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ELECTRICAL EFFECTS INDUCED AT THE BOUNDARY OF AN ACOUSTIC CAVITATION ZONE

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The passage of an ultrasonic wave through a liquid medium can produce cavitation. In this paper we describe the experiment made in order to determine the voltage induced in a cavitation zone and we establish two formulae for the calculus of this voltage, when the studied liquid was the water. The main result is the modeling of an electrical signal induced at the boundary of an acoustic cavitation zone, in water, using the ARIMA process.

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1. Introduction

When an acoustic wave propagates through a liquid containing microscopic gas inclusions, these "nucleation sites" can be mechanically activated, at which point they spawn free bubbles which then undergo highly energetic volume pulsations [1].

In a liquid, an ultrasonic field can carry along small bubbles or can produce cavitation bubbles. Their subsequent collapse/rebound cycle results in such effects as: pressure oscillations with frequency different from that of the stimulating ultrasonic field, sonoluminescence and rectified diffusion of gas dissolved in the liquid, emulsification of multiphase media and chemical reactions in cavitating liquids [2–7]. Erosion and unpassivation of solid boundary surfaces are phenomena of indisputable technical significance.

Some results concerning the electric pulses produced by cavitation are given in [8].

The apparition of a variable voltage induced between different points at the boundaries of an acoustic cavitation zone in water, benzene, diesel-oil *etc.* [7, 9–11] have been studied. We found out that this phenomenon occurred in all the analyzed liquids.

Our experiments have proved that the frequency of the induced electrical pulse corresponds to the frequency of the mechanical waves generated by the collapse of the cavitation bubbles.

In this paper we shall give a description of the experiment conducted in order to identify this voltage and we shall give some possible explanation of its generation mechanism.

The most important part of this work is the modeling of the signal collected by the data acquisition board, using the Box–Jenkins methods. In fact, we shall prove that an autoregressive integrated moving average process can describe with satisfactory accuracy the voltage occurring between two points at the cavitating zone boundaries in water.

2. Experimental set-up

In order to determine the voltage which appeared at the boundary of acoustic cavitation zone in a liquid, the experimental set-up given in figure 1 was used. It consists essentially of an ultrasonic generator and two vessels.

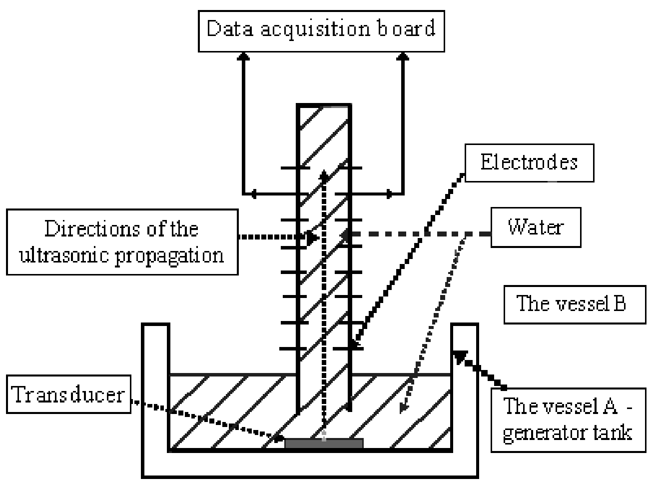


Fig. 1. The experimental set-up.

The ultrasonic generator operates at the frequency of 20 kHz and at three power levels: 80 W, 120 W and 180 W.

The vessel A is an ultrasonic generator tank. It contains a ceramic transducer, with the diameter of 3.7 cm, connected to a high frequency generator that excites the transducer to produce the ultrasound.

The vessel B, which was inserted into vessel A, contains pairs of electrodes of electrolytic copper, that measure the potential difference produced between different points at the cavitation zone boundaries.

The data acquisition board can be connected to different electrodes, which can be situated at distances between 5 and 61 cm from the ultrasound transducer. The connection can be made with circumferential electrodes at different levels or with pin electrodes, situated opposite to each other at the same level.

Also, one can work with vessels of different diameters.

The data acquisition board — CompuScope LITE — converts analog signals into digital data sequence with a resolution of 8 bits at a sampling rate of up to 10^6 MSample/s and stores the resulting digital pattern in the on-board memory.

The PC is then allowed to access the memory and retrieve the data for the further processing.

3. Results

The data used in the mathematical modeling were obtained in the following conditions:

— the voltage induced by cavitation was measured between pin electrodes, situated opposite to each other at the same level;

— the elevation of the electrodes above the liquid level was 10 cm;

— the diameter of the vessel B was 5 cm and the distance between the electrodes, 4.5 cm.

— the liquid contained in the vessels A and B was distilled water;

— the water temperature was 20° C;

— the ultrasonic generator power was 120 W;

— the collected signal was not amplified.

The values of the voltage induced by the cavitation and collected by the data acquisition board are represented in the figure 2. It can be seen that there exists a periodicity of the voltage and after the study of the values of captured signal, the determined period was 103 μs.

The powerful pulses occur every 50 μs, which is consistent with the ultrasonic transducer frequency. The pulses are interchangeably shifted to positive and negative side and it may be the reason why the period of the process is twice as long.

We could give the following explanation of the apparition of a potential difference between two points when an ultrasound propagates through a liquid [16].

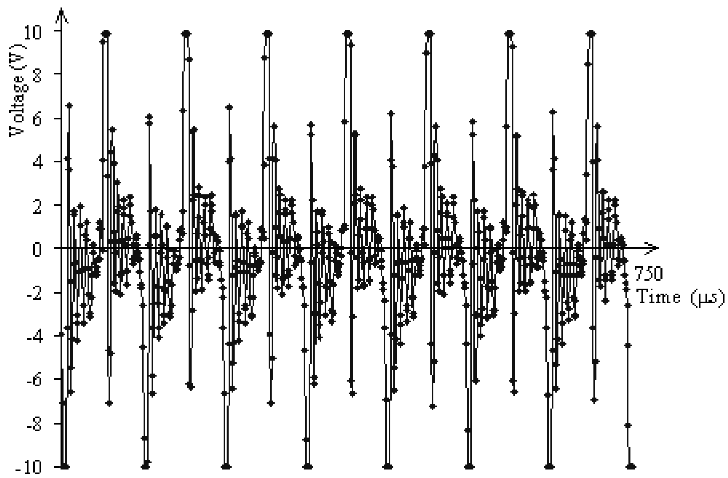


Fig. 2. The values of the voltage induced in water.

At the contact area between the cavitation bubble and the liquid, an electrical double layer appears, as an effect of the electric charge cumulation at the internal and the external surfaces of the bubble. At a certain moment, some bubbles collapse. Then, the following processes occur: the reappearance of the bubbles, the electrical charging by breaking the binding between the molecules, simultaneously with an increase in their size and a change in their spatial arrangement, so that, after a certain time, a new collapse happens.

Since in the disruption process the pressure of equilibrium at the interior and at the exterior of the cavitation bubble is persevered, the bubbles do not collapse all at the same moment. As a result, an alternating current appears between the measurement electrodes.

The frequency of the voltage induced by the bubbles is equal to the frequency of the appearance and the breaking of the bubbles.

To explain in detail the phenomenon, we can follow two ideas.

I. The appearance of cavitation bubbles in the ultrasonic field takes place concomitantly with their superficial electrical charging. The cavitation bubble moves in the ultrasonic field with the frequency of this field, but shows also an inherent oscillating motion, determined by the acoustic pressure, the surface tension and the internal gas and vapor pressure. The movements of the bubbles are oscillating movements of some electric loading.

Since the frequency of bubble motion shows harmonic and subharmonic components, the induced field is characterized by the existence of these components.

II. Another explanation uses the knowledge of the physics of plasma, such as the appearance of some spatial structure of electrical charge distribution, bounded by a double layer. The appearance, the development and the collapse of the cavitation bubbles are analogous to that of the double-layer in plasma; the existence of these phenomena in plasma was proved.

Thus, it can also be supposed that the cavitation bubbles dynamics induces detectable voltage oscillations in liquid.

Depending on the pressure variation and the surface tension value, some bubbles can be destroyed faster or slower which results in variable frequency and amplitude of the induced voltage.

Our experiments have proved that the voltage induced by the ultrasonic field depends on the liquid nature, the ultrasonic field frequency, the distance between the electrodes and the power of the ultrasonic generator.

Let us denote by:

a — atomic spacing;
D — electrode spacing;
e — single electron charge;
k — parameter which depends on the liquid nature;
n — number of free electrons on the cavitation bubble surface;
r — radius of the cavitation bubble;
M — molar mass of the liquid;
N_0 — Avogadro's constant;
U — induced voltage;
α — the liquid polarizability;
ε_0 — absolute dielectric permeability;
ε_r — relative dielectric permeability;
ρ — the liquid density;
σ — density of the superficial electric charge.

In [11] we derived the following formula for the induced voltage:

$$U = k\frac{neaD}{\alpha N_0}\,, \tag{1}$$

where

$$n = \frac{4\pi\sigma r^2}{e} \tag{2}$$

and

$$\alpha = \frac{3\varepsilon_0 M\left(\varepsilon_r - 1\right)}{N_0\rho\left(\varepsilon_r + 2\right)}\,. \tag{3}$$

For the distilled water $k = 100$ and $\alpha = 76 \times 10^{-38}\left[\frac{\mathrm{Cm}^2}{\mathrm{V}}\right]$.

It is known that the radius of a cavitation bubble is between 10^{-10} m and 10^{-2} m [1]. The values of the induced voltage calculated from (1) for different values of r correspond with those experimentally determined.

At the cavitation bubble apparition, a double layer of electrical dipoles is formed. At the collapse moment, the electrical charges are disseminated in the zone of the waves generated by the acoustic cavitation, producing a potential difference that can be calculated from (1). Therefore the voltage induced between two points of the acoustic cavitation zone depends on the number of free electrons on the cavitation bubble surface, the single electron charge and the radius of the cavitation bubble.

The results of the measurements of the caloric and acoustic power of some liquids can be seen in [12].

4. Mathematical modeling

An attempt to use the ARIMA terms to describe the process of voltage variations between two points at the acoustic cavitation zone (Fig. 2) is presented below. Due to the periodicity of voltage variation curve, the study was performed for a single period only.

The following definitions are known [14, 15].

Definition 1. *A time series is a realization of a stochastic process or a sequence of values that show the volume variation of a statistic population or of a specified characteristics level, related to the time.*

Definition 2. *A discrete time process is a sequence of random variables* $X_t, t \in \boldsymbol{Z}$.

Definition 3. *A discrete time process* $X_t, t \in \boldsymbol{Z}$ *is called stationary if*:

$$\forall t \in \boldsymbol{Z}, M(X_t^2) < \infty, \tag{4}$$

$$\exists \mu \in \boldsymbol{R}, \forall t \in \boldsymbol{Z}, M(X_t) = \mu, \tag{5}$$

$$\exists \gamma : \boldsymbol{R}^+ \to \boldsymbol{R}, \forall t \in \boldsymbol{Z}, \forall h \in \boldsymbol{Z}, \ \mathrm{Corr}(X_t,\, X_{t+h}) = \gamma(h)\,, \tag{6}$$

where M(X) is the expected value of the random variable X, μ *is a constant, Corr(X, Y) is the correlation of the random variables* X *and* Y *and* γ *is a real function.*

Definition 4. *A stationary process* $\xi_t, t \in \boldsymbol{Z}$ *is called a white noise if* $\gamma(h) = 0$, *for* $h \neq 0$, $M(\xi_t) = 0$ *and* $D^2(\xi_t) = \sigma^2 = \gamma(0)$, $\forall t \in \boldsymbol{Z}$.

Definition 5. *The function defined on* $\boldsymbol{Z}$, *by*:

$$\rho(h) = \frac{\mathrm{Corr}(X_t, X_{t+h})}{\sqrt{D^2(X_t) D^2(X_{t+h})}} = \frac{\gamma(h)}{\gamma(0)}$$

is called the autocorrelation function (ACF) of the process $X_t, t \in \boldsymbol{Z}$.

Definition 6. *If $X_t, t \in \boldsymbol{Z}$ is a stationary process, the function defined by*:

$$\tau(h) = \frac{\mathrm{Corr}(X_t - X_t^*, X_{t-h} - X_{t-h}^*)}{D^2(X_t - X_t^*)},\ h \in \boldsymbol{Z}_+$$

is called the partial autocorrelation function (PACF), where X_t^* (X_{t-h}^*) is the affine regression of X_t (X_{t-h}) with respect to $X_{t-1}, \dots, X_{t-h+1}$.

Definition 7. *Let p, d, q be natural numbers, $(\varphi_1, \dots, \varphi_p)$ and $(\theta_1, \dots, \theta_p)$ finite sequences of real coefficients, I — identity function and $X_t, t \in \boldsymbol{Z}_+$ a time process. Consider*

$$\begin{aligned}
B(X_t) &= X_{t-1}\,,\\
\Phi(B) &= I - \varphi_1 B - \cdots - \varphi_p B^p,\ \varphi_p \neq 0\,,\\
\Theta(B) &= I - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q,\ \theta_p \neq 0\,,\\
\Delta^d X_t &= (I - B)^d X_t\,.
\end{aligned}$$

The time process is called an autoregressive integrated moving average process of p, d, q orders and is denoted by ARIMA (p, d, q) *if*:

$$\Phi(B)\Delta^d X_t = \Theta(B)\xi_t\,,$$

where ξ_t, $t \in \boldsymbol{Z}_+$ is a white noise.

ξ_t, $t \in \boldsymbol{Z}_+$ is called the residual in the ARIMA *process.*

An autoregressive process of p order, denoted by AR(p), *is an* ARIMA *process of p,* 0, 0 *orders.*

A moving average process of q order, denoted by MA(q), *is an* ARIMA *process of* 0, 0, *q orders.*

An autoregressive moving average process of p and q orders, denoted by ARMA (p, q) *is an* ARIMA *process of p,* 0, *q orders.*

Remarks.

1. The AR, MA and ARMA processes are particular cases of the ARIMA process.

2. The autoregressive order p means that a series value is affected by the preceding p values (independently of one another). The degree of differencing d is the number of times that a time series is transformed by taking differences between series values and their predecessors. The number q is the order of moving average of the process.

3. In order to determine the type of the process, the form of the graph of the autocorrelation function of the process can be used.

(i) The ACF of an AR(p) process is an exponentially decreasing or a damped sine wave oscillation. The PACF of an AR(p) process is vanishing for all $h > p$ and $\tau(p) = \varphi_p$.

(ii) The ACF of an MA(q) process is vanishing for all $h > q$. The PACF of an MA(q) process is non-vanishing beginning at some lag value.

(iii) The ACF of an ARMA(p, q) process is a mixture of exponential decreasing curves and damped sine wave oscillation, when $p > q$; when $p < q$, the ACF is of the previous type for all $h > q - p$.

First, the autocorrelation function (ACF) of the voltage for the lags between 1 and 16 was studied and the confidence interval — at the confidence level 95% — was determined.

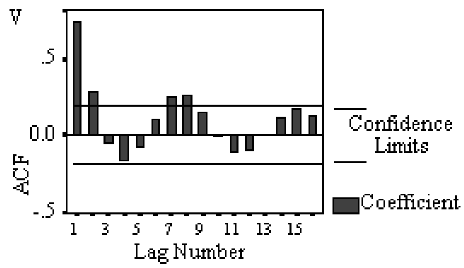


Fig. 3. The ACF of the voltage.

In the figures 3 and 4 it can be seen that some values of ACF and PACF fall outside the confidence interval. The forms of the graphs of ACF (figure 3) and PACF (figure 4) are of damped sine wave oscillation. They enable us to think that the process could be of ARMA type.

For the signal captured, we made also the Fourier analysis. The graph of the Fourier transform can be seen in figure 5.

To obtain the model the following fitting procedures were followed:

— identification *a priori* of the model, *e.g.* choosing of the parameters p, q, d of the ACF and/or PACF graph approximating functions;

— estimation of parameters, using the pseudo-maximum verosimility method, recursive methods or methods based on ACF function. At this stage the coefficients of the autoregressive and moving average polynomials are determined for each of the possible values of the triple (p, d, q);

— testing of the possible models parameters;

— identification *a posteriori*, *e.g.* identifying the best model.

The following model was determined:

$$\begin{aligned} U_t &= 1.3006553U_{t-1} - 0.7035790U_{t-2} + \xi_t - 0.6128040\xi_{t-1}\,, \\ U_0 &= 9.921875\,, \quad U_1 = 8.671875\,, \end{aligned} \tag{7}$$

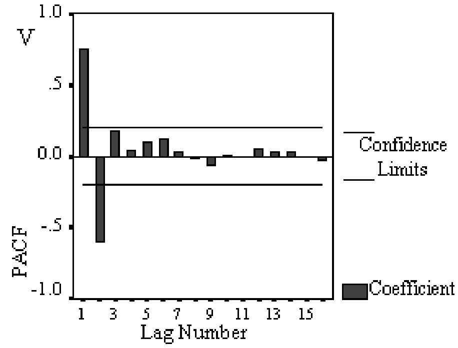


Fig. 4. The PACF of the voltage.

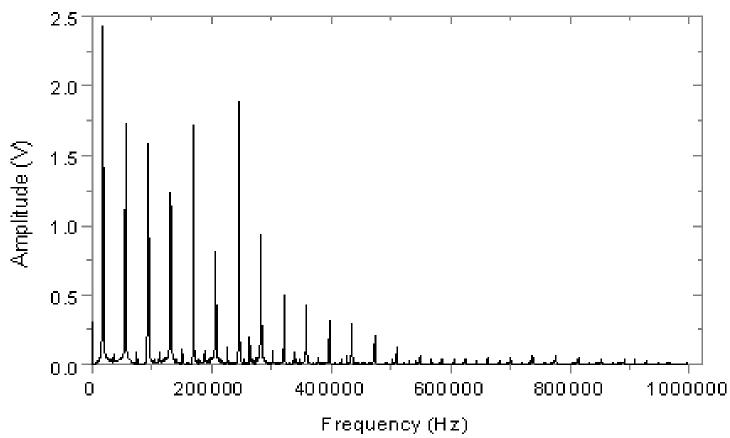


Fig. 5. The Fourier transform.

where $t \in \boldsymbol{N}, t \geq 2$, U_t is the voltage at the moment t and ξ_t, $t \in \boldsymbol{N}$ is the residual. This is the equation of an ARMA(2, 1) process, without a constant term. The model ARMA(2, 1), with a constant term, was also studied, but the significance test made for the coefficients, led us to reject the hypothesis that the constant was non-vanishing. To prove that the model was properly identified, the autocorrelation function and the partial autocorrelation function of the residuals were studied. The graphs of these functions can be seen in the figures 6 and 7.

The values of the autocorrelation function and of the partial autocorrelation function of the residuals keep within the confidence intervals, at 95% confidence level.

The values of the Box–Ljung statistics keep within the interval [0.007, 7.075], so they are less than $\aleph^2(102)$. The probabilities to accept the

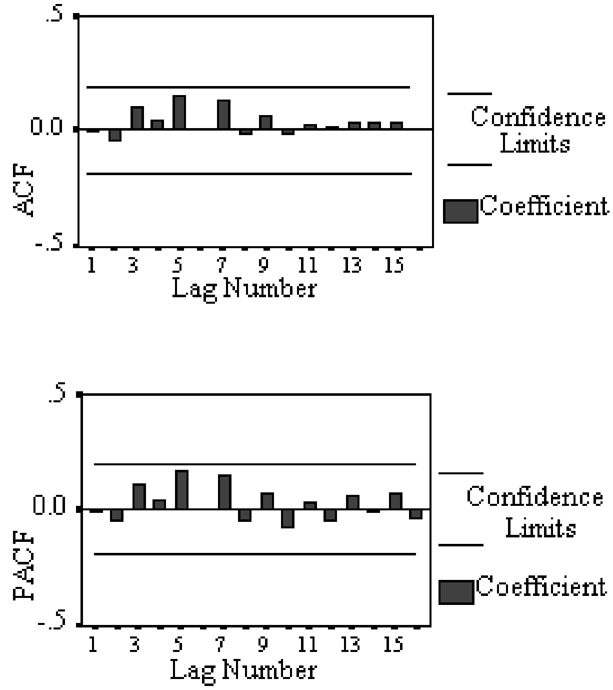
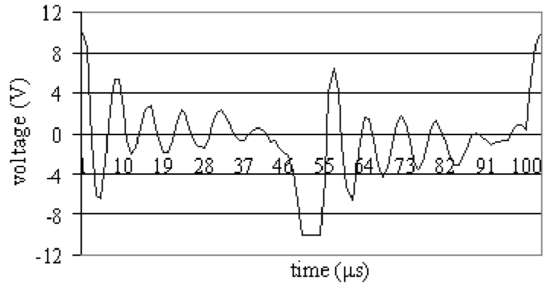


Fig. 6. The PACF for the residuals in the model ARMA(2,1)

hypothesis that the residuals form a white noise are around 90%. Therefore, the residuals form a white noise and the model is well selected. To compare the collected data and those calculated for a single period, using the ARMA model, one can confer figure 8. In its upper part the values of the signal are plotted and in its lower part, those calculated by means of Eq. (7) are shown. Since the differences between them are indistinguishable in many zones it was preferred to use two graphs instead of a single chart.

The mathematical model found out depends on the generator power. In the same conditions of the experiment, if the generator operates at 80W the model is of AR(2) type ([9]) and if the generator operates at 180W, the model is of ARIMA(2, 1, 0) type ([10]). So, in all the cases when the studied liquid was water, the autoregressive degree was 2.



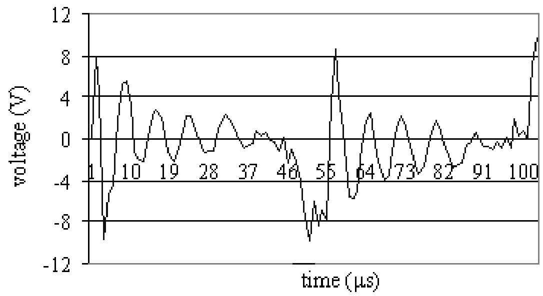


Fig. 7. The voltage experimentally determined and that calculated

5. Conclusions

Our experiments proved the apparition of voltage in water when an ultrasound wave passes through the liquid. It was proved experimentally that the voltage induced at the boundaries of an acoustic cavitation zone depends on the liquid, the intensity of the ultrasonic wave and the distance between the measurement electrodes.

It was proved that the electrical pulse depends also on the radii of cavitation bubbles and on the number of free electrons on the cavitation bubble surface.

We gave two possible explanations of the apparition of this phenomenon: one, based on the movement and the collapse of the cavitation bubbles and another based on the double layer theory.

Since the voltage variation between two points at the acoustic cavitation zone is a phenomenon produced in time and is a periodic one, when the studied liquid was the distilled water, the ARIMA method was appropriate to describe it.

This method could be used to model the phenomenon studied in this paper also for other liquids and to determine the acoustic logarithmic decrement. The results obtained till now [7, 10, 13] show that the calculated values coincide with the experimental data and with the data known from the literature.

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