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FISSION BARRIER HEIGHTS OF MEDIUM HEAVY AND HEAVY NUCLEI*

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The liquid-drop model which, in addition to the traditional terms, contains the first order curvature energy was successfully used for the description of binding energies of all known isotopes as well as the experimentally measured fission-barrier heights. It was shown that this new Lublin–Strasbourg Drop (LSD) described the available data better than other commonly used macroscopic models.

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1. Introduction

A majority of theoretical estimates of the masses of nuclei which are not far from stability agree generally well with the experimental data. However, the progress made in experimental nuclear physics over the last years to synthesize more exotic nuclei, like the discovery of isotopes close to the proton or neutron drip-lines or the synthesis of super-heavy elements, demands for a more careful checking of the theoretical models and may lead to some revision of their form or parameters.

The analysis made in Ref. [1] was based on the well known *traditional* liquid-drop formula of Myers and Świątecki for the nuclear binding energy (MS–LD) [2]. This model has been quite successful in reproducing nuclear masses, but it has also been known to overestimate the fission-barrier heights by up to about 10 MeV [3] in lighter nuclei. The MS–LD barriers are also higher than those evaluated by Sierk [4] within the Yukawa-folded macroscopic model. The aim of the investigation made in Ref. [1] was to simplify

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the description of the macroscopic part of the total binding energy used in the mass table by Moeller *et al.* [5], namely the Thomas–Fermi model [6]. Rather unexpectedly, a better description of the binding energies of known nuclei was found in [1] with a liquid drop model which in addition to the volume, surface and Coulomb terms contains a first order curvature-correction term, as compared to the one of Ref. [5]. That model known now as the Lublin–Strasbourg Drop (LSD) has simply replaced the Thomas–Fermi part of the nuclear binding energy of [5]. The remaining part of the energy, the so called microscopic energy, is simply taken from the mass tables of Ref. [5].

The aim of the present paper is to show that the LSD model describes well not only the ground-state binding energies but also the saddle-point masses, what proves its universal character. It is, in a way, quite surprising that such a simple liquid-drop type model can so accurately describe the features of nuclei.

2. Nuclear masses

We would like to demonstrate here that the above mentioned LSD model together with the Moeller microscopic corrections [5] is extremely successful in describing many features of nuclei. As compared to the classical liquid-drop model, the LSD contains, in addition, a curvature term proportional to $A^{1/3}$. Its parameters were adjusted to the bindings energies of all 2766 isotopes with proton and neutron numbers larger or equal to 8 which are included in the Strasbourg Chart of Nuclides [7].

The liquid-drop type mass formula used in Ref. [1] consists of the following terms:

$$M(Z,N;\mathrm{def}) = ZM_{\mathrm{H}} + NM_{\mathrm{n}} - b_{\mathrm{elec}}Z^{2.39} + b_{\mathrm{vol}}(1-\kappa_{\mathrm{vol}}I^2)A \tag{1}$$

$$+\, b_{\mathrm{surf}}(1-\kappa_{\mathrm{surf}}I^2)A^{\frac{2}{3}}B_{\mathrm{surf}}(\mathrm{def}) + b_{\mathrm{cur}}(1-\kappa_{\mathrm{cur}}I^2)A^{\frac{1}{3}}B_{\mathrm{cur}}(\mathrm{def}) \tag{2}$$

$$+\, b_{\mathrm{Coul}}\frac{Z^2}{A^{\frac{1}{3}}}B_{\mathrm{Coul}}(\mathrm{def}) + C_4\frac{Z^2}{A} + E_{\mathrm{micr}}(Z,N;\mathrm{def}) + E_{\mathrm{cong}}(Z,N)\,, \tag{3}$$

where $A = N+Z$, $I = (N-Z)/A$ and $b_{\mathrm{Coul}} = 3/5e^2/r_0^{1/3}$ and where B_{surf}, B_{cur}, B_{Coul} are respectively, the relative surface, first-order curvature and Coulomb energy (with respect to the sphere). The constant $M_{\mathrm{H}} = 7.289034\,\mathrm{MeV}$ is the hydrogen and $M_{\mathrm{n}} = 8.071431\,\mathrm{MeV}$ the neutron mass excess, while $b_{\mathrm{elec}} = 1.433\,\mathrm{eV}$ represents the binding energy of electron shells. The microscopic correction consists of shell, pairing and deformation energies and is taken from the mass tables of Moeller *et al.* [5]. The congruence energy introduced in Ref. [5] is of the form

$$E_{\mathrm{cong}} = 10\exp(-4.2|I|)\ \mathrm{MeV}\,. \tag{4}$$

The remaining parameters of Eq. (3) were adjusted in [1] to the experimental masses [7]. The r.m.s. deviation of experimental binding energies versus LSD model predictions is found to be 0.698 MeV, which is smaller than the ones found with other often much more elaborate theories, like the finite-range droplet [8] or the Thomas–Fermi (TF) model of Myers and Świątecki [5]. The fitted values of the eight free parameters are the following:

$$
\begin{array}{lll}
b_{\rm vol} = -15.4920\,{\rm MeV} & b_{\rm surf} = 16.9707\,{\rm MeV} & b_{\rm cur} = 3.8602\,{\rm MeV} \\
\kappa_{\rm vol} = 1.8601 & \kappa_{\rm surf} = 2.2938 & \kappa_{\rm cur} = -2.3764 \\
r_0 = 1.21725\,{\rm fm} & C_4 = 0.9181\,{\rm MeV} &
\end{array}
$$

from where the Coulomb coefficient is found as $b_{\rm Coul} = 0.809177\,{\rm MeV}$.

The differences between the LSD and the experimental masses for all the isotopes of the Strasbourg Chart of Nuclides [7] are presented in Fig. 1. One notices that for the majority of nuclei the absolute value of the difference does not exceed 0.5 MeV. The r.m.s. deviation of the TF estimates of Ref. [5] from the experimental masses is 0.757 MeV, which is 60 keV larger than the deviation corresponding to the LSD approach which indicates that a simple liquid-drop model including a term proportional to the nuclear curvature is able to give a better description of the macroscopic part of the nuclear binding energy. In both models the remaining discrepancy with the experimental data has its origin mostly in the microscopic part of the energy.

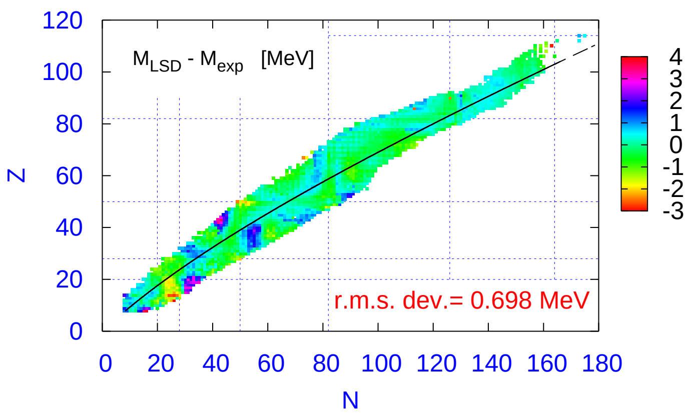

Fig. 1. LSD [1] and experimental [7] mass differences for 2766 known nuclides.

The differences between the theoretical masses obtained within the LSD [1] and the TF [5] models are plotted in Fig. 2 for the tin (left graph) and lead (right graph) chains of isotopes. One notices that within the limits of experimentally known isotopes (vertical dashed lines) both types of estimates differ by less than 0.5 MeV, while beyond these limits the difference increases significantly. There is some hope that future experiments, like those planned at FAIR at GSI or at SPIRAL 2 at GANIL will give an answer which model has a better predictive power.

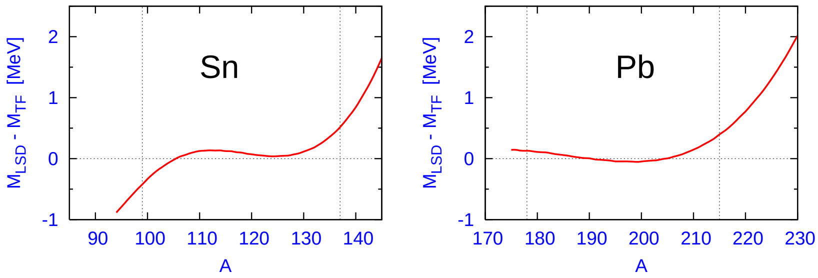

Fig. 2. Differences between the LSD and TF mass estimates of the tin (l.h.s.) and lead (r.h.s.) isotopes.

3. A rough estimates of fission-barrier heights

The fission-barrier heights $V_{\rm sadd}$ obtained with the macroscopic LSD approach and the corresponding experimental data are displayed in Fig. 3 as function of the fissility parameter. According to topographical theorem of Myers and Świątecki [6], one assumes that the fission-barrier hight is approximately equal to the difference between the macroscopic saddle-point mass and the ground-state mass [1]

$$V_{\rm sadd} = M_{\rm LSD}({\rm saddle}) - M_{\rm exp}({\rm g.s.})\,. \tag{5}$$

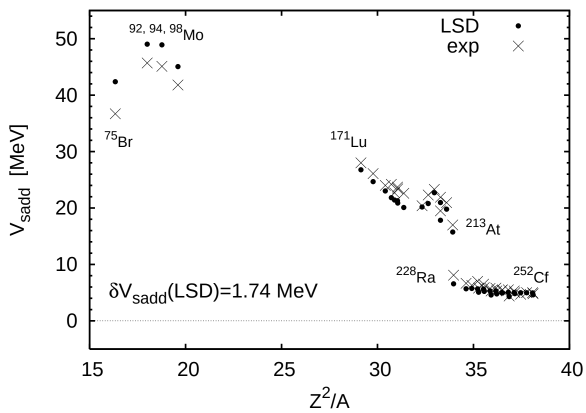

Fig. 3. Theoretical (points) and experimental (crosses) fission barrier heights $V_{\rm sadd}$ as a function of the fissility parameter.

In other words, one can say that, according to this theorem, the net-effect of the shell correction on the height of the saddle-point is, indeed, very small. In addition, a deformation dependence of the congruence energy (4) is taken into account as it was proposed in Ref. [6]. This last effect brings the LSD estimates for the barrier-heights significantly closer to the experimental data in the case of light isotopes ($A \leq 100$), while the fission barriers for heavy nuclei remain practically unchanged and agree well with the data [9]. The root-mean-square deviation of the theoretical barrier heights from experimental ones is found to be 1.74 MeV, but decreases by a factor of two when the four lightest nuclei are disregarded, *i.e.* when only nuclei with $Z > 70$ are considered.

It thus turns out that the liquid-drop model, which in addition to volume, surface and Coulomb energies contains the first order curvature term gives not only a very good description of ground-state masses but also a rather satisfactory prediction of fission-barrier heights. It should be noted that all the parameters of the LSD model were only fitted to the experimental ground-state energies and the correct reproduction of the fission-barrier heights can, therefore, be seen as an additional sign of the intrinsic consistency of the model.

4. Macroscopic–microscopic evaluation of the fission barrier heights

Within the macroscopic–microscopic model the nuclear binding energy consists of the macroscopic part and the shell and pairing corrections. In Ref. [10] the potential energy surface of even–even transactinide nuclei was evaluated using that kind of an approach. The calculation was performed in a four dimensional deformation space using the so-called modified Funny–

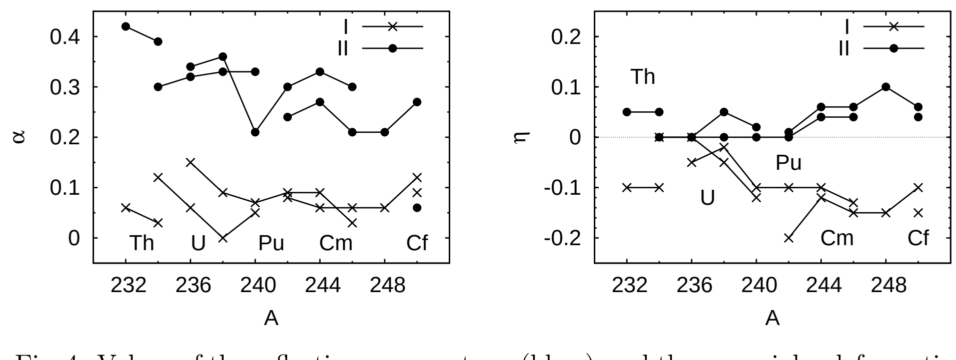

Fig. 4. Values of the reflection asymmetry α(l.h.s.) and the nonaxial η deformation parameters at the first (I) and the second saddle (II) of transactinide even–even nuclei.

Hills parametrisation of Ref. [11] to describe elongation (c), neck formation (h), left–right asymmetry (α) and nonaxiality (η) of fissioning nucleus. The macroscopic part of the binding energy was evaluated within the LSD model [1] and shell and pairing energies were obtained with the Yukawa-folded (YF) potential [3, 5].

The values of the reflection-asymmetry (α) and nonaxial (η) deformation parameters corresponding to the first (I) and the second (II) saddle-points are shown in Fig. 4 for 18 even–even transactinide isotopes for which the barrier heights are measured. It is seen that for a majority of considered nuclei both saddle points correspond to nonaxial and reflection asymmetric shapes. The total energy gain ΔV due to the α and η degrees of freedom on each of the barrier heights is shown in Fig. 5. It is seen that the first barriers are reduced by up to approximately 2 MeV while the reduction of the second barrier is much larger and reaches, in some cases, up to 7 MeV.

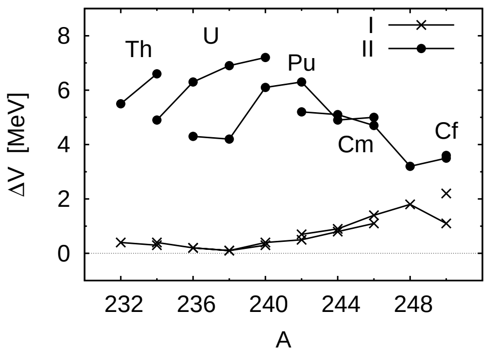

Fig. 5. Reduction of the first (I) and the second (II) saddle due to the reflection-asymmetric and nonaxial degrees of freedom.

It has to be stressed here that a part of this effect has its origin in the non-axial deformation of the *second* saddle. This is a new effect since in a majority of previous macroscopic–microscopic calculations the second saddle was supposed to be axially symmetric. The theoretical LSD+YF fission barrier heights are compared in Fig. 6 with the experimental data. One notices that for the heavier isotopes our approach slightly overestimates (by ≈ 1 MeV) the first as well as the second barrier (by ≈ 1 MeV), while the barrier heights of lighter transactinide nuclei are somewhat underestimated. We believe that the observed small discrepancies between the theoretical predictions of the fission barrier heights and the experimental data have their origin mostly in uncertainties in determining ground-state masses or, more precisely speaking, the magnitude of the shell and pairing corrections at the equilibrium point. To illustrate this point we display in Fig. 7 the difference between

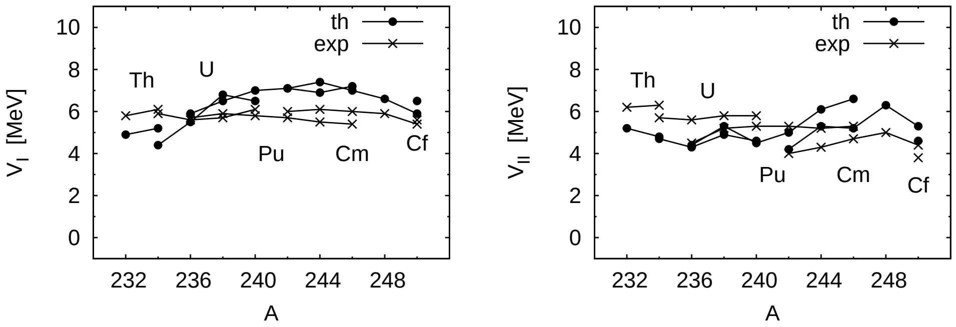

Fig. 6. Theoretical LSD+YF estimates (solid circles) of the first (I) and the second (II) fission barrier heights compared with experimental data (crosses).

the *pure* LSD saddle-point masses and the experimental ground-state masses. It is seen that such a phenomenological Ansatz (based on the topographical theorem [6]) reproduces very accurately the experimental values of the total fission barrier heights (the highest ones). The r.m.s. deviation of these two set of data for all 18 considered isotopes is 0.31 MeV whereas the largest deviation is 0.67 MeV.

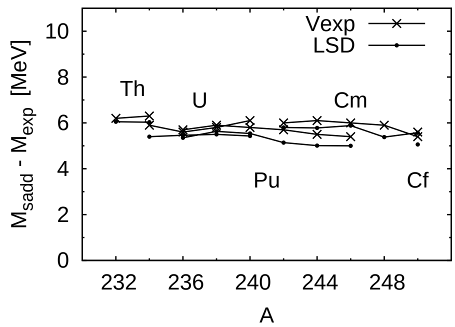

Fig. 7. The LSD barrier heights evaluated using the topographical theorem of Ref. [6] and the experimental ground-state masses, compared to the measured barrier heights $V_{\rm exp}$.

5. Summary and conclusions

The simple Lublin–Strasbourg Drop model describes well the binding energies of all known isotopes and predicts quite accurately the saddle-point masses of fissioning nuclei. We believe that the remaining error in the LSD estimates of the barrier heights (see Fig. 3) is probably due to some slight deficiency in the estimate of the microscopic corrections for the ground-state configuration. One should probably improve the way of evaluating the shell and pairing energy and better adjust the parameters of the single-particle potential and the pairing force. In addition, the recently developed new shell-correction method [12] by averaging in particle number space (instead of smoothing the single-particle energies as in the traditional Strutinsky prescription) predicts deeper minima for spherical nuclei what can change the estimates of the barrier heights of fissioning super-heavy isotopes [13].

It would also be interesting to compare the prediction of the LSD with the data for the neutron rich isotopes. The big discrepancy between the mass predictions in the LSD and Thomas–Fermi models observed in Fig. 2 should lead to different estimates of the fission as well as fusion barriers. In this respect the synthesis of very neutron rich nuclei at new experimental facilities in GANIL or GSI are impatiently awaited.

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