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FLUCTUATIONS IN HEAVY ION COLLISIONS*

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The physics of fluctuations in heavy ion collisions is discussed and a few examples for actual measurements are presented.

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1. Introduction

The study and analysis of fluctuations are an essential method to characterize a physical system. In general, one can distinguish between several classes of fluctuations. On the most fundamental level there are quantum fluctuations, which arise if the specific observable does not commute with the Hamiltonian of the system under consideration. These fluctuations probably play less a role for the physics of heavy ion collisions. Second, there are "dynamical" fluctuations reflecting the dynamics and responses of the system. They help to characterize the properties of the bulk (semi-classical) description of the system. Examples are density fluctuations, which are controlled by the compressibility of the system. Finally, there are "trivial" fluctuations induced by the measurement process itself, such as finite number statistics, *etc.* These need to be understood, controlled and subtracted in order to access the dynamical fluctuations which tell as about the properties of the system.

Fluctuations are also closely related to phase transitions. The well known phenomenon of critical opalescence is a result of fluctuations at all length scales due to a second order phase transition. First order transitions, on the other hand, give rise to bubble formation, *i.e.* density fluctuations at the extreme. Considering the richness of the QCD phase-diagram as sketched in Fig. 1 the study of fluctuations in heavy ions physics should lead to a rich set of phenomena.

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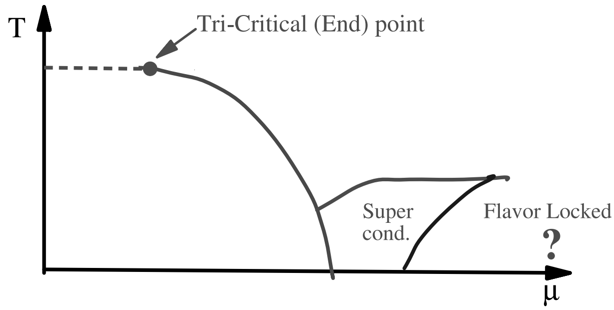

Fig. 1. Schematic picture of the QCD phase-diagram.

The most efficient way to address fluctuations of a system created in a heavy ion collision is via the study of Event-by-Event (E-by-E) fluctuations, where a given observable is measured on an event by event basis and the fluctuations are studied over the ensemble of the events. In most cases (namely when the fluctuations are Gaussian) this analysis is equivalent to the measurement of two particle correlations over the same region of acceptance [1]. Consequently, fluctuations tell us about the 2-point functions of the system, which in turn determine the response of the system to external perturbations.

In the framework of statistical physics, which appears to describe the bulk properties of heavy ion collisions up to RHIC energies, fluctuations measure so called susceptibilities of the system. These susceptibilities also determine the response of the system to external forces. For example, by measuring fluctuations of the net electric charge in a given rapidity interval, one obtains information on how this (sub)system would respond to applying an external (static) electric field. In other words, by measuring fluctuations one gains access to the same fundamental properties of the system as "table top" experiments dealing with macroscopic probes. In the later case, of course, fluctuation measurements would be impossible.

In addition, the study of fluctuations may reveal information beyond its thermodynamic properties. If the system expands, fluctuations may be frozen in early and thus tell us about the properties of the system prior to its thermal freeze out. A well known example are the fluctuations in the cosmic microwave background radiation, as first observed by COBE [2].

The field of Event-by-Event fluctuations is relatively new to heavy ion physics and ideas and approaches are just being developed. So far, most of the analysis has concentrated on transverse momentum and charge fluctuations.

Transverse momentum fluctuations should be sensitive to temperature/energy fluctuations [3, 4]. These in turn provide a measure of the heat capacity of the system [5]. Since the QCD phase transition is associated with a maximum of the specific heat, the temperature fluctuations should exhibit a minimum in the excitation function. It has also been argued [6, 7] that these fluctuations may provide a signal for the long range fluctuations associated with the tri-critical point of the QCD phase diagram. In the vicinity of the critical point the transverse momentum fluctuations should increase, leading to a maximum of the fluctuations in the excitation function.

Charge fluctuations [8, 9], on the other hand, are sensitive to the fractional charges carried by the quarks. Therefore, if an equilibrated partonic phase has been reached in these collisions, the charge fluctuations per entropy would be about a factor of 2 to 3 smaller than in an hadronic scenario.

2. Fluctuations in a thermal system

To a good approximation the system produced in a high energy heavy ion collision can be considered to be close to thermal equilibrium. Therefore let us first review the properties of fluctuations in a thermal system. Most of this can be found in standard textbooks on statistical physics such as *e.g.* [5] and we will only present the essential points here.

Typically one considers a thermal system in the grand-canonical ensemble. This is the most relevant description for heavy ion collisions since usually only a part of the system — typically around mid-rapidity — is considered. Thus the exchange of energy and conserved quantum numbers with the rest of the system, which serves as a heat-bath, is possible. There are, however, important exceptions when the number of conserved quanta is small. In this case an explicit treatment of these conserved charges is required, leading to a canonical description of the system [10] and to significant modifications of the fluctuations, as we shall dicuss below. Let us first discuss fluctuations based on a grand canonical ensemble and then later point out the differences if a canonical treatment is called for.

2.1. Fluctuations in a grand canonical ensemble

Assuming we are dealing with a system with one conserved quantum number (such as the electric charge, baryon number *etc.*) the grand canonical partition function is given by[1]

$$Z = \sum_{\mathrm{states}\, i} \left\langle i \right| \exp(-\beta(\hat{H} - \mu\hat{Q})) \left| i \right\rangle \equiv \mathrm{Tr}\ \left[\exp(-\beta(\hat{H} - \mu\hat{Q})) \right] , \qquad (1)$$

[1] We restrict ourselves to one conserved charge. Of course the are several conserved quantum numbers for a heavy ion collision. The extension of the formalism to multiple conserved charges is straightforward.

where $\beta = 1/T$ represents the temperature T of the system and Q is the conserved charge under consideration. Here the sum covers a complete set of (many particle) states. The relevant free energy F is related to the partition function via

$$F = -T \log Z \,. \tag{2}$$

For a thermal system, typical fluctuations are Gaussian [5] and are characterized be the mean dispersion

$$\left\langle (\delta X)^2 \right\rangle \equiv \left\langle X^2 \right\rangle - \left\langle X \right\rangle^2 \,, \tag{3}$$

with

$$\begin{aligned} \langle X \rangle &= \frac{1}{Z} \mathrm{Tr} \left[\hat{x} \exp(-\beta(\hat{H} - \mu \hat{Q})) \right] \equiv \mathrm{Tr}\left[\hat{x} \hat{\rho} \right], \\ \left\langle X^2 \right\rangle &= \frac{1}{Z} \mathrm{Tr} \left[\hat{x}^2 \exp(-\beta(\hat{H} - \mu \hat{Q})) \right] \equiv \mathrm{Tr}\left[\hat{x}^2 \hat{\rho} \right], \end{aligned} \tag{4}$$

where we have introduced the statistical operator

$$\hat{\rho} \equiv \frac{1}{Z} \exp(-\beta(\hat{H} - \mu \hat{Q})) = \exp(-\beta(\hat{H} - \mu \hat{Q} - F)) \,. \tag{5}$$

In the following, $\langle \ldots \rangle$ will always refer to thermal averages if not noted otherwise.

In particular fluctuations of quantities which characterize the thermal system, such as the energy or the conserved charges, can be expressed in terms of appropriate derivatives of the partition function. Of special interest in the context of heavy ion collisions are energy/temperature fluctuations, which are often related to the fluctuations of the transverse momentum as well as electric charge/baryon number fluctuations.

2.1.1. Fluctuations of the energy and of the conserved charges

As already pointed out at the beginning of this chapter when analyzing the system created in a heavy ion collisions, one usually studies only a small subsystem around mid rapidity. In a statistical framework, this situation is best represented by a grand canonical ensemble, where the exchange of conserved quantum number with the rest of the system is taken into account. The equilibrium state is then characterized by the appropriate conjugate variables, namely the temperature and the chemical potentials for the energy and the conserved quantities, respectively.

As a consequence, energy as well as the conserved charges may fluctuate and the size of fluctuations reveals additional properties of the matter, the so called susceptibilities, which characterize the response of the system to external forces.

For example, the fluctuation of the conserved charge in the subsystem under consideration is given by

$$\left\langle (\delta Q)^2 \right\rangle = T^2 \frac{\partial^2}{\partial \mu^2} \log Z = -T \frac{\partial^2}{\partial \mu^2} F\,. \tag{6}$$

Similarly, fluctuations of the energy can be expressed in terms of derivatives of the partition function with respect to the temperature

$$\left\langle (\delta E)^2 \right\rangle = \frac{\partial^2}{\partial \beta^2} \log Z = -T^3 \frac{\partial^2}{\partial T^2} F = T^2 C_V\,. \tag{7}$$

Note that the energy fluctuations are proportional to the heat capacity of the system. Thus one would expect that these fluctuations obtain a maximum as the system moves through the QCD phase-transition, which, among others, is characterized by a maximum of the heat capacity.

An alternative way [3,4] is to study temperature fluctuations, which are inversely proportional to the heat capacity [5]

$$\left\langle (\delta T)^2 \right\rangle = \frac{T^2}{C_V}\,. \tag{8}$$

However, for situation at hand, which is best described by a grand canonical ensemble, energy fluctuations appear to be the more appropriate observable.

The second derivatives of the free energy, which characterize the fluctuations, are usually referred to as susceptibilities. Thus we have the charge susceptibility

$$\chi_Q = -\frac{1}{V} \frac{\partial^2}{\partial \mu^2} F \tag{9}$$

and the "energy susceptibility"

$$\chi_E = -\frac{1}{V} \frac{\partial^2}{\partial T^2} F = c_v\,, \tag{10}$$

which is usually referred to as the specific heat. And just as the specific heat determines the response of the (sub)system to a change of temperature the charge susceptibility characterizes the response to a change of chemical potential. In case of electric charge, this would be the response to an external electric field. Consequently, by measuring the fluctuations one obtains information about some fundamental properties of the system, the susceptibilities, *i.e.* the responses to external forces.

In thermal field theory these susceptibilities are given by correlation functions of the appropriate operators. For example the charge susceptibility is given by the space-like limit of the static time–time component of the electromagnetic current–current correlator [11–14]

$$\chi_Q = \Pi_{00}(\omega = 0, q \to 0)\,, \tag{11}$$

with

$$\Pi_{\mu\nu}(k) = i \int d^4x \exp(-ikx)\, \langle \hat{\rho} T^*(j_u(x) j_u(0)) \rangle\ . \tag{12}$$

It is interesting to note, that the charge susceptibility is directly proportional to the electric mass [11]

$$m_{\mathrm{el}}^2 = -e^2 \chi_Q\,. \tag{13}$$

Equation (11) allows to calculate the electric mass in any given model (see *e.g.* [13–17] and in particular in Lattice QCD [18, 19]. Since dilepton and photon production rates are given in terms of the imaginary part of the same current–current correlation function — taken at different values of ω and q — model calculations for these processes will also give predictions for the charge susceptibility, which then can be compared with lattice QCD results. As shown in [13] an extraction of the χ_Q from Dilepton data via dispersion relations, however, is not possible. For that one needs also information for the space-like part Π_{00} which is not easily accessible by heavy ion experiments.

Charge fluctuations are of particular interest to heavy ion collisions, since they provide a signature for the existence of a de-confined Quark Gluon Plasma phase [8, 9]. Let us, therfore, discuss charge fluctuations in more detail.

Consider a classical ideal gas of positively and negatively charged particles of charge $\pm q$. The fluctuations of the total charge contained in a subsystem of N particles is then given by

$$\begin{aligned} \left\langle (\delta Q)^2 \right\rangle &= q^2 \left\langle (\delta(N_+ - N_-))^2 \right\rangle \\ &= q^2 \left[\left\langle (\delta N_+)^2 \right\rangle + \left\langle (\delta N_-)^2 \right\rangle - 2 \left\langle \delta(N_- N_+) \right\rangle \right]\ . \end{aligned} \tag{14}$$

Since correlations are absent in an ideal gas, $\langle \delta(N_- N_+) \rangle = 0$. Furthermore, for a classical ideal gas

$$\left\langle (\delta N)^2 \right\rangle = \langle N \rangle \tag{15}$$

and, therefore,

$$\left\langle (\delta Q)^2 \right\rangle = q^2 \left\langle N_+ + N_- \right\rangle = q^2 \left\langle N_{\mathrm{ch}} \right\rangle\ , \tag{16}$$

where $N_{\rm ch} = N_+ + N_-$ denotes the total number of charged particles. Taking quantum statistics into account modifies the results somewhat, since the number fluctuations are not Poisson anymore (see *e.g.* [20])

$$\left\langle \delta(N)^2 \right\rangle = \langle N \rangle \left(1 \pm \int \frac{d^3p}{(2\pi)^3} n_\pm(p) \right) \equiv \omega_\pm \langle N \rangle \ . \tag{17}$$

Here, $(+)$ refers to Bosons and $(-)$ to Fermions, and $n_\pm(p)$ represents the respective single particle distribution functions. For the temperatures and densities reached in heavy ion collisions, however, the corrections due to quantum statistics are small. For a pion gas at temperature $T = 170\,{\rm MeV}$ $\omega_\pi = 1.13$ [21].

Obviously, charge fluctuations are sensitive to the *square* of the charges of the particles in the gas. This can be utilized to distinguish a Quark Gluon Plasma, which contains particles of fractional charge, from a hadron gas where the particles carry unit charge. Charge fluctuations per particle should be smaller in a Quark Gluon Plasma than in a hadron gas. The appropriate observable to study is the charge fluctuations per entropy. To illustrate this point let us consider a noninteracting pion gas and Quark–Gluon gas in the classical approximation. Corrections due to quantum statistics and due to the presence of resonances are discussed in detail in [8, 9, 22]. In a neutral pion gas the charge fluctuations are due to the charged pions, which are equally abundant

$$\left\langle (\delta Q)^2_{\pi-{\rm gas}} \right\rangle = \langle N_{\pi^+} \rangle + \langle N_{\pi^-} \rangle \ , \tag{18}$$

whereas in a Quark Gluon Plasma the quarks and anti-quarks are responsible for the charge fluctuations

$$\left\langle (\delta Q)^2 \right\rangle_{\rm QGP} = Q_u^2 \langle N_u \rangle + Q_d^2 \langle N_d \rangle = \frac{5}{9} \langle N_q \rangle \ , \tag{19}$$

where $N_q = N_u = N_d$ denotes the number of quarks *and* anti-quarks. For a classical ideal gas of mass-less particles the entropy is given by

$$S = 4 \langle N \rangle \tag{20}$$

and thus we have for a pion gas

$$S_{\rm pion-gas} = 4 \left(\langle N_{\pi^+} \rangle + \langle N_{\pi^-} \rangle + \langle N_{\pi^0} \rangle \right) , \tag{21}$$

and for a Quark Gluon Plasma

$$S_{\rm QGP} = 4 \left(\langle N_u \rangle + \langle N_d \rangle + \langle N_g \rangle \right) = 4 \left(2 \langle N_q \rangle + \langle N_g \rangle \right) \ , \tag{22}$$

where N_g denotes the number of gluons. Therefore, the ratio of charge fluctuation per entropy in a pion gas is

$$\frac{\left\langle (\delta Q)^2 \right\rangle}{S}_{\rm pion-gas} = \frac{1}{6}\,, \tag{23}$$

whereas for a 2-flavor Quark Gluon Plasma it is

$$\frac{\left\langle (\delta Q)^2 \right\rangle}{S}_{\rm QGP} = \frac{1}{24}\,. \tag{24}$$

Consequently, the charge fluctuations per degree of freedom in a Quark Gluon Plasma are a factor of four smaller than in a pion gas. Hadronic resonances, which constitute a considerable fraction of a hadron gas reduce the result for the pion gas by about 30 % [9,22], leaving still a factor 3 signal for the existence of the Quark Gluon Plasma.

The above ratio, $\left\langle (\delta Q)^2 \right\rangle / S$ can also be calculated using lattice QCD [18], and, above the critical temperature, the value agree rather well with that obtained in our simple Quark Gluon Plasma model here [9]. More recent calculations for the charge susceptibility [19] give a somewhat smaller value, which would make the observable even more suitable.

Unfortunately, present lattice calculation are not available for this ratio below the critical temperature. Here one has to resort to hadronic model calculations. This has been done in [14,15] using either a virial expansion, a chiral low energy expansion or an explicit diagrammatic calculation. In all cases, the ratio is slightly increased due to interactions, thus enhancing the signal for the Quark Gluon Plasma.

The question then remains, how to measure this ratio in an actual experiment. This has been discussed in [9], where is was proposed to study the fluctuations of the ratio of positively over negatively charged particle

$$D = \langle N_{\rm ch} \rangle \left\langle \left(\delta \frac{N^+}{N^-} \right)^2 \right\rangle \simeq 4\, \frac{\left\langle (\delta Q)^2 \right\rangle}{\langle N_{\rm ch} \rangle} \sim \frac{\left\langle (\delta Q)^2 \right\rangle}{S}\,. \tag{25}$$

For a pion gas, $D_{\rm pion-gas} = 4$ whereas for a QGP, $D_{\rm QGP} \simeq 1$–1.5, where the uncertainty arises from relating the entropy S with the number of charged particles $\langle N_{\rm ch} \rangle$. Hadronic resonances introduce additional correlations, which reduce the value of the pion gas to $D_{\rm hadron-gas} \simeq 3$, but still a factor of 2 larger then the value for the QGP.

The key question of course is, how can these reduced fluctuations be observed in the final state which consists of hadrons. Should one not expect that the fluctuations will be those of the hadron gas? The reason, why it should be possible to see the charge fluctuations of the initial QGP has to do with the fact that charge is a conserved quantity. Imagine one measures in each event the net charge in a given rapidity interval Δy such that

$$\Delta y_{\rm coll} \ll \Delta y \ll \Delta y_{\rm max}\,, \tag{26}$$

where $\Delta y_{\rm max}$ is the width of the total charge distribution and $\Delta y_{\rm coll}$ is the typical rapidity shift due to hadronization and re-scattering in the hadronic phase. If, as it is expected, strong longitudinal flow develops already in the QGP-phase, the number of charges inside the rapidity window Δy for a given event is essentially frozen in. And if $\Delta y \gg \Delta y_{\rm coll}$ neither hadronization nor the subsequent collisions in the hadronic phase will be very effective to transport charges in and out of this rapidity window. Thus, the E-by-E charge-fluctuations measured at the end reflect those of the initial state, when the longitudinal flow is developed. Ref. [23] arrives at the same conclusion on the basis of a Fokker–Planck type equation describing the relaxation of the charge fluctuation in a thermal environment.

In Fig. 2 we show the results of an URQMD calculation [24], where the variable D is plotted versus the size of the rapidity window Δy. For large Δy the results have to be corrected for charge conservation effects; if all charges are accepted, global charge conservation leads to vanishing fluctuations (open symbols in Fig. 2). This can be easily corrected for (for details see [24]). The resulting values for D are shown as full symbols in Fig. 2. They agree nicely with the prediction for the resonance gas,

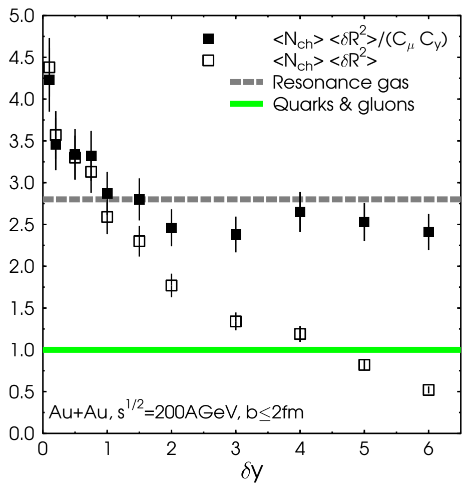

Fig. 2. Charge fluctuations as for different rapidity windows. Open symbols: without correction for global charge conservation; full symbols: with correction for global charge conservation.

as the should, since the URQMD model does not contain any partonic degrees of freedom. For small Δy the correlations imposed by the resonances are lost, because only one of the decay products is accepted. As a result we see an increase of D. For very small Δy, when $\langle N \rangle \simeq 1$, the ratio D is not well defined for events with $\langle N_- \rangle = 0$, and, therefore, cannot serve as a observable. Alternative observables, measuring the same quantity have been proposed and studied in [25].

First results from experiments have been reported. Phenix [26], which measures with a small rapidity acceptance, finds charge fluctuations consistent with a resonance gas. Also, CERES, NA49 as well as STAR have reported preliminary results [27]. All are consistent with either a pion gas or a resonance gas. No indication for a QGP so far. These findings have prompted ideas, that possibly a constituent quark plasma, without gluons, has been produced [28]. However, the measurement of additional observables would be needed in order to distinguish this from a hadronic gas.

But maybe the present range of Δy is so small, that the charge fluctuations have time to assume the value of the resonance gas. Thus a detailed analysis of D as a function of Δy is needed, before any firm conclusions can be drawn.

2.2. Fluctuations in a canonical ensemble

As pointed out in the beginning of this chapter, once the number of conserved quanta is small, *i.e.* of the order of one per event, the grand canonical treatment, where charges are conserved only on the average, is not adequate anymore. Instead the description needs to ensure that the quantum number is conserved explicitly in each event. Since the deposited energy is still large and is distributed over many degrees of freedom, the canonical ensemble is the ensemble of choice.

Obviously the fluctuations of the energy is identical to the grand canonical ensemble, but fluctuations of particles which carry the conserved charge are affected. As an example, let us consider the fluctuations of Kaons in low energy heavy ion collisions. At 1–2 AGeV bombarding energy, only very few kaons are being produced $\langle N_K \rangle \simeq 0.1$ [29], which makes an explicit treatment of strangeness conservation necessary. For simplicity, let us consider Kaons and Lambdas/Sigmas[2] as the only particles carrying strangeness. In the canonical ensemble, where strangeness is conserved explicitely, the partition function is given by [10]

[2] In the following we will denote both Lambdas and Sigmas as Lambdas, but include the appropriate degeneracy factor to take the sigmas into account.

$$Z_c = Z^0_{\rm rest} \sum_{n=0}^{\infty} \frac{1}{n!^2} \left(Z^0_K Z^0_\Lambda \right)^n \; , \tag{27}$$

where $Z^0_{\rm rest}$ is the standard (grand canonical) partition for all the other non strange particles and Z^0_K , Z^0_Λ are the single particle partition functions for Kaons and Lambdas, respectively,

$$Z^0_K \; = \; 2V \int \frac{d^3p}{(2\pi)^3} \exp(-\beta E_K) = N^0_K \, , \tag{28}$$

$$Z^0_\Lambda \; = \; 4V \int \frac{d^3p}{(2\pi)^3} \exp(-\beta(E_\Lambda - \mu_B) = N^0_\Lambda \, . \tag{29}$$

Here, the degeneracy factor of $d = 2,4$ for Kaons and Lambdas, respectively, take into account the presence of the K^0 and Σ particles. Note that Z^0 is simply the number of particles in the grand canonical ensemble in the limit of vanishing strange chemical potential. Given the above partition function, the probability P_n to find "n" Kaons is given by [30–32]

$$P_n = \frac{\varepsilon^n}{I_0(2\sqrt{\varepsilon})(n!)^2} \, , \tag{30}$$

where I_0 is the modified Bessel function and

$$\varepsilon = Z^0_K Z^0_\Lambda \, . \tag{31}$$

Given the probabilities Eq. (30), one can easily calculate the fluctuations

$$\begin{aligned} \langle N_K \rangle \; &= \; \sqrt{\varepsilon} \frac{I_1(2\sqrt{\varepsilon})}{I_0(2\sqrt{\varepsilon})} \, , \\ \left\langle N_K^2 \right\rangle \; &= \; \varepsilon \, , \end{aligned} \tag{32}$$

so that the second factorial moment

$$F_2 \equiv \frac{\langle N(N-1) \rangle}{\langle N \rangle^2} = \frac{1}{2} + \frac{\varepsilon}{6} + \ldots = \frac{1}{2} + \frac{\langle N_K \rangle}{6} + \ldots \, . \tag{33}$$

This is to be contrasted with the grand canonical result, which follows in the limit of $\varepsilon \gg 1$. In this case,

$$\left\langle N^2 \right\rangle_{\rm g.c.} - \langle N \rangle_{\rm g.c.} = \langle N \rangle^2_{\rm g.c.} \; , \tag{34}$$

so that

$$F_{2,{\rm g.c.}} = 1 \, . \tag{35}$$

Thus for $\langle N \rangle \ll 1$ the effect of explicit strangeness conservation reduces the second factorial moment by almost a factor of two. This suppression of the factorial moment due to explicit charge conservation can be utilized to measure the degree of equilibration reached in these collisions.

In Fig. 3, the time evolution of F_2 is shown for several initial kaon numbers. In all cases, F_2 quickly rises close to $F_2 \simeq 1$ before it settles at the final equilibrium value of $F_2^{\rm eq} \simeq 1/2$. Thus, by measuring F_2 one can directly determine how close to chemical equilibrium the system has developed, before it freezes out. In principle a similar measurement can also be done at higher energies for charmed mesons. To which extent this is technically feasible is another question.

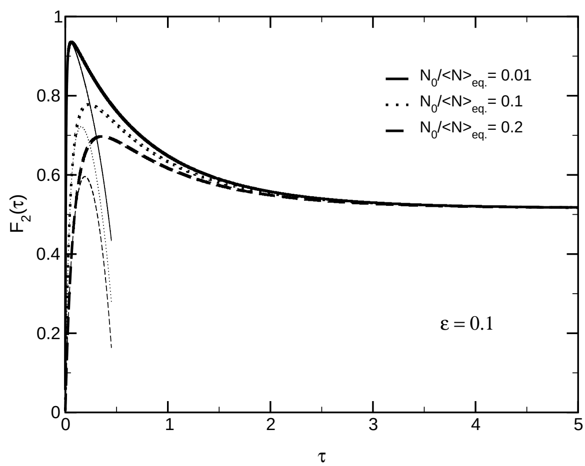

Fig. 3. Time evolution of F_2 for various initial kaon numbers.

2.3. Phase transitions and fluctuations

As already mentioned in the introduction, the QCD phase diagram is expected to be rich in structure. Besides the well known and much studied transition at zero chemical potential, which is most likely a cross over transition, a true first order transition is expected at finite quark number chemical potential. It has been argued [6] that the phase separation line ranges from zero temperature and large chemical potential to finite temperature and smaller chemical potential where it ends in a critical end-point (E) (see Fig. 1). Here the transition is of second order. There is also a first attempt to determine the position of the critical point in Lattice QCD [33].

As discussed in [6], the associated mass-less mode carries the quantum numbers of σ-meson, *i.e.* scalar iso-scalar, whereas the pions remain massive due to explicit chiral symmetry breaking as a result of finite current quark masses.

Fluctuations are a well known phenomenon in the context of phase transitions. In particular, second order phase transitions are accompanied by fluctuations of the order parameter at all length scales, leading to phenomena such as critical opalescence [5]. However, since the system generated in a heavy ion collision expands rather rapidly, critical slowing down, another phenomenon associated with a second order phase transition, will prevent the long wavelength modes to fully develop. In [34] these competing effects have been estimated and authors arrive at a maximum correlation length of about $\xi \simeq 3\,\mathrm{fm}$ if the system passes through the critical (end) point of the QCD phase diagram.

In [7] the authors argue that if the system freezes out close to the critical end-point, the long range correlations introduced by the mass less σ-modes lead to large fluctuations in the pion number at small transverse momenta. In the thermodynamic limit, this fluctuations would diverge, but in a realistic scenario, where the long wavelength modes do not have time to fully develop, the fluctuations a limited by the correlation length. In [34] it is estimated that a correlation length of $\xi \simeq 3\,\mathrm{fm}$ will result in $\sim$ 5–10 % increase in fluctuations of the mean transverse momentum, which should be observable with present day large acceptance detectors such as STAR and NA49. Since the precise position of the critical point is not well known, what is needed is a measurement of the excitation function of these fluctuations.

If the system undergoes a first order phase transition, bubble formation may occur. Since each bubble is expected to decay in many particles this leads to large multiplicity fluctuations in a given rapidity interval [35, 36].

Fluctuations of particle ratios, on the other hand, should be reduced due to the correlations induced by bubble formation [22].

3. Conclusions

In this contribution we have discussed in some detail the physics of fluctuations in the context of heavy ion collisions. As this is a developing field, this should be considered as a snapshot of our present understanding rather then a balanced review. We have argued that fluctuations are indeed a new tool to investigate the properties of the matter created in these collisions. As an example we have shown how charge fluctuations can be utilized to detect the presence of a Quark Gluon Plasma. The fluctuations of kaons at low energy collisions, on the other hand, may help us to pin down the question of equilibration in these systems.

With the availability of large acceptance detectors, the measurement of many fluctuating quantities will become possible providing novel insights into the properties of the systems created in these collisions.

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