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# STATISTICAL HADRONIZATION WITH EXCLUSIVE CHANNELS IN $e^+e^-$ ANNIHILATION*

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We present a systematic analysis of exclusive hadronic channels in $e^+e^-$ collisions at centre-of-mass energies between 2.1 and 2.6 GeV within the statistical hadronization model. Because of the low multiplicities involved, calculations have been carried out in the full microcanonical ensemble, including conservation of energy-momentum, angular momentum, parity, isospin, and all relevant charges. We show that the data is in an overall good agreement with the model for an energy density of about 0.5 GeV/fm$^3$ and an extra strangeness suppression parameter $\gamma_S \sim 0.7$, essentially the same values found with fits to inclusive multiplicities at higher energy.

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## 1. Introduction

The statistical approach to multi-hadron production in $e^+e^-$annihilations has a long story. Early works date back to the 70s [1,2], with different versions of the model and different observables examined in the relevant analyses, such as inclusive yields, multiplicity distributions *etc.* All of these calculations involved simplifying assumptions and drastic approximations, mostly because of the lack of computing power, so that in practice it was very difficult to confirm or rule out *the* statistical model, also in view of it being conceived as a full alternative to a dynamical model. Nowadays, with QCD being the accepted theory of strong interactions, the Statistical Hadronization Model (SHM) has resurged as a model of hadronization, it

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has a framework based on quantum statistical mechanics [3], and it has been extensively and successfully applied to the analysis of inclusive multiplicities in elementary [4] and relativistic heavy ion collisions [5].

It would be now desirable to quantitatively test the model on observables which are more sensitive to the form of the matrix element than average multiplicities. For this purpose, we compare the production rates of exclusive channels in $e^+e^-$collisions at low energy with the predictions of the SHM.

Exclusive channels in $e^+e^-$collisions have been measured at low centre-of-mass energy ($< 4\,\mathrm{GeV}$). At such a low energy, QCD is in the full non-perturbative regime and one can assume that, unlike at higher energy where clusters are two or more (jettiness), only one hadronizing massive cluster at rest in centre-of-mass frame of the collision is formed. The price to be paid is that, in calculating the model predictions, none of the relevant conservation laws, including energy-momentum, intrinsic angular-momentum and parity, as well as internal symmetries, can be neglected.

To carry out this calculation, we take advantage of the formalism developed in two previous papers of ours [6,7], where the microcanonical partition function of an ideal multi-species relativistic gas was calculated enforcing the conservation of the maximal set of observables pertaining to space-time symmetries (energy-momentum, spin, helicity, parity).

The work presented in this paper is described in detail in Ref. [8]. We summarize here only the main results.

## 2. Analysis of $e^+e^-$ collisions at low energy

The rates of exclusive hadronic channels can be measured only in low energy collisions (say $< 5\,\mathrm{GeV}$) because the large multiplicity of the final state at high energy makes a full identification of particles impossible. There have been, in the past, some attempts to reproduce hadron multiplicities and some multi-pion (kaon) differential cross sections in low energy $e^+e^-$collisions [2] by using statistical–thermodynamical or statistical-inspired models. Yet, in none of those calculations the full set of conservation laws has been taken into account, because of the lack of a proper formulation of the microcanonical ensemble with intrinsic angular momentum and the involved numerical calculations. This is a serious drawback because, when dealing with exclusive channels, all conservation laws play a major role and this is the main point of our analysis.

As discussed in the Introduction, at low energy the formation of a single cluster at rest in the centre-of-mass frame of an $e^+e^-$collision is assumed. Its mass will therefore coincide with $\sqrt{s}$ and the other quantum numbers will be those of the initial state. Particularly, in $e^+e^-$collision, the hadron production is dominated by the diagram with an intermediate virtual photon, so that the hadronizing cluster is assigned with a spin, parity and C-parity $J^{\mathrm{PC}} = 1^{--}$. On the other hand, isospin is not conserved in electromagnetic interaction and it is therefore unknown; in the Vector Dominance Model (VDM) this depends on the coupling of the photon to different resonances, but we will be working in a mass region above 2 GeV, far from known resonance region (see discussion in the following). Therefore, we will assume an unknown statistical mixture of $I = 0$ and $I = 1$ initial state, neglecting interference terms, and introducing a free parameter $\mathrm{I}_0$ such that for the mixed state

$$\mathrm{I}_0|0,0\rangle\langle 0,0| + (1-\mathrm{I}_0)|1,0\rangle\langle 1,0| \,.$$

Finally, the geometry of the cluster needs to be fixed. We assume a spherical shape and a volume given by

$$V = \frac{M}{\rho} = \frac{\sqrt{s}}{\rho}\,, \tag{1}$$

where $M$ is the mass and $\rho$ the energy density; this is taken to be a free parameter to be determined by comparing the model with the data.

Motivated by observations concerning hadron abundances at high energy, we allow deviation from the full statistical equilibrium of channels involving particles with strange valence quarks. This is done by introducing in the analysis the extra-strangeness suppression parameter $\gamma_{\mathrm{S}}$. For its definition here to be in agreement with the formulae of inclusive multiplicities of hadrons in the canonical and grand-canonical ensembles, one just needs to multiply the microcanonical weight $\Omega_{\{N_j\}}$ of a channel by $\gamma_{\mathrm{S}}^{s_j}$, $s_j$ being the number of valence strange quarks of each particle

$$\Omega_{\{N_j\}} \to \left[\prod_{j=1}^{K} \left(\gamma_{\mathrm{S}}^{s_j}\right)^{N_j}\right] \Omega_{\{N_j\}}\,. \tag{2}$$

The $\gamma_{\mathrm{S}}$ factor also applies to neutral mesons with hidden strange quark content like $\eta$, $\phi$ *etc.* Since the wavefunction of such particles is, in general, a superposition like $C_u u\overline{u} + C_d d\overline{d} + C_s s\overline{s}$ with $|C_u|^2 + |C_d|^2 + |C_s|^2 = 1$, only the component $s\overline{s}$ of the wavefunction is suppressed, *i.e.* we multiply by

$$\left[|C_s|^2\,\gamma_{\mathrm{S}}^2 + \left(1 - |C_s|^2\right)\right]\,,$$

for each neutral meson. To calculate $|C_s|^2$, we have used mixing angles quoted by the Particle Data Book [9].

The branching ratios, masses and widths of hadrons and resonances needed to calculate the contribution of parent channels (see Ref. [8] for an extensive discussion) to the examined channel rate have also been taken from the latest issue of the Particle Data Book [9]. All hadrons up to a mass of 1.8 GeV for mesons and 1.9 for baryons have been included for the generation of parent channels.

For our data points, we want to avoid the region of resonances and we do not want to get over the charm quark production threshold and this constrains the examined energy interval to about 2–3 GeV.

Much data in this energy interval has been lately provided by the BABAR experiment which has measured the cross sections of several multi-hadronic channels in $e^+e^-$collisions at several centre-of-mass energies with the method of initial state radiation. We have chosen four energy points, that is $\sqrt{s} = 2.1, 2.2, 2.4$ and 2.6 GeV and added to the available BABAR measurements older measurements performed by experiments at $e^+e^-$colliders run at the same centre-of-mass energies and collected in a nice review paper [10].

In order to compare the calculation with the data of exclusive channels rate, given in terms of a cross section, we have introduced a normalization free parameter $A(\sqrt{s})$

$$\sigma_{\{N_j\}} = A(\sqrt{s})\omega_{\{N_j\}}\,. \tag{3}$$

Finally, we have fitted all available measurements of exclusive channels rates at a given energy to the SHM with four free parameters: $\rho$, $\gamma_{\mathrm{S}}$, $A$ and $I_0$. The fit minimizes the $\chi^2$

$$\chi^2 = \sum_{\{N_j\}_{\mathrm{measured}}} \frac{\left(\sigma^{\mathrm{exp}}_{\{N_j\}} - \sigma^{\mathrm{theo}}_{\{N_j\}}\right)^2}{\Delta^2_{\mathrm{exp}} + \Delta^2_{\mathrm{theo}}}\,, \tag{4}$$

where the sum runs over measured channels; $\Delta_{\mathrm{exp}}$ is the experimental error and $\Delta_{\mathrm{theo}}$ is the theoretical uncertainty on the cross sections respectively. The latter is the sum in quadrature of the statistical error, owing to the finite statistics in Monte Carlo integration and the systematic error stemming from the uncertainty on branching ratios of resonances.

The fit results are summarized in Table I.

TABLE I

Summary of the fit results to multi-hadronic exclusive channels at different centre-of-mass energies.

| $\sqrt{s}$ [GeV] | $\rho$ [GeV/fm$^3$] | $\gamma_{\mathrm{S}}$ | $\chi^2$/d.o.f. |
|---|---|---|---|
| 2.1 | $0.24 \pm 0.17$ | $0.66 \pm 0.22$ | 93.4/16 |
| 2.2 | $0.36 \pm 0.20$ | $0.86 \pm 0.22$ | 82.6/14 |
| 2.4 | $0.44 \pm 0.30$ | $0.78 \pm 0.36$ | 55.4/17 |
| 2.6 | $0.56 \pm 0.36$ | $0.62 \pm 0.47$ | 44.9/12 |

## 3. Conclusions

Although the fit quality is not perfect in terms of statistical test, looking at the deviations between data and model [8], we can fairly conclude that the statistical hadronization model is able to satisfactorily reproduce most exclusive multi-hadronic channels measured in $e^+e^-$collisions at low energy. Especially at 2.4 GeV, all measured channel rates lie within 2.5 standard deviations from the model values, which is quite remarkable taking into account the obvious fact that exclusive channels are a very stringent test for any model, certainly much more than inclusive multiplicities, and that the fits were done with only 4 free parameters.

To fairly judge the quality of the agreement between model and data, it is worth keeping in mind that the analysis we have presented in this work still relies on several approximations, so that one may hope that a more thorough calculation will result in a better agreement between model and data.

Overall, the most interesting outcome of the analysis are the values of the fitted energy density $\rho$ and strangeness suppression parameter $\gamma_{\rm S}$, shown in Table I, around 0.5 GeV/fm$^3$ and 0.7 respectively. These values are essentially the same obtained with the analysis of inclusive hadronic multiplicities at high energy [3].

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