MULTIDIMENSIONAL SKYRME-DENSITY-FUNCTIONAL STUDY OF THE SPONTANEOUS FISSION OF $^{238}$U*

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We determined the spontaneous fission lifetime of $^{238}$U by a minimization of the action integral in a three-dimensional space of collective variables. Apart from the mass-distribution multipole moments $Q_{20}$ (elongation) and $Q_{30}$ (left–right asymmetry), we also considered the pairing-fluctuation parameter $\lambda_2$ as a collective coordinate. The collective potential was obtained self-consistently using the Skyrme energy density functional SkM*. The inertia tensor was obtained within the nonperturbative cranking approximation to the adiabatic time-dependent Hartree–Fock–Bogoliubov approach. The pairing-fluctuation parameter $\lambda_2$ allowed us to control the pairing gap along the fission path, which significantly changed the spontaneous fission lifetime.

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This study of spontaneous-fission lifetimes is based on the energy-density-functional (EDF) theory and relies on the collective potential and inertia determined within the adiabatic time-dependent Hartree–Fock–Bogoliubov (ATDHFB) approach. In practical calculations, we use the Skyrme EDF parametrization SkM* [1] and density-dependent pairing. The methodology adopted in this work strictly follows Refs. [2–4].

The ATDHFB inertia is calculated as

$$\mathcal{M}_{ij}^C \equiv \frac{1}{2\dot{q}_i\dot{q}_j} \sum_{\alpha\beta} \left( \frac{F_{i\alpha}^* F_{j\beta} + F_{i\alpha} F_{j\beta}^*}{E_\alpha + E_\beta} \right),$$  \hspace{1cm} (1)

where $\dot{q}_i$ and $\dot{q}_j$ represent time derivatives of the collective coordinates. The sum is evaluated over all quasiparticle states and $E_\alpha$ denotes the quasiparticle energy. Matrices $F^i$ are obtained from

$$-F^{i*} = \left( B^T \frac{\partial \rho}{\partial q_i} A + B^T \frac{\partial \kappa}{\partial q_i} B - A^T \frac{\partial \kappa^*}{\partial q_i} A - A^T \frac{\partial \rho^*}{\partial q_i} B \right) \dot{q}_i,$$  \hspace{1cm} (2)

where $A$ and $B$ are the Hartree–Fock–Bogoliubov (HFB) matrices, obtained self-consistently from the constrained HFB equations. The particle and pairing densities, $\rho$ and $\kappa$ respectively, are determined uniquely from $A$ and $B$.

The total Routhian is

$$H'_{HFB} = \hat{H}_{HFB} - \sum_{l=2,3} q_l \hat{Q}_{l0} - \sum_{\tau=p,n} \left( \lambda_\tau \hat{N}_\tau - \lambda_{2\tau} \left( \hat{N}_\tau^2 - \langle N_\tau^2 \rangle \right) \right),$$  \hspace{1cm} (3)

where $\hat{H}_{HFB}$ is the HFB Hamiltonian, $\hat{Q}_{20}$ and $\hat{Q}_{30}$ are quadrupole and octupole moments, respectively, and $\hat{N}_\tau$ is particle-number operator. The terms associated with $\lambda_{2\tau}$ modify the pairing correlations of the system [2, 5] that can be assessed through the average pairing gaps

$$\Delta_\tau = \frac{\text{Tr}' \hat{\Delta}_\tau \rho_\tau}{\text{Tr} \rho_\tau},$$  \hspace{1cm} (4)

where $\hat{\Delta}_\tau$ is the pairing field and $\text{Tr}' A = \sum_n A_{n\bar{n}}$, with bar over $n$ indicating the time-reversed state.

Calculations presented in this work were performed in a three-dimensional (3D) collective space, where moments $Q_{20}$ and $Q_{30}$ control axial nuclear shapes and $\lambda_2 = \lambda_{2p} = \lambda_{2n}$ allows for simultaneously changing proton and neutron pairing correlations. An early discussion of the effect of pairing fluctuations on fission dynamics was presented, for example, in Refs. [6, 7].
(see Ref. [2] for a comprehensive list of references). Although the potential energy $V$ increases as the pairing gap deviates from the HFB value, the collective inertia behaves as $\sim 1/\Delta^2$ and, therefore, the minimum-action path favors stronger pairing correlations [2].

In this contribution, we carry out a comparative study of $^{238}\text{U}$, assuming axial geometry. The role and importance of other degrees of freedom, such as triaxiality [2], will be discussed elsewhere.

Potential energy surfaces shown in Fig. 1 allow us to study competition between the deformation and pairing effects. It turns out that the pairing fluctuations are more important around the first saddle than in the ground-state energy minimum. As it is shown in Fig. 2, with increasing pairing, the potential energy increases, whereas the mass tensor, in general, decreases. Such a competition significantly affects the fission lifetimes. For example, our 2D calculations (along the $\lambda_2 = 0$ path) yield $T_{\text{SF}} = 2.34 \times 10^{21}$ y, while the 3D calculations including pairing predict $T_{\text{SF}} = 3.63 \times 10^{17}$ y, which is closer to the experimental value of $8.2 \times 10^{15}$ y. This is consistent with findings of recent Refs. [8, 9] based on Gogny–EDF framework.

In summary, we performed a preliminary axial-symmetry study of spontaneous fission of $^{238}\text{U}$, in which pairing fluctuations were treated dynamically by minimizing the collective action. Using the microscopic input based on the ATDHFB approach, we obtained a fair agreement with experiment.
Fig. 2. The quadrupole diagonal inertia (solid line) and potential energy (dashed line) as functions of the pairing-fluctuation parameter $\lambda_2$. The multipole moments ($Q_{20} = 55 \text{ b}$ and $Q_{30} = 0$) correspond to the fission barrier. The vertical line marks the value of $\lambda_2^{\text{opt}}$ that corresponds to the calculated dynamical fission path.

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