamical generation of exotic quantum numbers in the $\eta'\pi$ system. Experiments using COSY-11 and at ELSA, GSI and JLab have taught us much about η'-production dynamics from nucleons and nuclei, and the comparison of η' interactions with the corresponding π^0 and η processes.

We thank V. Metag, M. Nanova and K. Suzuki for helpful discussions. This work was supported by the Polish National Science Centre through grant No. 2011/03/B/ST2/01847.

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