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ON QUASIBOUND N^*–NUCLEI

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The possibility for the existence of unstable bound states of the $S11$ nucleon resonance $N^*(1535)$ and nuclei is investigated. These quasibound states are speculated to be closely related to the existence of the quasibound states of the eta mesons and nuclei. Within a simple model for the NN^* interaction involving a pion and eta meson exchange, N^*–nucleus potentials for N^*–^{3}He and N^*–^{24}Mg are evaluated and found to be of a Woods–Saxon-like form which supports two to three bound states. In the case of N^*–^{3}He, one state bound by only a few keV and another by 4 MeV is found. The results are however quite sensitive to the $NN^*\pi$ and $NN^*\eta$ vertex parameters. A rough estimate of the width of these states, based on the mean free path of the exchanged mesons in the nuclei, leads to very broad states with $\Gamma \sim 80$ and 110 MeV for N^*–^{3}He and N^*–^{24}Mg respectively.

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1. Introduction

The $S11$ nucleon resonance $N^*(1535)$ has always been considered a crucial ingredient in the search for the elusive eta-mesic nuclei [1, 2]. Analyses of an anticipated eta-mesic nucleus model the eta–nucleon interaction to proceed via the formation of an $N^*(1535)$ resonance which repeatedly decays, regenerates and propagates within the nucleus until it eventually decays into a free meson and nucleon. Such a picture makes one ponder if a quasibound state of the N^* and nucleus might also exist. The idea of a "bound" state of an N^* and a nucleus is conceptually similar to that of a Δ and a nucleus which was indeed investigated in the past. In an experiment performed at

MAMI [3], the reaction $^{12}\mathrm{C}(e,e'\,\Delta^0)^{11}\mathrm{C} \to {}^{12}\mathrm{C}(e,e'\,p\pi\,)^{11}\mathrm{C}$ was investigated and the authors claimed to have found evidence for two narrow peaks which they interpreted as $^{12}\mathrm{C}_\Delta$ states. The authors distinguished the reaction with two scenarios: *(i)* a "quasifree" Δ^0 is produced from a bound neutron and it flies off in the forward direction and decays in such a way that the decay particles are produced in the forward direction in the laboratory frame and *(ii)* a bound Δ^0 is produced and the whole nucleus takes the momentum transfer such that the Δ^0 moves much slower and the decay products can, in principle, come out in any direction. The forward direction decay products were then excluded in order to look for the bound Δ^0. Though the authors did claim to have found a narrow Δ^0 bound nucleus, and a theoretical calculation by Walcher [4] tried even to explain its existence, these works were criticized in [5] due to the importance of the non-mesonic Δ decay, namely, $\Delta N \to NN$ (they found the width to be around 100 MeV) and the idea, in general, remained mostly ignored.

Coming back to the discussion of the N^*–nuclei, in the present work we shall investigate the possibility for the existence of N^*–^{3}He and N^*–^{24}Mg unstable bound states within a one-meson exchange model for the elementary NN^* interaction. Though not very obvious, these states could possibly be related to the formation of η–^{4}He and η–^{25}Mg quasibound nuclei. In the next section, we present the N^*–nucleus potentials and the method to locate the possible bound states of this potential.

2. N^*–nucleus potential

Since the N^*–N interaction is not well-known and the existence of such a baryon resonance-nuclear state is as such not really known, in the present work, we will try to make a simple estimate to see if any further sophisticated calculation is worth following. With this in mind, we shall use (a) a one-meson exchange $NN^* \to NN^*$ interaction which is scalar and does not involve the spin-dependent parts, and (b) the N^*–nucleus potential which is obtained by folding the elementary NN^* interaction with a nuclear density. Neglecting the spin-dependent parts is not a drastic assumption as we will see below. Since the $N^*(1535)$ is a negative parity baryon, indeed in the one-pion and -eta exchange diagrams, the spin-dependent terms are suppressed as compared to the leading scalar terms.

2.1. Elementary $NN^* \to NN^*$ potential

The diagrams which we shall consider are shown in Fig. 1. We consider an N^* which is neutral. The calculation for a positively charged N^* can be repeated in a similar way. We shall not consider diagrams involving the $N^*N^*\pi$ or $N^*N^*\eta$ couplings which are hardly known. Apart from this fact, for such diagrams, the potential turns out to be spin-dependent (and so also suppressed as compared to the leading term in the potential of Fig. 1).

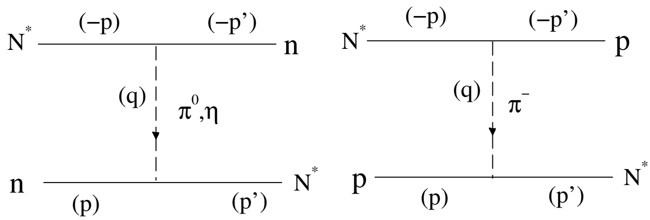

Fig. 1. Elementary $NN^* \to NN^*$ processes considered in this work.

The πNN^* and ηNN^* couplings (with $N^*(1535, 1/2^-)$) are given by the following interaction Hamiltonians [7]:

$$\begin{aligned} \delta H_{\pi NN^*} &= g_{\pi NN^*} \bar{\Psi}_{N^*} \vec{\tau} \Psi_N \cdot \vec{\Phi}_\pi + \text{h.c.}\,, \\ \delta H_{\eta NN^*} &= g_{\eta NN^*} \bar{\Psi}_{N^*} \Psi_N \cdot \Phi_\eta + \text{h.c.} \end{aligned} \tag{1}$$

Starting with, say, the diagram for the $N^*n \to nN^*$ process in Fig. 1 and using the standard Feynman diagram rules with the non-relativistic approximation for the spinors

$$u_i = \sqrt{2m_i} \begin{pmatrix} w_i \\ \frac{\vec{\sigma}_i \cdot \vec{p}_i}{2m_i c} w_i \end{pmatrix}, \tag{2}$$

we can write the amplitude as

$$\frac{g^2_{xNN^*} \bar{u}_{N^*}(\vec{p}\,') \, u_n(\vec{p}) \, \bar{u}_n(-\vec{p}\,') \, u_{N^*}(-\vec{p})}{q^2 - m_x^2}, \tag{3}$$

where $x = \pi$ or η and $q^2 = \omega^2 - \vec{q}^{\,2}$ is the four momentum squared carried by the exchanged meson ($q = p' - p$ as shown in the figure). Here, for example,

$$\bar{u}_n \left(-\vec{p}\,'\right) \, u_{N^*}(-\vec{p}) = N \left(1 \, - \, \frac{\vec{\sigma}_n \cdot \vec{p}\,' \vec{\sigma}_{N^*} \cdot \vec{p}}{4 m_N m_N^* c^2}\right) \tag{4}$$

and we drop the second term in the brackets which is spin-dependent as well as $1/c^2$ suppressed. The potential in momentum space obtained from the above amplitude is given as:

$$v_x(q) = \frac{g^2_{xNN^*}}{q^2 - m_x^2} \left(\frac{\Lambda_x^2 - m_x^2}{\Lambda_x^2 - q^2}\right)^2, \tag{5}$$

where the last term in brackets has been introduced to take into account the off-shellness of the exchanged meson. The momentum transfer $q^2 = \omega^2 - \vec{q}^{\,2}$

in the present calculation is approximated simply as $q^2 \simeq -\vec{q}^2$. The neglect of the energy transfer in the elastic $NN^* \to NN^*$ process is not necessarily justified but introducing a finite energy transfer gives rise to poles in (5) thus making the calculation of the N^*–nucleus potential a formidable task. Hence, restricting ourselves to a calculation within this approximation, we Fourier transform the potential in (5) to obtain the potential in r-space. The Fourier transform of (5) can be calculated analytically and we get

$$v_x(r) = \frac{g_{xNN^*}^2}{4\pi}\left[\frac{1}{r}\left(e^{-\Lambda_x r} - e^{-m_x r}\right) + \frac{\Lambda_x^2 - m_x^2}{2\Lambda_x}\, e^{-\Lambda_x r}\right]. \tag{6}$$

The elementary potentials for two different parameter sets of the coupling constants for the πNN^* and ηNN^* vertices are shown in Fig. 2.

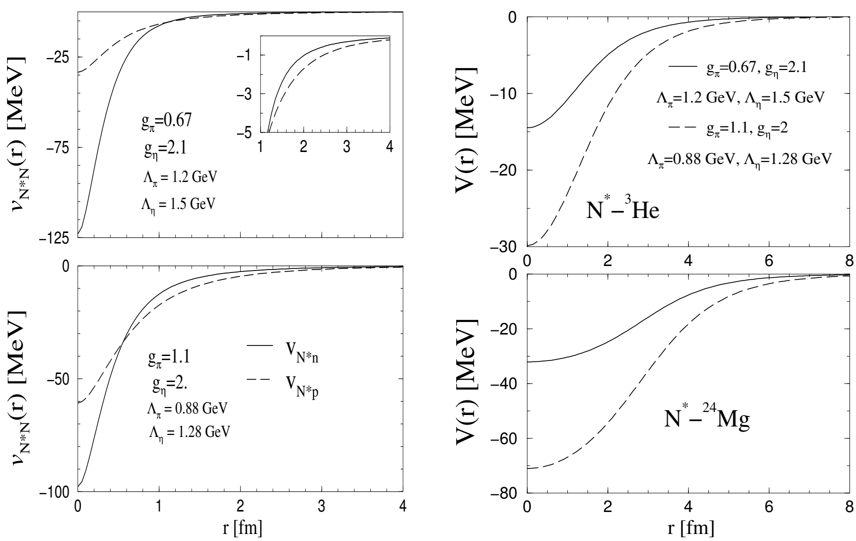

Fig. 2. Elementary $NN^* \to NN^*$ potentials with π and η exchange (left) and N^*–nuclear potentials (right).

2.2. N^*–^{3}He and N^*–^{24}Mg potentials

Once the elementary potential has been defined, we use the folding model

$$V(R) = \int d^3r\, \rho(r)\, v\left(\left|\vec{r} - \vec{R}\right|\right), \tag{7}$$

to construct the N^*–nucleus potential $V(R)$ and write

$$\begin{aligned} V(R) &= V_p(R) + V_n(R) \\ &= Z \int d^3r\, \rho_p(r)\, v_p\left(\left|\vec{r} - \vec{R}\right|\right) + N \int d^3r\, \rho_n(r)\, v_n\left(\left|\vec{r} - \vec{R}\right|\right), \end{aligned} \tag{8}$$

where Z and N are the number of protons and neutrons, $v_n(r) = v_{\pi^0}(r) + v_\eta(r)$ and due to the isospin factor appearing in the π^- exchange diagram (see Fig. 1 and Eq. (1)), $v_p(r) = v_{\pi^-}(r)\vec{\tau}_1 \cdot \vec{\tau}_2$. We also assume $\rho(r) = \rho_n(r) = \rho_p(r)$ with $\rho(r)$ normalized to 1. After performing the angle integration, the above integral reduces, for example, to

$$V_n(R) = \frac{-2\pi A}{R} \int \left\{ \frac{e^{-m_x(|r-R|)} - e^{-m_x(r+R)}}{m_x} - \frac{e^{-\Lambda_x(|r-R|)} - e^{-\Lambda_x(r+R)}}{\Lambda_x} \right.$$
$$\left. +B\left[\left(\frac{r+R}{\Lambda_x} + \frac{1}{\Lambda_x^2}\right) e^{-\Lambda_x(r+R)} - \left(\frac{|r-R|}{\Lambda_x} + \frac{1}{\Lambda_x^2}\right) e^{-\Lambda_x|r-R|}\right] \right\} r\, dr\, \rho(r)\,,$$

where $A = g^2_{xNN^*}/4\pi$ and $B = (\Lambda_x^2 - m_x^2)/2\Lambda_x$.

In the case of the ^{3}He nucleus, the nuclear density $\rho(r)$ is a sum of Gaussians [8] and the above integral can, in principle, be done analytically. However, such an attempt leads to lengthy expressions with error functions and exponentials which are not particularly enlightening and hence we rather perform the integral numerically. The density for ^{3}He is taken from [8] and that for ^{24}Mg is assumed to have a standard Woods–Saxon form. The N^*–nuclear potentials (in Fig. 2) can be fitted reasonably well to the Woods–Saxon forms of potentials. This facilitates the search for bound states of this potential.

3. Bound states of the N^*–nucleus potential

The Schrödinger equation for the Woods–Saxon potential can be reduced to one for the hypergeometric functions [9] and a condition for the existence of bound states can be found. For a Woods–Saxon potential of the type

$$V(r) = -\frac{V_0}{1 + e^{\frac{r-R}{a}}}\,, \tag{9}$$

the Schrödinger equation

$$\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} + \frac{2m}{\hbar^2}(E - V)u = 0 \tag{10}$$

may be transformed to the independent variable $y = 1/[1+e^{r-R/a}]$ to obtain a hypergeometric differential equation. After some lengthy algebra [9], one obtains the following condition for bound states:

$$\frac{\lambda R}{a} + \Psi - 2\phi - \arctan\frac{\lambda}{\beta} = (2n-1)\frac{\pi}{2}\,, \qquad n = 0, \pm 1, \pm 2, \ldots \tag{11}$$

where

$$\frac{2mE}{\hbar^2}\,a^2 = -\beta^2\,; \qquad \frac{2mV_0}{\hbar^2}\,a^2 = \gamma^2\,; \qquad \lambda = \sqrt{\gamma^2-\beta^2}$$

and $\phi = \arg\Gamma(\beta + i\lambda)$; $\Psi = \arg\Gamma(2i\lambda)$.

3.1. Binding energies of the N^*–nuclei

The N^*–nucleus potentials shown in Fig. 2 can be very well fitted using a Woods–Saxon form and the condition in (11) used to determine if a bound state of the N^*–nucleus potential exists and at what energy. The table below gives the parameters of the Woods–Saxon fits to the N^*–nucleus potentials and the energies of the bound states obtained using (11). The results are tabulated for two-parameter sets of the coupling constants for the πNN^* and ηNN^* vertices.

TABLE I

	$g_{\pi NN^*} = 0.67$, $g_{\eta NN^*} = 2.1$ $\Lambda_\pi = 1.2$, $\Lambda_\eta = 1.5$ GeV [10] E, V_0 [MeV], a, R [fm]	$g_{\pi NN^*} = 1.1$, $g_{\eta NN^*} = 2$ $\Lambda_\pi = 0.88$, $\Lambda_\eta = 1.28$ GeV [11]
N^*–^{3}He	$E = -0.03$ $V_0 = 18$, $a = 0.8$, $R = 1.3$	$E = -3.9$ $V_0 = 37$, $a = 0.84$, $R = 1.4$
N^*–^{24}Mg	$E = -17.1, -1.8$ $V_0 = 34$, $a = 0.9$, $R = 2.9$	$E = -47.6, -20.8, -2.6$ $V_0 = 76$, $a = 0.98$, $R = 2.9$

3.2. Estimate of the widths

Given that the N^*–nucleus is not expected to be a "bound" state (with infinite lifetime) but rather an unstable- or quasi-bound state, we also give a rough estimate of its width using a procedure similar to that of Ref. [4]. Assuming an average mean free path of the π (or η) to be given by $\langle l(\omega)\rangle = (\rho\,\sigma(\omega))^{-1}$ and also assuming that the N^* was produced, say, at the centre of the nucleus, the number of times that the meson rescatters is given by

$$N(\omega) = g_{\rm corr}\left(\frac{R}{\langle l(\omega)\rangle}\right)^2 = g_{\rm corr}\,[R\rho\sigma(\omega)]^2\;, \tag{12}$$

where we assume, as in [4], that the geometric factor $g_{\rm corr}$ is to be multiplied if it is assumed that the N^* is homogeneously produced over the nucleus. Starting with the amplitude as a function of the energy ω as

$$G(\omega) = G_0\frac{\hbar}{\sqrt{2\pi}}\frac{-i}{(\omega-\omega_0-\epsilon)+i(\Gamma/2)} \tag{13}$$

and taking into account that the meson does not propagate as a plane wave between rescatters in the nucleus (after being produced and absorbed due to the N^* decay), $|G(\omega)|^2$ is found to be

$$|G(\omega)|^2 = G_0^2 \frac{\hbar^2}{2\pi} \frac{1}{(\omega - \omega_0 - \epsilon)^2 + (\Gamma/2)^2} \frac{\sin^2((N(\omega)+1)\phi(\omega)/2)}{\sin^2(\phi(\omega)/2)}, \tag{14}$$

where $\phi(\omega)$ is the phase advance experienced by the propagating meson and is given by

$$\phi(\omega) = \arctan\left(\frac{\omega_0 + \epsilon - \omega}{\Gamma/2}\right) \tag{15}$$

and

$$\sigma(\omega) = \sigma_0 \frac{(\Gamma/2)^2}{(\omega - \omega_0 - \epsilon)^2 + (\Gamma/2)^2}. \tag{16}$$

Here, ω_0 is the difference of the N^* and N masses (~ 597 MeV). In Fig. 3, we see a plot of the function $|G(\omega)|^2$ (normalized to its peak value) as a function of ω for different values of the cross section parameter σ_0. The peak position is shifted from 597 MeV due to the meson phase factor as well as the binding energy, ϵ, of the N^* in the nucleus. As we can see, the distributions become narrow for increasing values of the cross sections. In the same figure, to the right, we see the full width at half maximum as a function of σ_0 for the N^*–^{3}He and N^*–^{24}Mg nuclei. The absorption cross section parameter, σ_0, depends on the magnitude of the cross sections in $\pi N \to \pi N$ and $\eta N \to \eta N$ scattering in the N^*-resonance region. These cross sections are of the order of 3 fm^2, for example, for $\pi^- p \to \pi^- p + \pi^0 n$

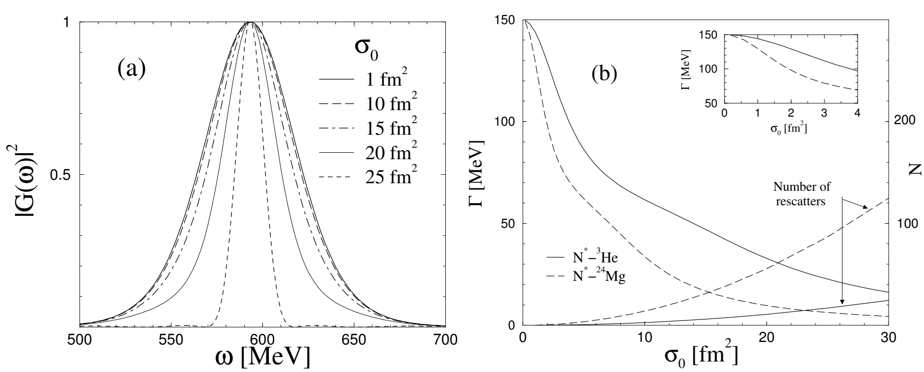

Fig. 3. (a) Energy dependence of the normalized resonance curve $|G(\omega)|^2$ for different values of σ_0 and (b) the full width at half maximum (Γ) of the resonance curves as a function of σ_0. The maximum number of rescatterings (N) of the exchanged meson are also shown as a function of σ_0 (with the scale on the right-hand side).

in the N^*-resonance region. In Fig. 3, we also see the maximum number of rescatters that the meson would perform before leaving the nucleus at each value of σ_0. It appears from the figure that for the range of relevant cross sections, the meson will not even rescatter once and, in this case, the state would be broad (for example, at $\sigma_0 = 3$ fm^2, $\Gamma \sim 80$ MeV for N^*–^{3}He and about 110 MeV for N^*–^{24}Mg). It seems only consistent that if the cross sections are bigger, then there are more rescatters and the state is longer lived (small Γ) as seen in the figure. The curves in Fig. 3 are not very sensitive to the binding energy of the N^*–nucleus.

To summarize, we can say that within the simple model calculation done here, very broad states of N^*–^{3}He and N^*–^{24}Mg may exist. If an eta-mesic nucleus is visualized in the form of an eta meson propagating inside the nucleus via the formation, decay and regeneration of the N^* resonance, the above could imply the existence of broad eta-mesic nuclear states [12].

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