PRODUCTION OF GLUON JETS IN *PP* COLLISIONS BY DOUBLE POMERON EXCHANGE IN THE LANDSHOFF–NACHTMANN MODEL

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Using the Landshoff–Nachtmann two-gluon exchange model of the pomeron, the double pomeron exchange contribution to production of gluon pairs in the central region of rapidity is calculated. The results are compared with those for production of quark–antiquark pairs.

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1. Introduction

Production of the Higgs boson by double pomeron exchange was recently studied by several authors [1]. Unfortunately the published theoretical estimates of the cross-sections are widely different and thus do not permit to obtain reliable predictions needed for future experiments. This reflects our present limited understanding of the nature of the diffractive (pomeron) reactions.

One way to reduce this ambiguity is to calculate the cross-section for other double pomeron exchange processes and compare them with existing data. In the present paper we study this problem using the Landshoff–Nachtmann model of the pomeron [2] to calculate the production of two large transverse momentum gluon jets shown in Fig. 1. In this way we hope to obtain a better estimate of the Higgs production studied in this model some time ago [3].

Our calculations follow closely the method used in [4,5] where the cross-section for production of heavy quark–antiquark pairs was calculated. The results for two gluon jets, together with those for quark jets, allow to obtain the full cross-section for double-diffractive jet production in the Landshoff–Nachtmann model.
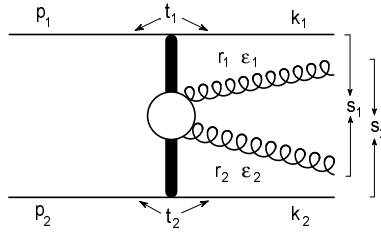
Fig. 1. Production of two gluons by double pomeron exchange.

2. The relevant diagrams

In the Landshoff–Nachtmann model [2] the pomeron is described as an exchange of two "nonperturbative" gluons, which takes place between a pair of quarks of colliding hadrons. Thus to calculate the cross-section for reaction $pp \longrightarrow pp + gg$ by double pomeron exchange we have to take into account twelve diagrams presented in Fig. 2.
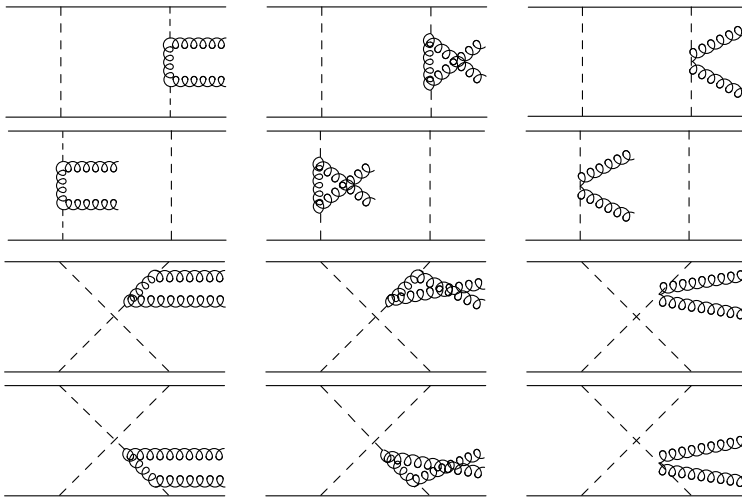

Fig. 2. The diagrams which have to be taken into account in the process of the gluon pair production by double pomeron exchange. The dashed lines represent the exchange of the nonperturbative gluons.

Other diagrams, like these presented in Fig. 3, either do not contribute significantly because of large transverse momentum on one of the quark internal lines or simply vanish because pomeron is the color singlet.
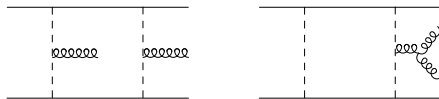
Fig. 3. The examples of the diagrams which do not contribute.

Following [3] we assume that up to the highest powers of s/s_1 and s/s_2, the total amplitude given by diagrams in Fig. 2 is of the form:

$$M_{fi} = M_{fi}|_{t_1=t_2=0} \left(\frac{s}{s_1}\right)^{\alpha_2-1} \left(\frac{s}{s_2}\right)^{\alpha_1-1} . \tag{1}$$

Here $\alpha_1 = \alpha(t_1)$, $\alpha_2 = \alpha(t_2)$ and $\alpha(t) = 1 + \epsilon + \alpha't$ is the pomeron Regge trajectory with $\epsilon \approx 0.08$, $\alpha' = 0.25$ GeV^{-2} (s_1, s_2, t_1, t_2 are marked in Fig. 1). So, in order to calculate the amplitude for production of two gluons by double pomeron exchange, it is enough to take diagrams from Fig. 2 at $t_1 = t_2 = 0$.

Even with this simplification, however, the direct evaluation of all diagrams of Fig. 2 is still tedious. Fortunately, as shown in [6] and exploited in [3], it turns out that up to terms of the order ϵ, the matrix element $M_{fi}|_{t_1=t_2=0}$ is equal to the sum of the following three diagrams:
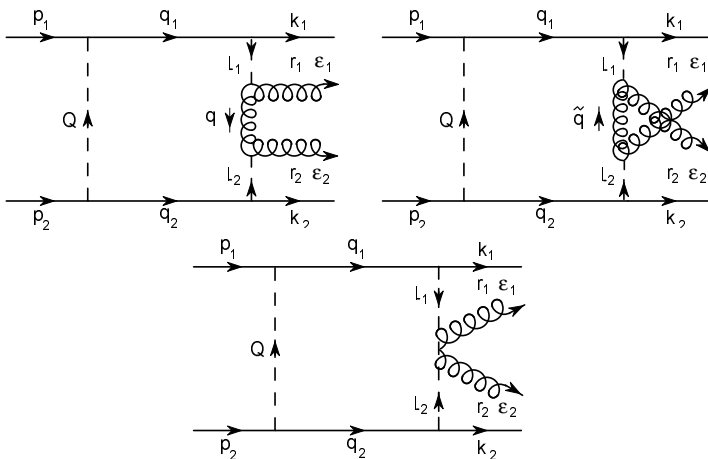

Fig. 4. Putting the quarks lines q_1 and q_2 on shell is equivalent to calculating the amplitude for production of gluon pair by double pomeron exchange at $t_1 = t_2 = 0$.

where the lines q_1 and q_2 are put on shell and $t_1 = t_2 = 0$.

The calculation is performed in two steps. First, we calculate $M_{fi}|_{t_1=t_2=0}$ for colliding quarks, given by the sum of the diagrams presented in Fig. 4 and use (1) to introduce the Regge behavior. To obtain the cross-section the presence of three quarks in the proton must be taken into account.

3. Matrix element

We take the masses of the light quarks to be approximately zero and, following [3], use the Sudakov parametrization for momenta shown in Fig. 4:

$$Q = \frac{x}{s}p_1 + \frac{y}{s}p_2 + v,$$

$$k_1 = x_1 p_1 + \frac{y_1}{s}p_2 + v_1,$$

$$k_2 = \frac{x_2}{s}p_1 + y_2 p_2 + v_2,$$

$$r_2 = x_g p_1 + y_g p_2 + v_g, \tag{2}$$

where v, v_1, v_2, v_g are two-dimensional four-vectors.

Let us express $M_{fi}|_{t_1=t_2=0}$ as the following sum:

$$M_{fi}|_{t_1=t_2=0} \equiv M_{fi}^{(3)} + M_{fi}^{(4)}, \tag{3}$$

where $M_{fi}^{(3)}$ is the sum of the first two diagrams presented in Fig. 4, and $M_{fi}^{(4)}$ being last diagram in Fig. 4. Following closely the method used in [5] we obtain (see Appendix):

$$M_{fi}^{(4)} = -A\frac{4}{12}\delta_{s,c}^{if}\delta_{uw}\pi\tau s\varepsilon_1\varepsilon_2, \tag{4}$$

$$M_{fi}^{(3)} = -A\frac{4}{12}\delta_{s,c}^{if}\delta_{uw}\int d^2\vec{v}f(\vec{v})\exp\left(\frac{-\vec{v}^2}{\tau}\right), \tag{5}$$

where:

$$f(\vec{v}) \equiv \frac{1}{q^2}\Big(s^2\varepsilon_1\varepsilon_2\left(\delta_2 - y_g\right)x_g - 2s\varepsilon_1 v\varepsilon_2 v$$

$$+2s\varepsilon_1 v\varepsilon_2 p_2\left(\delta_2 - y_g\right) - 2s\varepsilon_2 v\varepsilon_1 p_2\left(\delta_2 - y_g\right)\Big)$$

$$+\frac{1}{\bar{q}^2}\left(s^2\varepsilon_1\varepsilon_2\left(\delta_1 - x_g\right)y_g - 2s\varepsilon_1 v\varepsilon_2 v + 2s\varepsilon_2 v\varepsilon_1 p_2 y_g - 2s\varepsilon_1 v\varepsilon_2 p_2 y_g\right). \tag{6}$$

Here $A = i(D_0 G^2)^3 g^2/(2\pi)^2 G^2$, G and g are the non-perturbative and perturbative quark–gluon couplings, $\tau = \mu^2/(1+x_1+y_2)$. $D(p^2) = D_0\exp\left(p^2/\mu^2\right)$ with $\mu \approx 1$ GeV and $G^2 D_0 \approx 30$ GeV$^{-1}\mu^{-1}$, is the non-perturbative gluon propagator. By δ_1 and δ_2 we denote $1-x_1$ and $1-y_2$,

respectively. The indices u and w denote the color of the produced gluons. ε_1 and ε_2 are the polarization vectors of gluons chosen to satisfy the relations:

$$\sum_{\lambda=1,2} \varepsilon_i^\mu(\lambda) \varepsilon_i^\nu(\lambda) = -g^{\mu\nu} + \frac{p_1^\mu r_i^\nu + p_1^\nu r_i^\mu}{p_1 r_i} \qquad i = 1,2. \qquad (7)$$

So that, in addition to the constraints $\varepsilon_1 r_1 = \varepsilon_2 r_2 = 0$, we have further constraints: $\varepsilon_1 p_1 = \varepsilon_2 p_1 = 0$.

Performing the integration in (5) is a rather difficult task. Following [5] we expand the integral over $d^2\vec{v}$ in power series:

$$\int d^2\vec{v} f(\vec{v}) \exp\left(\frac{-\vec{v}^2}{\tau}\right) = a\pi\tau + \sum_j \frac{\pi\tau^2}{2} a_{jj} + \dots, \qquad (8)$$

where a and a_{jj} are coefficients of expansion of $f(\vec{v})$:

$$a = f(\vec{v} = 0), \qquad a_{jj} = \frac{1}{2} \frac{\partial^2 f(\vec{v})}{\partial \vec{v}^j \partial \vec{v}^j}|_{\vec{v}=0}. \qquad (9)$$

One can check that in the first term of (8) $M_{fi}^{(3)}$ and $M_{fi}^{(4)}$ exactly cancel each other. So we are forced to calculate the second term. After rather lengthy calculations we obtain:

$$M_{fi}|_{t_1=t_2=0} = -A\frac{4}{12}\delta_{s,c}^{if}\delta_{uw}\pi\tau^2\left\{-\frac{1}{x_g y_g}\varepsilon_1\varepsilon_2 + \frac{2}{sx_g y_g}\frac{\delta_2}{\delta_1}\varepsilon_1 p_2\varepsilon_2 p_2\right\}. \qquad (10)$$

Taking the square of the amplitude (10), averaging and summing over spins, colors and polarizations one arrives at the simple formula:

$$\overline{\left|M_{fi}|_{t_1=t_2=0}\right|^2} = \frac{H}{x_g^2 y_g^2},$$

where:

$$H = \frac{8}{(12)^2} 2 \left(\frac{4\pi\left(D_0 G^2\right)^3 \mu^4}{9\left(2\pi\right)^2}\right)^2 \left(\frac{g^2}{G^2}\right)^2. \qquad (11)$$

Factors 8 and $\left(\frac{1}{12}\right)^2$ are related to the summing over colors of produced gluons and to the square of color factor of diagrams represented by $M_{fi}^{(3)}$, respectively.

Taking into account (1) where $(s/s_1) = (1/\delta_2)$, $(s/s_2) = (1/\delta_1)$, $t_1 = -\vec{v}_1^2$ and $t_2 = -\vec{v}_2^2$ we obtain:

$$\overline{|M_{fi}|^2} = \frac{H}{(x_g y_g)^2 (\delta_1\delta_2)^{2\epsilon} \delta_1^{2\alpha' t_1} \delta_2^{2\alpha' t_2}} \exp\left(2\beta\left(t_1 + t_2\right)\right). \qquad (12)$$

The factor $\exp\left(2\beta\left(t_1 + t_2\right)\right)$ takes into account the effect of the momentum transfer dependence of the non-perturbative gluon propagator with $\beta = 1$ GeV^{-2} [7,8].

4. Cross-section

Having calculated $\overline{|M_{fi}|^2}$ we can write the formula for the cross-section:

$$\sigma = \frac{81}{2s\left(2\pi\right)^8 2!} \int \overline{|M_{fi}|^2}\left[F\left(t_1\right)F\left(t_2\right)\right]^2 dPH, \tag{13}$$

where the factor 81 takes into account the presence of three quarks in each proton, 2! is an identical final state particle phase space factor[1] and $F\left(t\right)$ is the nucleon formfactor approximated by:

$$F\left(t\right) = \exp\left(\lambda t\right) \tag{14}$$

with $\lambda = 2$ GeV^{-2}. Differential phase–space factor dPH has the form:

$$
\begin{aligned}
dPH \;=\; & d^4k_1\delta\left(k_1^2\right)d^4k_2\delta\left(k_2^2\right)d^4r_1\delta\left(r_1^2\right)d^4r_2\delta\left(r_2^2\right) \\
& \times\Theta\left(k_1^0\right)\Theta\left(k_2^0\right)\Theta\left(r_1^0\right)\Theta\left(r_2^0\right)\delta^{(4)}\left(p_1 + p_2 - k_1 - k_2 - r_1 - r_2\right).
\end{aligned} \tag{15}
$$

It turns out that a substantial part of the integrations can be performed analytically [5]. Denoting by $E_{\min}^{\mathsf{T}}$ the minimum value of transverse energy of the produced gluon (we integrate over $E^{\mathsf{T}} \geq E_{\min}^{\mathsf{T}} \neq 0$) and by Δ the maximum value of the energy loss of the initial hadrons ($\delta_{1,2} < \Delta \sim 0.1$), we obtain:

$$
\begin{aligned}
\sigma\left(E_{\min}^{\mathsf{T}}\right) \;=\; & C\int\limits_0^{h(\Delta)} dx\, x\left(1 - x^2\right)^{2\epsilon}\left(\ln\frac{1 + x}{1 - x} + \frac{2x}{1 - x^2}\right) \\
& \times\frac{1}{\frac{\lambda + \beta}{\alpha'} + \frac{1}{2}\ln\frac{1 - x^2}{\delta^2}}\ln\frac{\frac{\lambda + \beta}{\alpha'} + \ln\frac{\Delta\left(1 - x^2\right)}{\delta^2}}{\frac{\lambda + \beta}{\alpha'} + \ln\frac{1}{\Delta}},
\end{aligned} \tag{16}
$$

where $h\left(\Delta\right) = \sqrt{1 - \delta^2/\Delta^2}$, $\delta^2 = 4(E_{\min}^{\mathsf{T}})^2/s$ and:

$$C = 9\pi\left(\frac{s}{4(E_{\min}^{\mathsf{T}})^2}\right)^{2\epsilon}\left(\frac{\pi^2\left(D_0 G^2\right)^3 \mu^4}{18\left(2\pi\right)^6 E_{\min}^{\mathsf{T}}\alpha'}\right)^2\left(\frac{g^2}{G^2}\right)^2.$$

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[1] We would like to thank J.R. Cudell for a correspondence about this point.

The major uncertainty in above result is the value of the non-perturbative coupling constant G.

Finally, let us remind the result for quark–antiquark production by double pomeron exchange [4]:

$$
\sigma \left((k_{\min}^{\mathsf{T}})^2\right) = \widetilde{C} \int_0^{\widetilde{h}(\Delta)} dx\, x \left(1 - m^2 \frac{1 - x^2}{m^2 + (k_{\min}^{\mathsf{T}})^2}\right) \left(1 - x^2\right)^{1+2\epsilon}
$$

$$
\times \left(\ln \frac{1+x}{1-x} + \frac{2x}{1-x^2}\right)
$$

$$
\times \frac{1}{\frac{\lambda+\beta}{\alpha'} + \frac{1}{2} \ln \frac{1-x^2}{\widetilde{\delta}^2}} \ln \frac{\frac{\lambda+\beta}{\alpha'} + \ln \frac{\Delta\left(1-x^2\right)}{\widetilde{\delta}^2}}{\frac{\lambda+\beta}{\alpha'} + \ln \frac{1}{\Delta}}, \tag{17}
$$

where $\widetilde{h}(\Delta) = \sqrt{1 - \widetilde{\delta}^2/\Delta^2}$, $\widetilde{\delta}^2 = 4\left(m^2 + (k_{\min}^{\mathsf{T}})^2\right)/s$, m is the mass of the produced quark, and:

$$
\widetilde{C} = \frac{1}{3}\pi \left(\frac{s}{4\left(m^2 + (k_{\min}^{\mathsf{T}})^2\right)}\right)^{2\epsilon} \left(\frac{\pi^2 \left(D_0 G^2\right)^3 \mu^4 m}{18 \left(2\pi\right)^6 \alpha' \left(m^2 + (k_{\min}^{\mathsf{T}})^2\right)}\right)^2 \left(\frac{g^2}{G^2}\right)^2.
$$

By $k_{\min}^{\mathsf{T}}$ we denote the minimum value of transverse momentum of the produced quark.

5. Numerical results and discussion

Let us remind the following numbers that appear in (16) and (17): $\epsilon = 0.08$, $\alpha' = 0.25$ GeV^{-2}, $\mu = 1$ GeV, $G^2 D_0 = 30$ GeV$^{-1}\mu^{-1}$, $\lambda = 2$ GeV^{-2}, $\beta = 1$ GeV^{-2}, $\Delta = 0.1$. The minimum value of transverse energy of the produced gluon $E_{\min}^{\mathsf{T}}$ is chosen at 5, 7 and 10 GeV. At Tevatron energy, $\sqrt{s} = 1.8$ TeV, the results for the cross-section σ (multiplied by $\left(G^2/4\pi\right)^2$) are as follows:

TABLE I

The numerical results for the cross-section for production of gluon pairs by double pomeron exchange. The calculation is performed at Tevatron energy $\sqrt{s} = 1.8$ TeV.

$\sqrt{s}$ [TeV]	$E_{\min}^{\mathsf{T}} = 5$ GeV	$E_{\min}^{\mathsf{T}} = 7$ GeV	$E_{\min}^{\mathsf{T}} = 10$ GeV
1.8	$\left(G^2/4\pi\right)^2 \sigma = 3$ μb	$\left(G^2/4\pi\right)^2 \sigma = 0.9$ μb	$\left(G^2/4\pi\right)^2 \sigma = 0.3$ μb

The running coupling constant $g^2/4\pi$ was evaluated at $2E^{\mathsf{T}}_{\min}$, *i.e.* 0.17, 0.15 and 0.14 for $E^{\mathsf{T}}_{\min} = 5, 7, 10$ GeV, respectively.

The dependence of the cross-section *versus* the center mass energy $E_{\mathrm{CM}} = \sqrt{s}$ for the range of $\sqrt{s}$ taken from 0.5 TeV to 40 TeV is shown in Fig. 5.
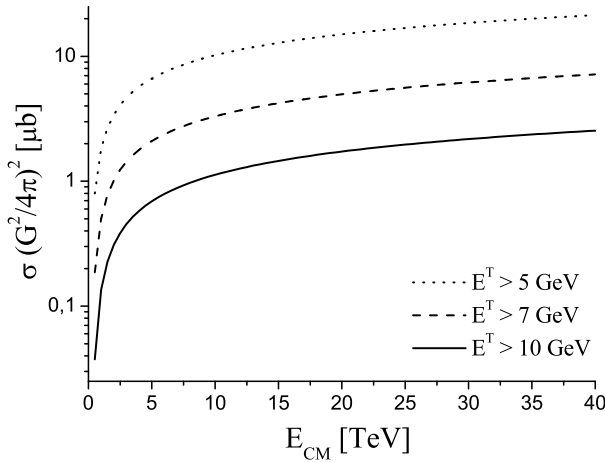

Fig. 5. Double pomeron exchange contribution to the cross-section for the gluon pairs production process *versus* the center mass energy.

In order to obtain the total cross-section for production of jets by double pomeron exchange, we have to consider the quark–antiquark production process. It is obvious that for the case in which the minimum value of transverse momentum of the produced quark $k^{\mathsf{T}}_{\min} = 0$, the relations hold: $\sigma_{u\bar{u}} \simeq \sigma_{d\bar{d}} > \sigma_{s\bar{s}} \gg \sigma_{c\bar{c}} > \sigma_{b\bar{b}} > \sigma_{t\bar{t}}$. The values for $\sigma_{c\bar{c}}$, $\sigma_{b\bar{b}}$ and $\sigma_{t\bar{t}}$ are determined by (17) (this expression is reliable for heavy quarks production process). For more details see [4, 5].

In the experiment we observe jets with some minimal $E^{\mathsf{T}}_{\min} \sim$ GeV, however, and these relations are not valid. At Tevatron energy $\sqrt{s} = 1.8$ TeV and $E^{\mathsf{T}}_{\min} = 7$ GeV we have:

TABLE II

The numerical results for the cross-section for production of heavy quark pairs by double pomeron exchange. The calculation is performed at Tevatron energy $\sqrt{s} = 1.8$ TeV and $E^{\mathsf{T}}_{\min} = 7$ GeV.

$\sqrt{s}$ [TeV]	$\left(G^2/4\pi\right)^2 \sigma_{c\bar{c}}$ [nb]	$\left(G^2/4\pi\right)^2 \sigma_{b\bar{b}}$ [nb]	$\left(G^2/4\pi\right)^2 \sigma_{t\bar{t}}$ [nb]
1.8	0.3	3.1	0

As the masses of the heavy quarks we take $m_c = 1.2$ GeV, $m_b = 4.2$ GeV, $m_t = 175$ GeV (there is not enough energy to produce a $t\bar{t}$ pair). The running coupling constant $g^2/4\pi$ was evaluated at $2E^\mathsf{T}_{\min}$, *i.e.* 0.15, 0.15 and 0.1 for $c\bar{c}$, $b\bar{b}$ and $t\bar{t}$, respectively.

The dependence of the cross-section on the center of mass energy $E_{CM} = \sqrt{s}$ for the range of $\sqrt{s}$ taken from 0.5 TeV to 40 TeV is shown in Fig. 6.
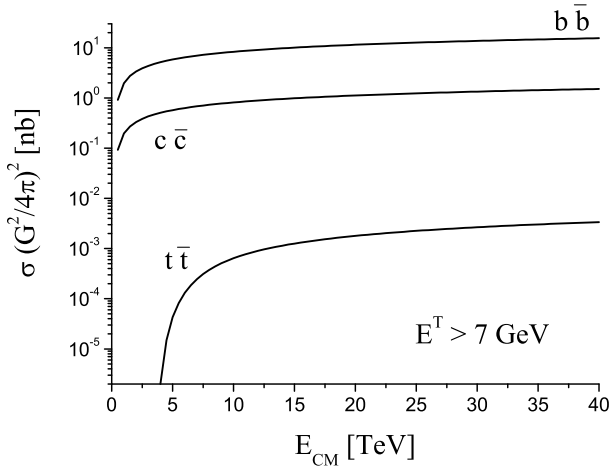

Fig. 6. Double pomeron exchange contribution to the cross-section for production of heavy quark pairs process *versus* the center mass energy.

As we see $\sigma_{b\bar{b}} > \sigma_{c\bar{c}}$. This is a consequence of the fact that we observe the jets with minimal $E^\mathsf{T}_{\min} = 7$ GeV. Since there are very few light quarks with $E^\mathsf{T}_{\min} \sim$ GeV, the cross-section for production $u\bar{u}\ d\bar{d}\ s\bar{s}$ pairs with such minimal energy is negligible. To summarize, the cross-section for quark–antiquark production with minimal $E^\mathsf{T}_{\min} \sim$ GeV is fully dominated by the $c\bar{c}$ and $b\bar{b}$ pairs production.

One sees that $q\bar{q}$ contribution to production of jets is about three orders of magnitude smaller than that of gg contribution. Similar observation was first made in Ref. [9].

Our cross sections are calculated for the case where both initial protons are scattered quasi-elastically. If one allows for diffractive dissociation of the incident hadrons *i.e.* if one puts $\lambda = 0$ in (14) the cross sections increase about half order of magnitude.

6. Conclusions

We have calculated the cross-section for gluon pair production by double pomeron exchange in the Landshoff–Nachtmann model. The obtained results were compared with those for production of quark–antiquark pairs calculated in the same model. It turns out that cross-section for jet production is strongly dominated by gg production.

Appendix

In this appendix some main steps leading to the expressions (4) and (5) are more explicitly shown.

Let us denote the color structure of the diagrams represented by $M_{fi}^{(3)}$ as follows:
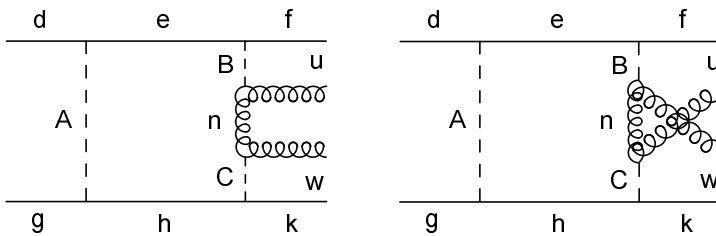

Fig. 7. The capital letters denote color indices of the non-perturbative gluons and small letters color indices of quarks and perturbative gluons.

Performing color calculations for the first diagram in Fig. 7 (one has to remember that pomeron does not change color of colliding quarks) we have:

$$\frac{1}{4}\left(\lambda_{fe}^{B}\lambda_{ed}^{A}\right)_{\text{singlet}}\frac{1}{4}\left(\lambda_{kh}^{C}\lambda_{hg}^{A}\right)_{\text{singlet}}f_{Bun}f_{nwC}=-\frac{1}{12}\delta_{fd}\delta_{kg}\delta_{uw}, \qquad (18)$$

what is a consequence of:

$$\left(\lambda_{fe}^{B}\lambda_{ed}^{A}\right)_{\text{singlet}}=\frac{2}{3}\delta^{AB}\delta_{fd}\,,$$
$$\left(\lambda_{kh}^{C}\lambda_{hg}^{A}\right)_{\text{singlet}}=\frac{2}{3}\delta^{CA}\delta_{kg}\,. \qquad (19)$$

One can check that for the second diagram the color factor is the same.

The matrix element $M_{fi}^{(3)}$ is given by (see Fig. 4):

$$
\begin{aligned}
M_{fi}^{(3)} &= -\frac{A}{12}\delta_c^{if}\delta_{uw}\int d^4 Q\,\delta\left(q_1^2\right)\delta\left(q_2^2\right)D\left(Q^2\right)D\left(l_1^2\right)D\left(l_2^2\right)\\
&\times\bar{u}\left(k_1\right)\gamma^\beta\not{q}_1\gamma^\nu u\left(p_1\right)\bar{u}\left(k_2\right)\gamma^\lambda\not{q}_2\gamma_\nu u\left(p_2\right)\\
&\times\left\{V_{\beta\xi\delta}\left(l_1,-r_1,-q\right)\frac{-g^{\delta\rho}}{q^2}V_{\rho\sigma\lambda}\left(q,-r_2,l_2\right)\varepsilon_1^\xi\varepsilon_2^\sigma\right.\\
&\left.+V_{\beta\xi\delta}\left(l_1,-r_2,\tilde{q}\right)\frac{-g^{\delta\rho}}{\tilde{q}^2}V_{\rho\sigma\lambda}\left(-\tilde{q},-r_1,l_2\right)\varepsilon_1^\sigma\varepsilon_2^\xi\right\},\qquad (20)
\end{aligned}
$$

where:

$$
\begin{aligned}
V_{\mu_1\mu_2\mu_3}\left(k_1,k_2,k_3\right) &= g_{\mu_1\mu_2}\left(k_1-k_2\right)_{\mu_3}+g_{\mu_2\mu_3}\left(k_2-k_3\right)_{\mu_1}\\
&+g_{\mu_3\mu_1}\left(k_3-k_1\right)_{\mu_2}.\qquad (21)
\end{aligned}
$$

Here $A=iD_0^3 G^4 g^2/(2\pi)^2$, G and g are the non-perturbative and perturbative quark–gluon couplings, $D\left(p^2\right)=D_0\exp\left(p^2/\mu^2\right)$ with $\mu\approx 1$ GeV and $G^2 D_0\approx 30$ GeV$^{-1}\mu^{-1}$ is the non-perturbative gluon propagator. The fact that we put inner quark lines on shell is expressed by $\delta\left(q_1^2\right)\delta\left(q_2^2\right)$.

After some approximations, based on the fact that integrand in (20) is exponentially damped in $\vec{v}^2$, what allows to consider $\vec{v}$ as small, we obtain (for more details see [5]):

$$
M_{fi}^{(3)}=-A\frac{4}{12}\delta_{s,c}^{if}\delta_{uw}\int d^2\vec{v}f\left(\vec{v}\right)\exp\left(\frac{-\vec{v}^2}{\tau}\right),
$$

where:

$$
\begin{aligned}
f\left(\vec{v}\right) &= \left\{p_1^\beta V_{\beta\xi\delta}\left(l_1,-r_1,-q\right)\frac{-g^{\delta\rho}}{q^2}V_{\rho\sigma\lambda}\left(q,-r_2,l_2\right)p_2^\lambda\varepsilon_1^\xi\varepsilon_2^\sigma\right.\\
&\left.+p_1^\beta V_{\beta\xi\delta}\left(l_1,-r_2,\tilde{q}\right)\frac{-g^{\delta\rho}}{\tilde{q}^2}V_{\rho\sigma\lambda}\left(-\tilde{q},-r_1,l_2\right)p_2^\lambda\varepsilon_1^\sigma\varepsilon_2^\xi\right\}.
\end{aligned}
$$

Substituting (21) and (2) in the above formula, using $p_1^2=p_2^2=0$ and $\varepsilon_1 r_1=\varepsilon_2 r_2=\varepsilon_1 p_1=\varepsilon_2 p_1=0$ what is a consequence of (7), we obtain (6).

Let us denote color structure of the diagram represented by $M_{fi}^{(4)}$ as follows:
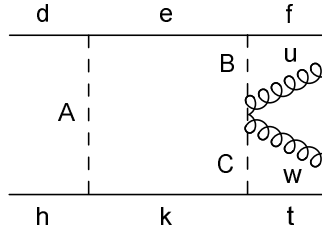
Fig. 8. The capital letters denote color indices of the non-perturbative gluons and small letters color indices of quarks and produced gluons.

The matrix element $M_{fi}^{(4)}$ is given by (see Fig. 4):

$$
\begin{aligned}
M_{fi}^{(4)} &= -A\frac{1}{4}\left(\lambda_{fe}^B\lambda_{ed}^A\right)_{\text{singlet}}\frac{1}{4}\left(\lambda_{tk}^C\lambda_{kh}^A\right)_{\text{singlet}}\int d^4Q\delta\left(q_1^2\right)\delta\left(q_2^2\right) \\
&\times D\left(Q^2\right)D\left(l_1^2\right)D\left(l_2^2\right)\bar{u}\left(k_1\right)\gamma^\beta\slashed{l}_1\gamma^\nu u\left(p_1\right)\bar{u}\left(k_2\right)\gamma^\lambda\slashed{l}_2\gamma_\nu u\left(p_2\right) \\
&\times \left\{f^{xBC}f^{xuw}\left(g_{\beta\xi}g_{\lambda\sigma}-g_{\beta\sigma}g_{\xi\lambda}\right)+f^{xBw}f^{xuC}\left(g_{\beta\xi}g_{\lambda\sigma}-g_{\beta\lambda}g_{\xi\sigma}\right)\right. \\
&\left.+ f^{xBu}f^{xCw}\left(g_{\beta\lambda}g_{\xi\sigma}-g_{\beta\sigma}g_{\xi\lambda}\right)\right\}\varepsilon_1^\xi\varepsilon_2^\sigma. \quad (22)
\end{aligned}
$$

Using (19) and performing the rest color calculations we obtain:

$$
\begin{aligned}
M_{fi}^{(4)} &= -\frac{A}{12}\delta_c^{if}\delta_{uw}\int d^4Q\delta\left(q_1^2\right)\delta\left(q_2^2\right)D\left(Q^2\right)D\left(l_1^2\right)D\left(l_2^2\right) \\
&\times\bar{u}\left(k_1\right)\gamma^\beta\slashed{l}_1\gamma^\nu u\left(p_1\right)\bar{u}\left(k_2\right)\gamma^\lambda\slashed{l}_2\gamma_\nu u\left(p_2\right) \\
&\times\left[2g_{\beta\lambda}g_{\xi\sigma}-g_{\beta\xi}g_{\lambda\sigma}-g_{\beta\sigma}g_{\xi\lambda}\right]\varepsilon_1^\xi\varepsilon_2^\sigma. \quad (23)
\end{aligned}
$$

After some approximations, fully described in [5] we have:

$$
M_{fi}^{(4)} = -A\frac{4}{12}\delta_{s,c}^{if}\delta_{uw}s\varepsilon_1\varepsilon_2\int d^2\vec{v}\exp\left(\frac{-\vec{v}^2}{\tau}\right),
$$

what is our result (4).

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