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NON-IDENTICAL PARTICLE CORRELATION ANALYSIS IN THE PRESENCE OF NON-FEMTOSCOPIC CORRELATIONS*

ADAM KISIEL

Faculty of Physics, Warsaw University of Technology
Koszykowa 75, 00-662 Warszawa, Poland

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Femtoscopy is a technique of using correlations of two emitted particles to estimate the space-time extent of the source produced in heavy-ion collisions. Correlations of two non-identical particles have a unique additional feature of being sensitive to the difference in average emission position of the two particle types. For pion–kaon pairs, the femtoscopic signal arises from the Coulomb interaction between particles. Its strength is comparable to the magnitude of effects of non-femtoscopic origin. In this work, we identify main sources of these background correlations. We propose a robust method to estimate them and account for their influence in the femtoscopic analysis of experimental data. We validate the proposed correction method on a data sample generated with the THERMINATOR 2 model and provide a recipe for experimentalists.

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1. Introduction

In heavy-ion collisions at RHIC and LHC energies, a system is produced which is well-described by hydrodynamics. If "traditional" identical particle femtoscopy is used to measure such a system, the so-called "lengths of homogeneity" mechanism leads to the decrease of the measured system size with pair transverse momentum. Such decrease is observed universally in all experimental data for heavy-ion collisions [1]. The correlations of non-identical particles were proposed as a femtoscopic tool in [2]. In contrast to "traditional" femtoscopy, this type of measurement presents a unique additional possibility to extract the difference in average emission positions and times of two types of particles (later referred to as "emission asymmetry", or

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simply "asymmetry"). Such difference is also naturally occurring for systems where matter is behaving collectively [3–7]. The emission asymmetry is a signature of collectivity which is independent of the "lengths of homogeneity" effect. It serves as an additional critical cross-check. In particular, the fall of radii with momentum can also be explained if a static system with temperature gradients is emitting particles. However, no emission asymmetry is produced in this scenario. Therefore, it is important to experimentally verify if the emission asymmetry is indeed observed, to distinguish between these two scenarios. This has been done so far at RHIC [8] and at lower collision energies [9].

In this work, we consider the non-identical particle correlations at the LHC energies, with the help of the THERMINATOR 2 model [10]. In particular, we show that the elliptic flow phenomena produce a non-trivial background for non-identical particle femtoscopy. We discuss the origin of the background in detail and propose a robust method for accounting for it in realistic experimental scenarios. We also verify the validity of this procedure with dedicated simulations.

2. Non-identical particle femtoscopy formalism

The formalism of non-identical particle femtoscopy has been described in detail in [7]. Here, we only briefly remind the main elements which are of particular relevance to this work.

In theoretical formulation, a two-particle correlation function C_{AB} for a pair of particles of type A and B is connected with the "emission function" of the source S_{AB}, describing the probability to emit a given particle pair with momenta p_A and p_B from two given emission points x_A and x_B, respectively

$$C_{AB}\left(\vec{k^*}\right) = \int S\left(\vec{p_A}, \vec{x_A}, \vec{p_B}, \vec{x_B}\right) \left|\Psi_{AB}\left(\vec{k^*}, \vec{r^*}\right)\right|^2 \mathrm{d}^4x_A \mathrm{d}^4x_B\,, \tag{1}$$

where k^* is a momentum of the first particle in the rest frame of the pair, r^* is the space-time separation of the two emission points and Ψ_{AB} is the function describing the interaction, which we discuss below. In this function, all relevant interactions for a given pair should be included. We consider pairs consisting of a charged pion and a charged kaon. For such a pair, Coulomb interaction is dominant. Strong interaction is also present, but is expected to be small, therefore we neglect it. With such an assumption, Ψ_{AB} becomes the Bethe–Salpeter amplitude for the pair [7]

$$\Psi_{AB} = \sqrt{A_C(\eta)}\left[e^{-ik^*r^*}F(-i\eta, 1, i\xi)\right]\,, \tag{2}$$

where A_C is the Gamov factor, $\xi = k^*r^*(1+\cos\theta^*)$, $\eta = 1/(k^*a_C)$, and F is the confluent hypergeometric function. θ^* is the angle between $\vec{k^*}$ and $\vec{r^*}$, and a_C is the Bohr radius which is equal to ±248.52 fm for the pion–kaon pair. The function calculated according to Eq. (1) shows a strong correlation effect[1] at low k^* (positive for opposite-charge pairs, negative for same-charge pairs) and asymptotically approaches unity (the "no-correlation" value) for large k^*.

In the experiment, such a correlation function is measured by collecting pairs of particles from data. In particular, all pions of a given charge are combined with all kaons of a given charge and a distribution of their k^* is created. If both particles come from the same event, they are stored in the so-called "signal" histogram S. If each of the two particles is taken from different event, they form the so-called "background" distribution B. The correlation function is then simply: $C = NS/B$, with the normalization factor N calculated in such a way that C is at unity at large k^*. This way of constructing the "background" distribution, called "mixing", ensures that the single-particle acceptance effects are divided out in the procedure of the calculation of C.

It is possible to imitate experimental data analysis procedures using Monte Carlo models such as THERMINATOR 2. However, there is one important limitation. No current model implements the two-particle interaction in the particle emission process. Therefore, correlation functions from models, calculated in the way described above, would show no correlation. The effect of Final State Interaction is added in a so-called "afterburner" or "weighting" procedure, where a pair in S is stored with an additional weight equal to $|\Psi_{AB}|^2$. The model correlation function constructed in this way most closely resembles an experimental one. In a "blind" test, such a function can be treated as experimental in order to validate the analysis methods.

3. THERMINATOR 2 model simulations

This work is based on simulations in the THERMINATOR 2 model [10], which was selected because it incorporates the necessary collectivity phenomena. It is an event and particle generator, and for each particle its space-time creation point is known, which is essential for femtoscopy analysis. In particular, the hypersurfaces from the (3+1)D hydrodynamics code [11], generated for the Pb–Pb collisions at LHC energies at selected collision centralities were used. The event sample was the same as the one used in [12], which was shown to describe very well the space-time and momentum observables from the ALICE Collaboration. The samples for three centrality classes are used: 5–10%, 10–20%, and 30–40%.

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[1] The correlation effect is equal to $C - 1$.

4. Correlation function representation

In the most general case, the correlation function $C(\vec{k^*})$ is a three-dimensional object[2]. It can be represented in several ways, one of them is a decomposition into spherical harmonics. It was shown in [7] that this representation has specific unique advantages for the representation of a pion–kaon correlation function. Only two major components of the representation — the $l = 0$, $m = 0$ component (or $\Re C_0^0$) and $l = 1$, $m = 1$ (or $\Re C_1^1$) contain the essential information about the direction averaged source size of the system and the emission asymmetry between the two particle types. In addition, the asymmetry, the main objective of this analysis, is reflected mainly in the $l = 1$, $m = 1$ component. We use this representation through this work.

5. Non-femtoscopic backgrounds

An example of the correlation function calculated for the THERMINAOR 2 sample at selected centrality according to the "experimental" procedure described above is shown in Fig. 1. It exhibits expected features. The

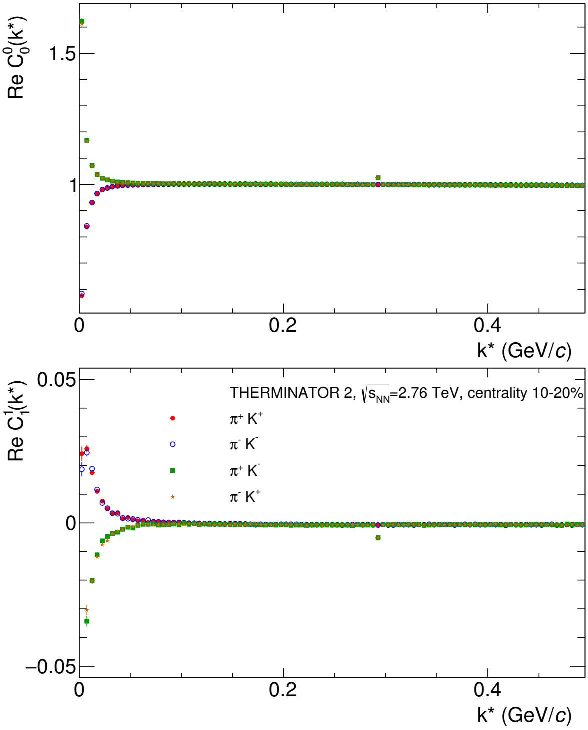

Fig. 1. Pion–kaon correlation functions for selected centrality.

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[2] The masses of the two particles are fixed, therefore the fourth component of relative four-momentum is not independent.

overall correlation effect represented by C_0^0 is positive for opposite-charge pairs and negative for same-charge pairs. The function is also close to unity at larger k^*. The asymmetry signal $\Re C_1^1$ clearly deviates from 0 in the correlation region. This is expected if indeed a non-zero asymmetry is present between pions and kaons. The fact that the asymmetry signal changes sign between same-sign and opposite-sign pairs is also consistent with this interpretation.

However, taking a closer look at the correlation, which is done in Fig. 2, one observes that C_0^0 is actually not flat at large k^*, while $\Re C_1^1$ has a non-zero negative value there. None of these effects are expected in a purely femtoscopic correlation function. It appears that our correlation contains not only femtoscopic correlations, but also a correlation coming from some other sources. The fact that this background is identical for all pair charge combinations suggests that global event-wide correlations are producing it.

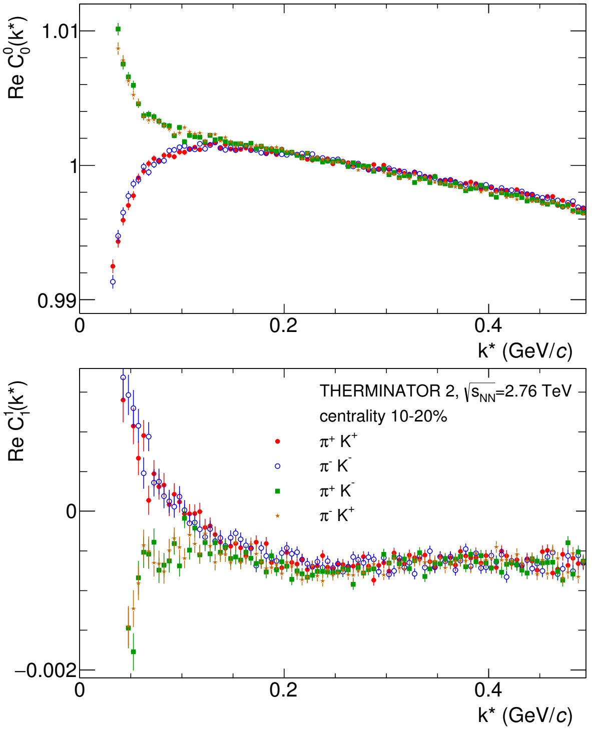

Fig. 2. Pion–kaon correlation functions for selected centrality, zoomed to emphasize the background effect.

One obvious candidate for the source of the non-femtoscopic correlations is the elliptic flow. Let us examine this hypothesis in detail. The non-femtoscopic correlations have a negative slope visible for large k^*. In other words, it is more probable to find a pair with large relative momentum in the case where a pion and a kaon are taken from two different events (the B sample) than in the case where both come from the same event (the S sample). When the elliptic flow is present in an event, all particles (including pions and kaons) are more likely emitted in a specific direction (in-plane) than in a direction perpendicular to it. This is a form of "collimation" of particles, which is another way of saying that the momenta of particles tend to point in the same direction. Therefore, their difference tends to be smaller, compared to the case when they are not collimated. In the case where we take "mixed" particles, both of them come from different events, so they do not share the same event plane. There is no "collimation" effect which makes combinations with larger momentum difference more likely. In conclusion, the hypothesis that "elliptic flow" is the source of the non-femtoscopic correlations seen in Fig. 2 is qualitatively reasonable.

In a model, we can test this hypothesis directly. We do this by slightly modifying the procedure of the correlation function construction. Before mixing the two particles, we rotate both events in such a way that their

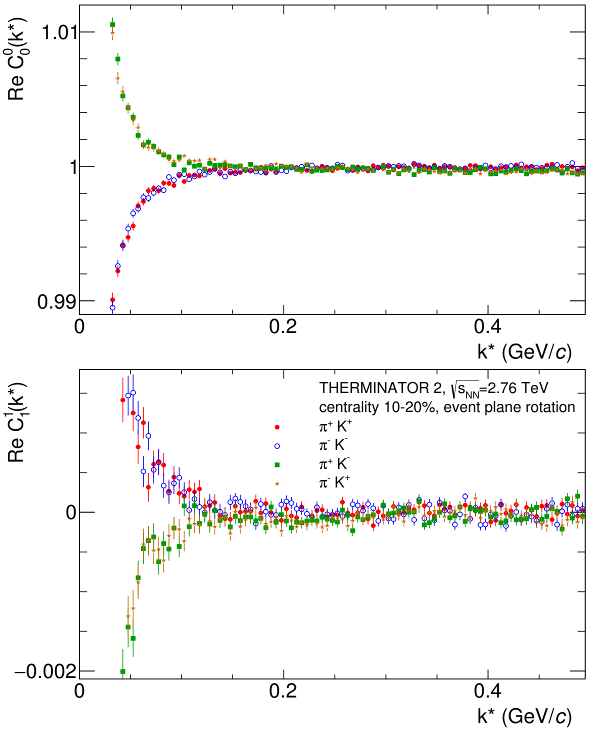

Fig. 3. Pion–kaon correlation function for selected centrality, with additional event plane rotation procedure applied (see the text for details), zoomed to emphasize the background region.

event planes point in the same direction. Then the "collimation" effect should apply both to S and B sample, and should be divided out in the correlation function. C should be "flat" in the large k^* region.

The correlation function calculated using this "rotation" procedure is shown in Fig. 3. The function is flat at large k^* for C_0^0 and at zero for $\Re C_1^1$, as expected for purely femtoscopic one. This is a strong evidence that indeed elliptic flow is causing the non-femtoscopic correlations.

6. Background correction procedure

We have shown that non-femtoscopic effects are present in the pion–kaon correlation function, and identified the main cause for such correlations. We have used a model-based calculation procedure which eliminates this correlation. However, such a procedure is usually not applicable in experimental analysis. The azimuthal angle acceptance is usually not perfectly uniform, in that case event rotation would break the fundamental assumption of the correlation function construction (the same single-particle acceptance for signal and background samples). There is also a finite resolution of the event plane angle determination. Therefore, we can instead use simulations to develop and test the "experimental" procedure to account for this effect in data. It is important to stress that this procedure will be data-driven and, as such, introduce a minimal theoretical systematic uncertainty on the measurement. The procedure will also exploit the specific unique feature of the non-identical particle correlation, where data for both same-sign and opposite-sign pairs are available, and they share the same "background" effect, while the signal for them is opposite.

In model calculations, we can isolate the background correlation with a straightforward modification of the procedure described above. Instead of the weight equal to $|\Psi|^2$ in the calculation of the correlation function, one simply puts unity. Then the femtoscopic part of this correlation is absent and only the non-femtoscopic "baseline" remains. Such a correlation is shown in Fig. 4. The baseline in C_0^0 appears, as expected, to be very similar for same-sign and opposite-sign pairs, with the only difference being a small shift in normalization between the two charge combinations. Similarly, in $\Re C_1^1$, the baselines are similar to each other. The shape of the baseline has a smooth dependence in C_0^0 and a more complicated shape in $\Re C_1^1$. Importantly, the shape of the background in the "femtoscopic" region is not a trivial extrapolation from the large k^* behaviour.

We now aim to characterize the baseline. A 6th order polynomial is necessary to fully capture the behaviour of the function. We have fitted a single polynomial to all 4 baseline functions for C_0^0 and another polynomial to all 4 baseline functions for $\Re C_1^1$. We only allowed the first polynomial

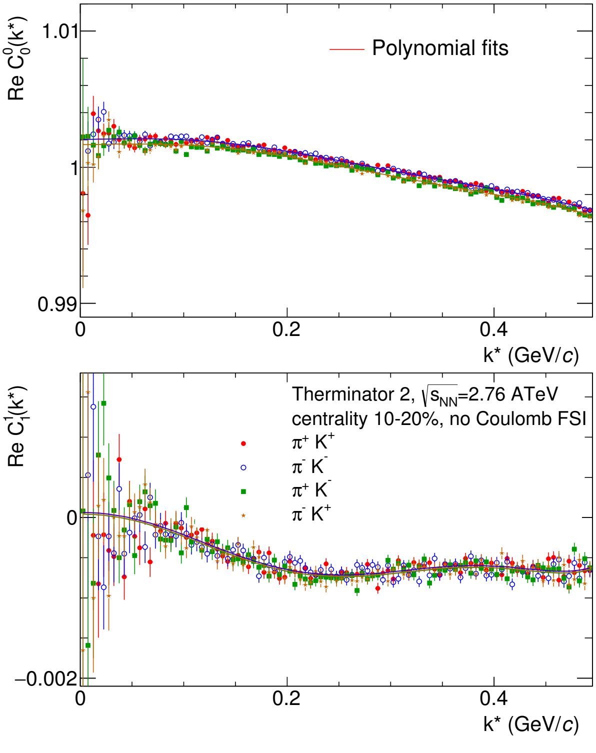

Fig. 4. Baseline correlation functions for selected centrality (see the text for details).

term to be different for each function, which resulted in 4 fitted functions which differ only by a slight vertical shift. We have plotted those functions on top of the MC ones in Fig. 4. The fits describe the data very well and the slight difference in the vertical direction between backgrounds for same-sign and opposite-sign pairs is only visible for C_0^0. That observation provides an important input for realistic experimental measurements. If a background is observed in the data, it can be assumed to have the same shape for all pair charge combinations. This provides a powerful constraint on the procedures which could allow to extract parameters of this background from data alone.

7. Extraction of femtoscopic parameters

In traditional femtoscopic analysis, the theoretical correlation function given by Eq. (1) is calculated with some assumption of the form of S and an analytical function is derived, which is then fitted to the experimental data. For a non-identical correlation function dominated by the Coulomb interaction, this procedure is not feasible. So we resort to the numerical integration of Eq. (1). We work in the Longitudinally Co-Moving System (LCMS), where the longitudinal momentum of the pair vanishes. We use the Bertsh–Pratt decomposition of the relative momentum into the "long" direction along the beam axis, "out" direction along the pair transverse momentum and "side", perpendicular to the other two. We assume that the source is a three-dimensional Gaussian. In addition to the size of the system, we also introduce the emission asymmetry between the two types of particles (here pions and kaons), which is expected to be non-zero only in the "out" direction [7]. S is then expressed as

$$S(\vec{r}) \approx \exp\left(-\frac{\left[r_{\mathrm{out}}-\mu_{\mathrm{out}}\right]^2}{2\sigma_{\mathrm{out}}^2}-\frac{r_{\mathrm{side}}^2}{2\sigma_{\mathrm{side}}^2}-\frac{r_{\mathrm{long}}^2}{2\sigma_{\mathrm{long}}^2}\right), \qquad (3)$$

where σ are the sizes of the system in the three directions and μ is the emission asymmetry. For a given set of parameters, a correlation function can be calculated and compared to "experimental" data. The procedure is then repeated for many sets of values of σ_{out} and μ. The results of the fit are the σ_{out} and μ parameter values for which the calculated function best describes the data. This fitting method is described in more detail in our previous work [7], where the procedure was also validated and shown to give correct results. When fitting only the two components of the correlation function spherical harmonics decomposition, only two parameters can be determined unambiguously. Therefore, in the fitting, we put $\sigma_{\mathrm{side}} = \sigma_{\mathrm{out}}$ and $\sigma_{\mathrm{long}} = 1.3\,\sigma_{\mathrm{out}}$, following the relations between these sizes for identical pions [13]. The fit procedure has then only two parameters: σ_{out} describing the overall size of the system, and μ_{out}, giving the magnitude of the emission asymmetry.

We apply the fitting procedure to the "reference" correlation functions which do not contain any background. The results of this fit are shown in Fig. 5 in the leftmost row. The size of the system as well as emission asymmetry grow in magnitude for more central collisions. The average reference size and asymmetry for each centrality are additionally highlighted with the vertical dashed lines. Clearly, results for all charge combinations are consistent within statistical uncertainty as expected.

Next, we proceed to fit the correlation function without attempting to correct for the background. The results are shown in the same figure, in the second row labelled "No corr.". The system size σ is in a reasonable agreement with the reference fit, however the asymmetry μ shows very large variations, up to 50% in magnitude. The variations are visibly stronger for least central collisions. This confirms our earlier observation, which identified elliptic flow as the main background source — in these collisions, the elliptic flow is the strongest, hence the strongest background and most sig-

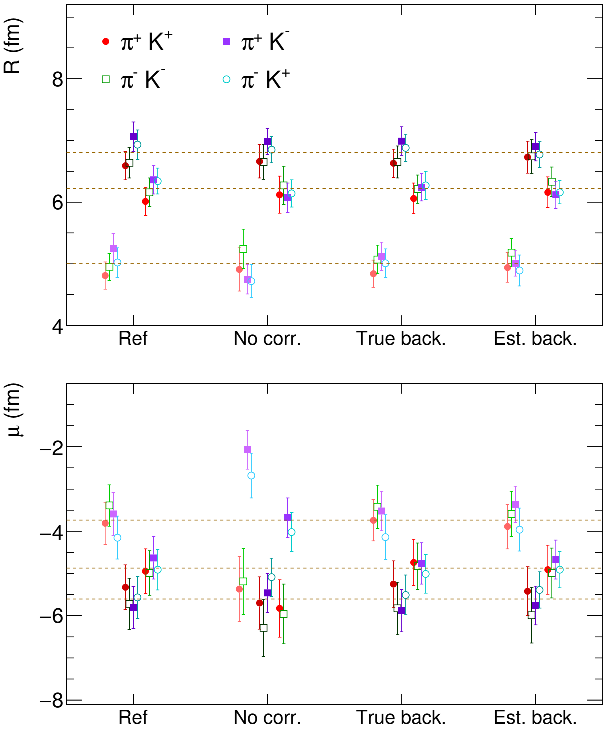

Fig. 5. Fit results for three centrality ranges (5–10%, 10–20%, 30–40%) for all pair-charge combinations, as a function of the procedure to account for the background (see the text for details). The lines represent the average of the reference fit values for each centrality.

nificant perturbation of the results. In addition, as expected, the asymmetry magnitude is shifted in opposite directions for pairs of the same charge (increased magnitude) and opposite charges (decreased magnitude). Such large variations of the fit results are unacceptable, therefore a procedure to account for the background must be proposed.

We take the polynomial fits to the pure background correlation function described above and we subtract the background magnitude from both the C_0^0 and $\Re C_1^1$ functions. We then refit those functions. The results are shown in the third row of Fig. 5, labelled “True back.”. The correction procedure is behaving as desired — the results are again consistent with the “reference” fit and values for all pair-charge combinations are consistent with each other for all centralities. This shows that when the true background is known, it is additive with the correlation effect and simple subtraction is the correct procedure to account for it.

In the experimental data, however, it is not possible to measure a "pure" background, it is always convoluted with the femtoscopic effect. Moreover, we have shown that the background shape is non-trivial in the region of the femtoscopic effect, therefore it is also not possible to characterize background outside of the femto region and then extrapolate. A dedicated procedure must be proposed which estimates the background in the full k^* range, using only the full correlation functions for all charge combinations.

Fortunately, for non-identical pion–kaon pairs, the correlation effect in C_0^0 is positive for opposite-charge pairs ("os") and negative for same-charge pairs ("ss"). If they both sit on top of the same background, it should be possible to extract it reliably. For the Coulomb interaction in a given k^* bin, the correlation function values are approximately connected by $C_{\rm ss} = 1/C_{\rm os}$ when there is no background. The experimental correlation functions $C_{\rm ss}^{\rm E}$ and $C_{\rm os}^{\rm E}$ contain both the Coulomb effect and the background. We are, therefore, trying to find such a value of the background G, for which

$$\left(C_{\rm ss}^{\rm E} - G\right) = 1/\left(C_{\rm os}^{\rm E} - G\right) \, . \tag{4}$$

Following our experience with pure background, we propose that G is a 6th order polynomial. In the minimization procedure, we look for such functional form of G for which the "$++$" correlation corrected for background according to Eq. (4) is as close as possible to the inverse of "$+-$" corrected in the same way, "$++$" is close to the inverse of "$-+$", "$--$" is close to the inverse of "$+-$", and "$--$" is close to the inverse of "$-+$" simultaneously. We employ a χ^2 test, where each k^* bin enters with the weight determined by the statistical errors of the correlation functions. We only allow for the normalization of G to change between the charge combinations, while the shape is the same for all four combinations. A similar procedure is carried out for $\Re C_1^1$, but here the same-sign and opposite-sign effects should have the same magnitude after background subtraction and opposite sign. We test this "experimentalist's" procedure on our "full" correlation function. We obtain new estimates of the background. We again correct the full correlations for this background with a simple subtraction and refit the corrected correlations. The results of the fit are shown in the fourth row of Fig. 5, labelled "Est. back.". The fit results are again satisfactory: they are in agreement with the "reference" values and the results for all charge combinations are consistent with each other. The procedure works for all centralities, including those with large backgrounds. It can, therefore, be directly used by experimentalists in the analysis of data on non-identical particle correlations. This procedure introduces minimal theoretical systematic uncertainty. In fact, with the accuracy of the Monte Carlo studies shown in this work, we observe that the procedure does not introduce any systematic shift (the "reference" and "corrected" results are consistent with each other). We estimate that the theoretical systematic uncertainty coming from the proposed background estimation and correction procedure is smaller than 5%.

8. Conclusions

Realistic simulation of the pion–kaon correlation functions were performed in the THERMINATOR 2 simulated events for the LHC energies at selected centralities. In addition to the femtoscopic effect, the correlations also contain significant backgrounds. The main source of this background was identified to be the particle collimation associated with elliptic flow. We have proposed a data-driven method to correct for this background. This procedure has been shown to work and to introduce a systematic uncertainty not larger than 5%. It can be directly applied in the upcoming measurement of non-identical particle correlations at the LHC and RHIC.

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