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TWO-PHOTON TRANSITION FORM FACTORS OF PSEUDOSCALAR MESONS IN THE PHOKHARA AND THE EKHARA GENERATORS*

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The implementation of the newly developed two-photon transition form factors for pseudoscalar mesons in the PHOKHARA and the EKHARA generators is discussed. The forthcoming developments in the PHOKHARA generator are shortly reviewed.

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1. Introduction

The two-photon transition form factors of the pseudoscalar mesons are important for a precise determination of theoretical predictions of the light-by-light (LbL) contribution to the anomalous magnetic moment of the muon $(g-2)_\mu$, where one observes the discrepancy between the measured value [1] and the Standard Model predictions [2–4]. The main source of the error in the evaluation of $(g-2)_\mu$ comes from the hadronic part. The biggest contribution to its value and its error is the hadronic vacuum polarization (HVP), while the LbL contribution is the second largest source of the uncertainty. Unfortunately, we are not able to calculate these contributions with acceptable error from the first principles. Instead, we have to rely on effective models in the case of LbL, or the dispersive integral approach in the case of HVP.

In these proceedings, we report on the latest upgrade of PHOKHARA [5] and EKHARA [6, 7] Monte Carlo event generators related to the modeling of the newly developed [8] transition form factors of pseudoscalar mesons, and we shortly review the short-term research program.

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2. The implementation in the PHOKHARA generator

The newly developed phenomenological model of the two-photon transition form factors of the π^0, η and η' has been implemented in the Monte Carlo event generator PHOKHARA. In our approach, we rely on the resonance chiral effective Lagrangian [9–11], which contains multi-octet contributions and takes into account the η–η' mixing scheme adopted from [12, 13]. This model is an extension of the model presented in [14] and allows to cover also the time-like region. The model parameters have been fixed by performing two different fits to experimental data. In the first case, the mixing parameters of η–η' have been fixed (fit1) according to [12, 13] and in the second case, they have been used as free parameters of the model (fit2). For more details, see [8].

Using this new model of the form factors, we have calculated amplitudes for the reactions $e^+e^- \to P\gamma(\gamma)$, $P = \pi^0, \eta, \eta'$. These amplitudes have been implemented in the PHOKHARA generator. For each of these channels, two modes are possible, the scan mode where the invariant mass of the $P\gamma$ system is fixed at the leading order (LO) and the radiative return mode, where the invariant mass of the $P\gamma$ at the LO depends on the energy of the photon(s) emitted from the initial states. The diagrams relevant for the considered processes in the scan mode include the leading order (LO) amplitude from figure 1 (a), virtual and soft photon corrections represented by diagram from figure 1 (b) and an additional emission of a hard photon from the initial state represented by the diagram in figure 1 (c). For the radiative return mode, the only contribution which we take into account comes from the class of diagrams from figure 1 (c).

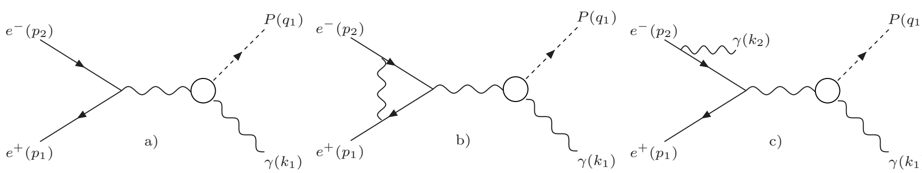

Fig. 1. Diagrams for the process $e^+e^- \to P\gamma(\gamma)$.

The next-to-leading order (NLO) cross section is given by the following formulae:

$$\sigma_{\rm NLO} = \sigma_{1\gamma}\left(1 + \Delta_{\rm soft,virt,1ph}\right) + \sigma_{2\gamma}\,, \tag{1}$$

where $\sigma_{1\gamma}$ is the LO cross section with one photon in the final state, and $\sigma_{2\gamma}$ is the cross section with two photons in the final state. Soft and virtual corrections have the following form [15]:

$$\Delta_{\mathrm{soft,virt,1ph}} = \frac{2\alpha}{\pi}\left(\log\left(2w\right)\left(\log\left(s/m_e^2\right)-1\right)+3\log\left(s/m_e^2\right)-1+\zeta_2\right) , \tag{2}$$

where $w = E_\gamma^{\max}/\sqrt{s}$ is the soft photon cutoff, $s = (p_1+p_2)^2$, $E_\gamma^{\max}$ is the maximal energy of the soft photon. The independence of the next-to-leading order cross section from the separation parameter w is ensured by contributions coming from the hard photon emission, which after the numerical integration exhibits the same logarithmic dependence on w as the soft part. We have checked numerically that varying the w within one order of magnitude, the obtained values of the σ_{NLO} are equal with an accuracy better than 0.01%.

In order to improve the efficiency of the Monte Carlo generation, we have used the following procedure to generate the 3-body Lorentz invariant phase space. First, we generate one of the invariants $Q_1^2 = (k_1+q_1)^2$ or $Q_2^2 = (k_2+q_1)^2$ with the probability 1/2 and using the multi-channel Monte Carlo technique we absorb peaks in Q_i^2 into changes of variables. We have taken into account peaks from different regions of the phase space: soft photon emission for which we have used the logarithmic change of the variables, the narrow resonances (ϕ, ω), where we have used the inverse trigonometric functions to absorb the Breit–Wigner form of the resonances and one channel, where the Q_i^2 have been generated flat to account for the behavior between the narrow resonances. One of the photon's polar angles is generated flat in the rest frame of Q_i and another one in the center-of-mass (CM) frame of electron–positron. For the photon generated in the CM frame of the initial particles, we use change of the variables from Eq. (29) of reference [5], which allows to absorb the collinear emission peaks. The azimuthal angles are generated flat. Based on the generated invariant and angles, we generate the momentum of one of the photon in the CM frame of electron–positron and the remaining momenta in the rest frame of Q_i. Then we perform boost from the rest frame of Q_i into the CM frame of the initial particles.

The PHOKHARA Monte Carlo event generator was used to simulate the cross section of the reaction $e^+e^- \to P\gamma(\gamma)$. The simulations have been performed using the version of the form factor model in which the parameters that describe the η–η' mixing have been fixed (fit1). The results for fit2 were presented in [8]. In figure 2, we show the effect of radiative corrections. To obtain these plots, we have used the following event selection: We require at least one photon with the energy bigger than 0.5 GeV. This photon and the pseudoscalar meson have to be observed in the angular range between 20 and 160 degrees. The radiative corrections are big. The reason of this is the fast falling of the pseudoscalar transition form factor with the increasing value of the virtual photon invariant mass. For the LO cross section, the form factor, which enters the final vertex is calculated at s, while for two-photon amplitude, the form factor is calculated at value of Q_1^2 or Q_2^2. The main contribution to the difference between LO and NLO cross sections comes from the region of Q_1^2 (Q_2^2) much smaller than s.

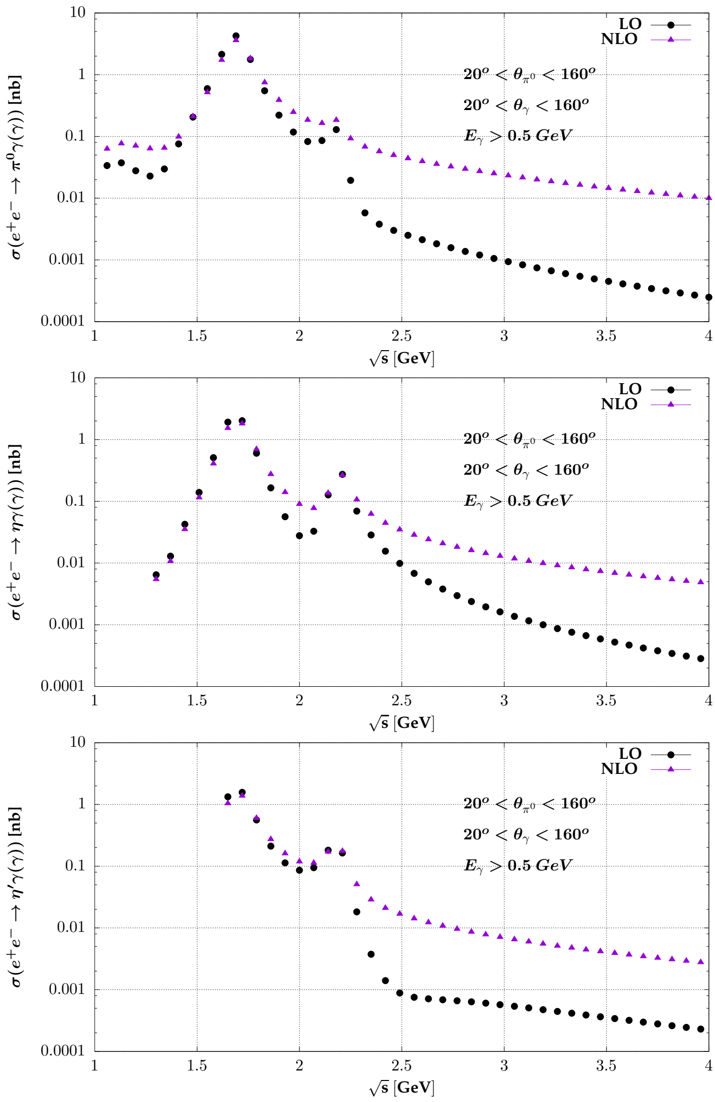

Fig. 2. The LO and the NLO cross section of the reaction $e^+e^- \to P\gamma(\gamma)$.

3. The implementation in the EKHARA generator

We have investigated the influence of the new model of the two-photon transition form factors of pseudoscalar mesons on the cross section of the reaction $e^+e^- \to e^+e^-P$, $P = \pi^0$, η, η'. This reaction have been simulated by the EKHARA generator. We investigate here the model dependence of the cross section on the second invariant. This was never tested experimentally and it influences the form factor measurement. In figure 3, we present the relative difference between the approximated cross section (approx), where we set one of the invariant in the transition form factor to zero and the complete cross section (full), where we take into account virtualities of both photons. In the case of the complete cross section, we limit the second virtual photon invariant mass square to $-t_1 < 0.18$ GeV^2 for π^0 and to $-t_1 < 0.38$ GeV^2 for η, η'. One can observe that corrections, which comes from the second invariant, are at the level of a few percents. The biggest influence of the non-zero value of t_1 is in the case of η. The differences between the old (2 octets) [14] and the new model (3 octets) [8] up to 1% can be observed in the case of η and η', while for π^0 the difference between the models is negligible.

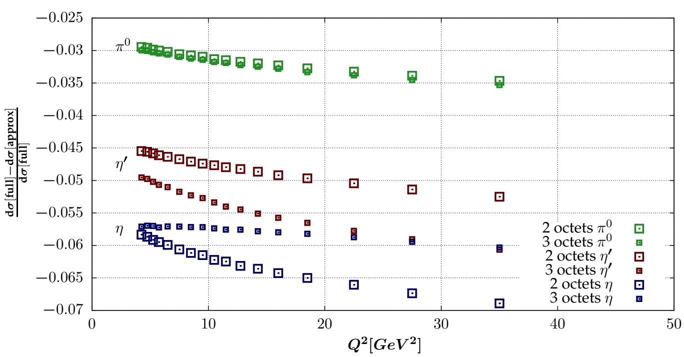

Fig. 3. Relative difference between the complete and the approximate cross section. See the text for details.

4. Forthcoming developments

We started to calculate and to implement in the PHOKHARA generator the radiative corrections to initial states at two-loop level for the process $e^+e^- \to$ hadrons $+\gamma$. The corrections of the order of α^2 will be added to the leading order amplitude and the corrections of the order of α will be added to the next-to-leading order amplitude using the leading logarithmic approximation. The virtual corrections in the leading logarithmic approximation were derived in [16]. For the soft photon emission, we will follow the method described in [17]. The amplitudes for the emission of 3 hard photons from initial states will be also added.

5. Conclusions

The new model of the transition form factors of the pseudoscalar mesons has been developed and implemented in the PHOKHARA and the EKHARA event generators. Using this new model and its implementation into the event generator PHOKHARA, we have investigated the impact of the radiative corrections on the $e^+e^- \to P\gamma$ cross section. Furthermore, we have studied the two-photon process with the pseudoscalar production in the case of non-vanishing values of both photon virtualities and reported about the forthcoming research.

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