ATTEMPTS AT EXPLAINING NEUTRINO MASSES AND MIXINGS USING FINITE HORIZONTAL SYMMETRY GROUPS*

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(Received October 29, 2013)

A brief discussion about the current status of the search for the possible finite symmetry of a leptonic mass matrix is presented. Possible extensions of the models of leptons that can describe the masses and mixing elements are discussed.

DOI:10.5506/APhysPolB.44.2353
PACS numbers: 12.15.Ff, 12.60.Fr, 14.60.Pq

1. Introduction

One of the most successful scientific theories — the Standard Model of particle physics — is not free from several open questions and still has a vast number of unsettled problems. Among others, one is related to the so-called hierarchy problem. It is not clear why quarks and leptons exist in three generations and why they possess characteristic mass spectra with hierarchical structures. Since the SM was formulated, it has long been known to have a plethora of unknown coupling constants in the Yukawa interaction. The masses and mixings of the three families of quarks and leptons result from the form of the respective Yukawa matrices that are formulated the flavour basis, and they are treated as free parameters of the model. The SM does not explain those parameters but introduces a mechanism by which all particles acquire masses, the so-called Higgs mechanism [1–3]. We do not know of any fundamental principle (as gauge symmetry) that allows the Yukawa Lagrangian and Higgs potential to be constrained in order to explain the fermion masses and mixing angles.

* Presented at the XXXVII International Conference of Theoretical Physics "Matter to the Deepest" Ustroń, Poland, September 1–6, 2013.

The global question is whether there is an organising principle that dictates the family structure of these Yukawa couplings. As symmetry plays a very fundamental role in particle physics, it seems to be natural to wonder if symmetry will really be able to open the door to the generation problem. In this short paper, we briefly summarise the present efforts in this field, the failures that have been observed and discuss possible future attempts to resolve the matter.

2. Family symmetries

In general, there are three approaches to this matter. The first one derives from a theory that assumes a substructure for the fundamental fermions — the preons theory; the second, which is based on the grand unification theories, and the third, the family symmetries, which are sometimes also called the horizontal symmetries. The discussion in this paper will be limited to the horizontal symmetries. Nowadays, all indications are that the Higgs particle, such as that envisaged by the MS, has been discovered [4, 5]. In the light of this empirical evidence and knowing the mass generation mechanism, we expect that such a symmetry will give the relations between the Yukawa constant matrices and the vacuum expectation values for the Higgs particle, thus reducing the number of unknown parameters.

Many ideas about the realisation of symmetry that arise from many different assumptions have been presented and published. For a large comprehensive review, see *e.g.* [6, 7]. From the methodological point of view, they can be classified into two groups of methods. The "bottom–up" methods in which one can try to find the best group symmetry from more and better experimental data, and the "top–down" methods in which, by assuming the symmetry group, one can compare the masses and mixing with experiments. Both are based on the relation between the mass matrix and the mixing matrix, and provide us with the tools with which to search for the symmetry.

Neutrino sector is an attractive region for examinations because mixing is relatively big comparing to quarks (quark mixing matrix is almost diagonal suggesting that crucial for us, non-diagonal elements, may come from perturbative effects). On the other hand, this region is sensitive for all unsolved problems related to neutrino physics. As an example, it is worth to mention that in the days when the reactor angle of neutrino mixing was thought to be zero and the atmospheric angle maximal, mixings could be taken to be tribimaximal (TBM) [8] and, therefore, could be explained by an A_4 finite discrete group. In recent years, we have still incomplete but new and better data [9, 10] from neutrino oscillation experiments. In particular, we know with the very good precision that the mixing angle θ_{13} is different from zero [11] and the atmospheric angle is possibly nonmaximal. Therefore, the TBM pattern is no longer valid. Many suggestions have been examined in an attempt to explain the new data.

3. Brief review

This section briefly presents the one possible "top–down" method that can be helpful in the search for the new realisation of symmetry [12].

Let us assume that there exists a discrete flavour symmetry called G_F and, moreover, that this is the same symmetry for up and down fermions in the SM fermion doublets. For each lepton and neutrino field $\Psi = \{L_{\mathrm{L}}, \nu_{\mathrm{R}}\}$, a three-dimensional representation of the G_F which is called A^{Ψ} exists. For each Higgs multiplet Φ (if we assume that more (N_d) Higgs doublets exist), an $(N_d \times N_d)$ dimensional representation of the G_F, the so-called A^{Φ}, exists. Let us apply such a symmetry to the Yukawa term

$$L_{\mathrm{Y}} = -\sum_{i=1}^{N_d} \bar{L}_{\chi\mathrm{L}} \left(\tilde{h}_i^{\nu}\right)_{\chi,\delta} \Phi_i \nu_{\delta\mathrm{R}} = L'_{\mathrm{Y}} \,, \tag{1}$$

where h_i are Yukawa constants.

By definition, such a transformation should leave the Lagrangian unchanged and each field transformation can be described as follows

$$L'_{\alpha\mathrm{L}} = \left(A^{\mathrm{L}}\right)_{\alpha,\chi} L_{\chi\mathrm{L}} \,, \qquad \nu'_{\beta\mathrm{R}} = (A^{\nu})_{\beta,\delta} \nu_{\delta\mathrm{R}} \,, \qquad \Phi'_i = (A^{\phi})_{i,k} \Phi_k \,, \tag{2}$$

where $A^{\mathrm{L}}, A^{\nu}, A^{\Phi}$ are the appropriate representation matrices. In this case, we assume that the left-handed charged lepton fields and the left-handed neutrino fields transform in the same way.

Invariance of the Lagrangian (1) leads to an invariance in the coupling constants: $\tilde{h}_i^{\nu} = h_i^{\nu}$, so

$$\sum_{i=1}^{N_d} \left(A^{\mathrm{L}\dagger} h_i^{\nu} \left(A^{\Phi}\right)_i A^{\nu} \right) = h_i^{\nu} \,. \tag{3}$$

In general, the mass matrix can be denoted as

$$M^{\nu}_{\alpha,\beta} = \frac{1}{\sqrt{2}} \sum_{i=1}^{N_d} v_i \left(h_i^{\nu}\right)_{\alpha,\beta} \,, \tag{4}$$

where v_i are the vacuum expectation values of the Higgs fields.

If there are only three active families of leptons and only one Higgs doublet is introduced, matrix (4) is simplified and takes the form

$$M^\nu = \frac{1}{\sqrt{2}} v h^\nu \,. \tag{5}$$

The flavour symmetry gives the equality between the neutrino mass matrices before and after symmetry transformation. In the matrix notation

$$M^{\nu\prime} = A^{\mathrm{L}\dagger} M^\nu A^\nu = M^\nu \,. \tag{6}$$

Assuming the existence of only one three-dimensional irreducible representation of G_F ($A^{\mathrm{L}} = A^\nu = A$), which is the same for both the lepton and neutrino fields, one can find the commutation relation

$$A^\dagger M^\nu A = M^\nu \Leftrightarrow [M^\nu, A] = 0 \,. \tag{7}$$

The representation matrix may be obtained with the help of a group of generators, G_i,

$$A = G_1^a G_2^b G_3^c \,. \tag{8}$$

From Eqs. (7) and (8), it follows that all of the generators of the group in the three dimensional flavour space commutate with M_ν

$$[M^\nu, G_i] = 0 \,. \tag{9}$$

In the physical base for the charged leptons, the matrix which diagonalize the neutrino mass matrix is equivalent to the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix

$$M^l = \mathrm{diag}(m_e, m_\nu, m_\tau) \Rightarrow U^l = I \Rightarrow U_{\mathrm{PMNS}} = U^{l\dagger} U^\nu = U^\nu \,. \tag{10}$$

Then, there is a simple relationship between the elements of the PMNS matrix that we know from the experiment. If we assume that $u_i (i = 1, 2, 3)$ are columns of the PMNS matrix then, the elements of the generators for the G_F group can be expressed by:

$$G_1 = u_1 u_1^\dagger - u_2 u_2^\dagger - u_3 u_3^\dagger \,, \tag{11}$$

$$G_2 = -u_1 u_1^\dagger + u_2 u_2^\dagger - u_3 u_3^\dagger \,, \tag{12}$$

$$G_3 = -u_1 u_1^\dagger + u_2 u_2^\dagger + u_3 u_3^\dagger \,. \tag{13}$$

Hence, we can easily identify the group. All of the finite dimension subgroups of the SU(3) group up to 511 order was examined in this way and gave no satisfactory agreement with the experimental data [13].

All of the above considerations with one Higgs doublet are correct under the assumption, that the horizontal symmetry that is satisfied at high energy is broken at the present energies. Without this assumption, we obtain that $U_{\mathrm{PMNS}} = I$ [14].

4. Open problems and conclusions

To date, within the frame of the SM with one Higgs doublet and the current values of the mixing angles, it has been impossible to explain the lepton generation problem at the accepted level. It is worth checking whether new and better knowledge about the neutrino sector parameters produces a better agreement. It is also worth re-examining the SM with one Higgs doublet using the up-to-date experimental data.

New data may come from (1) the solar, atmospheric, accelerator and reactor neutrino oscillations (for better determination of the mixing angles and for discovering the Dirac CP violating phase), (2) from the MiniBooNE experiment, which is investigating the existence of sterile neutrinos (namely, the number of neutrino generations), (3) experiments of neutrinoless double beta decay, which can indicate the type of neutrino mass spectrum (hierarchy, inverse hierarchy and mass degeneration) and two possible Majorana CP violating phases.

Due to the difficulties in resolving the generation problem with the present experimental data, further theoretical studies are necessary. Such studies should focus on: (1) theories with the richer Higgs bosons sector that is used for spontaneous symmetry breaking, such as the SM with two Higgs doublets, as well as triplet and singlet Higgs fields, (2) a situation in which sterile neutrinos also appear in addition to three active neutrinos (one or more), (3) an examination of how the broken CP symmetry in the lepton sector is able to influence the family symmetry between leptons. For example, if we assume that one additional Higgs doublet appears in the model as is the case in two Higgs doublet model [15–18] for which we have to resolve the system of equations for the Yukawa coupling elements

$$h^{1l} A^{*\Phi}_{1k} + h^{2l} A^{*\Phi}_{2k} = A^{\mathrm{L}} h^{kl} \left(A^{\mathrm{R}}\right)^{+} , \qquad k = 1, 2 \tag{14}$$

for charged leptons (where h^{1l}, h^{2l} are matrices of Yukawa couplings), and

$$h^{1\nu} A^{\Phi}_{1k} + h^{2\nu} A^{\Phi}_{2k} = A^{\mathrm{L}} h^{k\nu} \left(A^{\mathrm{L}}\right)^{T} , \qquad k = 1, 2 \tag{15}$$

for neutrinos ($h^{1\nu}, h^{2\nu}$ are matrices of Yukawa couplings and where A^{L} which was introduced in Eq. (2) is the same for left-handed charged leptons and left-handed neutrinos, but A^{R} is an additional unitary matrix introduced for right-handed charged leptons). A^{Φ} is a two-dimensional representation of the investigated flavour symmetry for the Higgs fields. It is worth mentioning that in the case of two Higgs doublets, the symmetry imposed on the Yukawa constant does not pass on the symmetries of the mass matrices. Therefore, the unitary matrices that diagonalize the charged lepton and neutrino mass matrices are not the same, the $U_{\mathrm{PMNS}} \neq I$, and there is no need for horizontal symmetry breaking at low energies.

To summarise: to date, it was impossible to find the symmetry to explain the relations between masses and mixing matrices for quarks and leptons. Examining the discrete finite groups up to the high orders produces no results. Taking into account one Higgs and desiring to have a preserved charged current, confront into the contradiction that $U_{\mathrm{PMNS}} = I$. Perhaps we can solve this problem by using a second Higgs. Our investigations into this issue are being continued.

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