POLARIZATION EFFECTS IN NEUTRINO–NUCLEON INTERACTIONS*

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Our recent investigations of the spin asymmetry observables in the charged current inelastic and quasielastic neutrino (antineutrino)–nucleon scattering are reviewed. The spin asymmetry observables contain full information about the structure of the electroweak neutrino–nucleon vertex. Hence, they can be used to constrain the cross-section models for the single-pion production in ν-nucleon scattering and they allow to study the axial content of the nucleon and the second class current contribution to the quasielastic scattering amplitudes.

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1. Motivation

The neutrino oscillation has been an object of investigation for several decades. The parameters measured in oscillation experiments are squared mass differences Δm_{ij}^2 which are manifesting in periodicity of oscillation, CP-violation phase δ obtained from matter/antimatter asymmetry and mixing angles θ_{12}, θ_{23}, θ_{13} which are visible in the oscillation probability. Still, two parameters θ_{23} and δ are poorly established and the great effort has been made to get to know them better. Progress in the experimental studies requires an improvement of the theoretical models describing the neutrino–nucleus cross sections for the neutrino energies characteristic for the long-baseline experiments [1].

We consider two processes which are detected and utilized to study the oscillation phenomenon in the neutrino long-baseline experiments, such as T2K [2], namely, the charged current quasielastic (CCQE) ν-nucleus scattering and the charged current inelastic ν-nucleus scattering with the single pion in the final state.

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Usually, the models for the ν-nucleus scattering take for the input the elementary neutrino–nucleon interaction. Hence in this paper, we consider the elementary ν-nucleon and $\bar{\nu}$-nucleon interactions, namely:

(i) the CCQE process

$$\nu + n \to \mu^- + p \,, \tag{1}$$

where n/p refers to neutron/proton;

(ii) the single pion production (SPP) in the charged current ν-nucleon scattering

$$\nu + N \to \mu^- + N' + \pi \,, \tag{2}$$

where N and N' denotes initial and final nucleon, respectively.

The CCQE ν-nucleon scattering is described by simple model in which the elementary nW^+p vertex is parametrized by three form factors [3]: two vector form factors, known from the electron–nucleon scattering analyses and one axial form factor, $F_{\rm A}$, which is obtained from the analysis of the neutrino–deuteron scattering data [4] (for recent review, see [5]). The axial part of the scattering amplitude gives a dominant contribution to the cross section for the CCQE scattering, hence, the precise knowledge of $F_{\rm A}$ is required [6]. Within the same simple formalism, it is possible to discuss the contributions which are not allowed by the Standard Model. They are described by the so-called second class currents (SCC), for a recent review, see [7].

The description of the SPP in the ν-nucleon scattering is more complicated than the CCQE approach. Indeed, the SPP model should take into consideration two mechanisms for the pion production: resonant (RES) and nonresonant. The latter gives a contribution to the so-called nonresonant background (NB). A freedom in the description of the SPP dynamics results in the model dependence of the theoretical predictions. Moreover, from the analysis of the unpolarized differential ν-nucleon cross-sections data, it is impossible to extract the RES and the NB contributions separately. Hence, to validate models new observables are desired [8].

The spin asymmetry observables contain additional, averaged to spin cross sections, information about the nature of the interactions and the form of the elementary vertex. In this review, we give a number of examples showing that the spin observables contain nontrivial information about RES and NB amplitudes in the SPP [8–10]. Additionally, our recent results for the spin asymmetry observables in the CCQE interactions are shortly presented. It is demonstrated that the target spin asymmetry is responsive to the axial form factor $F_{\rm A}$ and the SCC contributions.

The paper is organized as follows: Section 2 introduces the necessary formalism: it consists of a resume of the CCQE and the SPP models. Section 3 presents the results and conclusions.

2. Formalism

2.1. Spin asymmetry observables

The differential cross section for either (1) or (2) process reads

$$\mathrm{d}\sigma = \mathrm{d}\sigma_0 \left(1 + \mathcal{P}_l^\mu s_\mu^l + \mathcal{T}^\mu s_\mu^N + \mathcal{P}_{N'}^\mu s_\mu^{N'} + \mathcal{O}\left(s^2\right)\right) , \tag{3}$$

where s_α^X denotes the spin four-vector for lepton ($X = l$), initial nucleon ($X = N$), and final nucleon ($X = N'$). Any spin four-vector satisfies the property $s_\alpha^X s^{X\alpha} = -1$. The terms bi-linear and tri-linear in spin four-vectors in Eq. (3) are omitted. $\mathcal{P}_{N'}^\mu$ and $\mathcal{P}_l^\mu$ are the polarization four-vectors of the recoil nucleon and final lepton, respectively. The target spin asymmetry is denoted by $\mathcal{T}_N^\mu$.

Each of these four-vectors has three independent components. Indeed, the polarization and target asymmetry vectors can be written in the form of[1]

$$\mathcal{P}_{N'}^\mu = -\sum_{a=\mathrm{L,T,N}} \mathcal{P}_{N'}^a \xi_a^\mu , \quad \mathcal{P}_l^\mu = -\sum_{a=\mathrm{L,T,N}} \mathcal{P}_l^a \zeta_a^\mu , \quad \mathcal{T}^\mu = -\sum_{a=\mathrm{L,T,N}} \mathcal{T}^a \chi_a^\mu , \tag{4}$$

where

$$\mathcal{P}_{N'}^\mu \xi_{a,\mu} = \mathcal{P}_{N'}^a , \quad \mathcal{P}_l^\mu \zeta_{a,\mu} = \mathcal{P}_l^a , \quad \mathcal{T}^\mu \chi_{a,\mu} = \mathcal{T}^a . \tag{5}$$

The basis vectors are given in Appendix A.

The polarization observables of the final particles in the CCQE scattering have been studied by many authors. Let us mention only very recent papers by Bilenky and Christova [11] and Fatima *et al.* [7]. In the first work, the impact of the axial form factor on the polarization of the lepton and recoil nucleon has been studied. In the other, the authors investigate the SCC contribution to the polarization of the final lepton and recoil nucleon. In this paper, we focus on the discussion of the target spin asymmetry. This observable has been not discussed yet.

The polarization of the final lepton in the SPP in ν-nucleon scattering has been studied by several groups, for reference, see [8].

The first calculations of the polarization components of the recoil nucleon is given in Ref. [8]. In the same work, the impact of the RES and the NB contributions on the spin observables is studied. In our next paper [9], the

[1] Negative sign in front of the sums is a result of negative normalization of the spin four-vectors.

target spin asymmetry, its dependence on the resonance and nonresonant contributions, is discussed. In the next sections, the results of these studies are shortly summarised.

2.2. Quasielastic scattering

The most general form of the electroweak nW^+p vertex[2], reads [3]

$$\Gamma^{\mu}_{+}(q) = F_1^{\mathrm{V}}\gamma_\mu + i\sigma^{\mu\nu}q_\nu\frac{F_2^{\mathrm{V}}}{2M} - \left(F_{\mathrm{A}}\gamma_\mu + \frac{F_{\mathrm{P}}}{2M}q_\mu + i\sigma^{\mu\nu}q_\nu\frac{F_3^{\mathrm{A}}}{M}\right)\gamma_5\,. \qquad (6)$$

$F_{1,2}^{\mathrm{V}}$ is the vector form factor of the nucleon, whereas F_{A} is the axial form factor, $F_{\mathrm{P}}(Q^2) = 4M^2F_{\mathrm{A}}(Q^2)/m_\pi^2 + Q^2$, where Q^2 is the square of the four-momentum transfer. The axial form factor F_3^{A} contributes to the SCC.

We adopt the standard dipole parametrization of the vector form factors [3] and the axial form factor, namely

$$F_{\mathrm{A}}\left(Q^2\right) = \frac{1.2723}{\left(1+Q^2/M_{\mathrm{A}}^2\right)^2}\,, \qquad (7)$$

where M_{A} is the axial mass parameter. The time invariance is assumed which implies reality of all form factors. For the SCC axial form factor, we adopt the parametrization [7]

$$F_3^{A}\left(Q^2\right) = \frac{F_3^{A}(0)}{\left(1+Q^2/M_{\mathrm{A}}^2\right)^2}\,. \qquad (8)$$

2.3. Single pion production models

In order to discuss the dependence of the polarization observables on the RES and NB contributions, we consider two SPP models: HNV (Hernandez, Nieves, and Valverde) [12] and FN (Fogli and Nardulli) [13].

In the HNV model, SPP is described by seven Feynman diagrams: two resonance and five nonresonant, which are motivated by the nonlinear σ model.

The FN model is simpler than the HNV and it contains only four SPP amplitudes: one resonance and three nonresonant. This model was derived from the linear sigma model. Models HNV and FN differ in the treatment of the NB. However, one should also mention that in the FN model, the resonance contribution is described by the oversimplified hadronic current.

[2] It is assumed that the vector current is conserved.

3. Results and summary

The polarization observables in the CCQE contain additional, averaged to the spin cross sections, information about the axial form factor [14]. As an example, in Fig. 1, we plot the components of the target spin asymmetry for antineutrino–proton scattering. It can be noticed that varying the axial mass parameter value results in the visible change of the components of $\mathcal{T}^{\mu}$. Notice that, when the nucleon form factors are real, then the normal components of target spin asymmetry vanish.

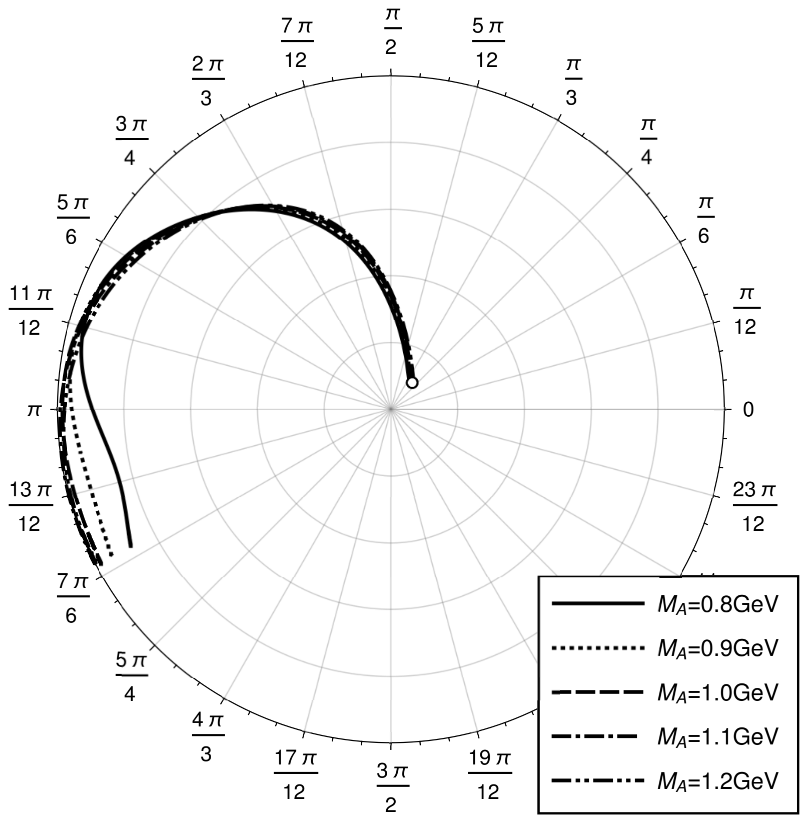

Fig. 1. The target spin asymmetry $\mathcal{T}_N$ in polar plot for $\bar{\nu}_\mu + p \rightarrow \mu^+ n$ scattering. The results are obtained for different axial mass parameters: $M_{\mathrm{A}} = 0.8$, 0.9, 1.0, 1.1, 1.2 GeV with $F_3^{\mathrm{A}(}0) = 0$. The radius corresponds to the $\mathcal{T} = \sqrt{\mathcal{T}_{\mathrm{L}}^2 + \mathcal{T}_{\mathrm{N}}^2 + \mathcal{T}_{\mathrm{T}}^2}$ but here $\mathcal{T}_N = 0$. An angle $\hat{\phi}$ is defined by $\cos(\hat{\phi}) = \mathcal{T}_N^L/\mathcal{T}$, $\sin(\hat{\phi}) = \mathcal{T}_N^T/\mathcal{T}$. The line length is parametrized by the antineutrino energy, then the open circle denotes the minimal energy, whereas the maximal energy is equal 5 GeV.

The asymmetry $\mathcal{T}^{\mu}$ is also sensitive to the SCC contribution, here described, only by the $F_3^{\rm A}$. It is illustrated in Fig. 2. Let us mention that the components of the target spin asymmetry calculated for the neutrino–neutron scattering are insensitive to the axial form factors $F_{\rm A}$ (see Fig. 3) and F_3^2.

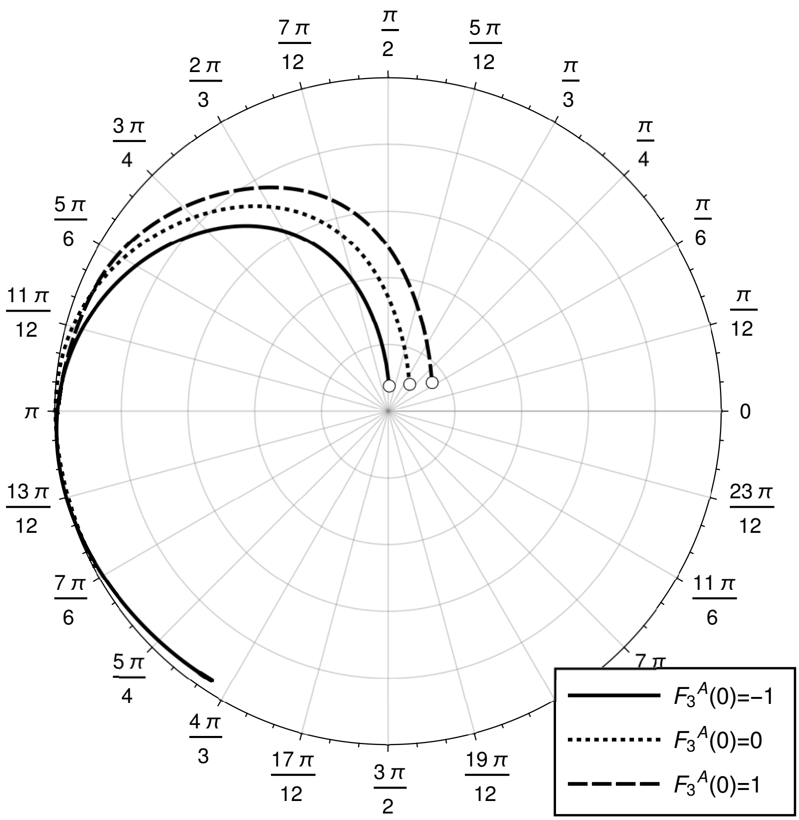

Fig. 2. Caption the same as in Fig. 1 but here the results are obtained for various values of $F_{\rm A}^3(0)$, with $M_{\rm A} = 1.0$ GeV.

As we have noticed above, the impact of the RES and the NB contributions on the polarization of the final lepton and recoil nucleon in the SPP processes was discussed in our paper [8]. It was shown that the normal component of the polarization of final lepton is given by the interference between the RES and the NB contributions. Therefore, its measurement would give information about the relative sign between the RES and the NB amplitudes as well as it would limit the model dependence in SPP approaches. The components of the polarization of the recoil nucleon are also sensitive to RES/NB contribution. Indeed, the dependence of resonance contribution to

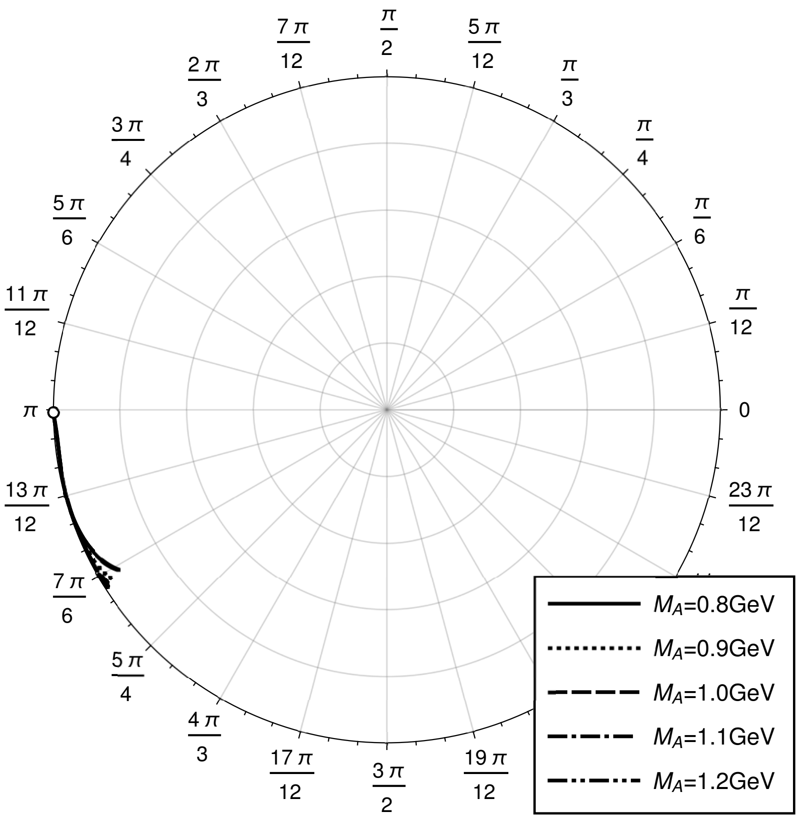

Fig. 3. Caption the same as in Fig. 1 but for $\nu_\mu + n \to \mu^- + p$.

the cross section on the pion scattering angle is sinusoidal and it is distorted by the NB contribution. Moreover, the normal component of the recoil nucleon, integrated over the pion variables, is dominated by the interference between the RES and the NB amplitudes, see Fig. 4. Measurement of this observable should significantly constrain the SPP models.

Eventually, in our last work [9], we showed that target spin asymmetry is also responsive to the details of the SPP models. We considered two polarizations of the target: normal and longitudinal to neutrino velocity. In both cases, the observables are sensitive to the RES and the NB contributions. We illustrate this property in Fig. 5, where the normal component of the target spin asymmetry is shown.

To summarize: the spin asymmetry observables contain information about the elementary electroweak nucleon vertex which is complementary to spin averaged quantities. Investigation of the spin properties in the neutrino–nucleon scattering should help in constraining theoretical models and searching for the signal from the physics beyond the Standard Model.

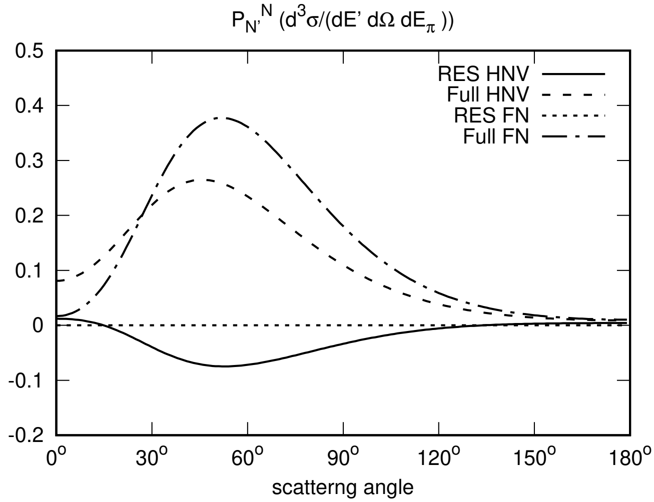

Fig. 4. The dependence of $\mathcal{P}^{N}_{N'}$ on the scattering angle (in the lepton scattering plane) calculated for $\nu_\mu + p \rightarrow \mu^- + p + \pi^+$ for the HNV and FN models and the neutrino energy $E = 0.7$ GeV, the energy transfer $\omega = 0.5$ GeV, the pion energy $E_\pi = 0.25$ GeV. The RES contributions are denoted by dotted (FN) and solid (HNV) lines, the full model result is represented by dashed/dash-dotted (HNV/FN) line.

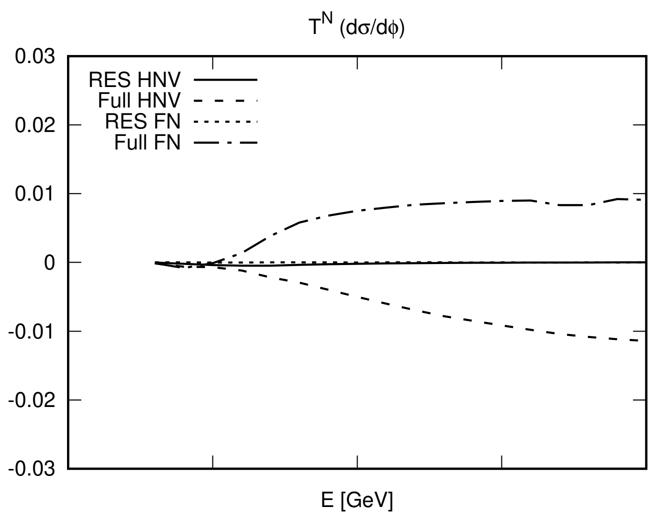

Fig. 5. The energy dependence of the normal component of the polarization asymmetry $\mathcal{T}^{\rm N}$ in the process $\nu_\mu + p \rightarrow \mu^- + p + \pi^+$ for the HNV and FN models. The RES contributions are denoted by dotted (FN) and solid (HNV) lines, the full model is denoted by dashed (HNV) and dash-dotted (FN) lines.

The calculations have been carried out in Wroclaw Centre for Networking and Supercomputing (http://www.wcss.wroc.pl), grant No. 268. A part of the algebraic calculations presented in this paper has been performed using the FORM language [15] and FeynCalc package [16, 17].

Appendix A

Spin basis vectors

Let $\boldsymbol{k}$ and $\boldsymbol{p}$ denotes the neutrino and the target momenta, whereas $\boldsymbol{k}$, $\boldsymbol{p}'$ and $\boldsymbol{k}_\pi$ represent the momenta of the final lepton, the recoiled nucleon and the pion, respectively. We keep the notation $E_p = \sqrt{\boldsymbol{p}^2 + M^2}$, where M is the particle's mass.

Longitudinal, transverse and normal spin vectors for the recoil nucleon N' read

$$\xi^\mu_{\rm L} = \frac{1}{M}\left(|\boldsymbol{p}'|, \frac{E_{p'}\boldsymbol{p}'}{|\boldsymbol{p}'|}\right), \quad \xi^\mu_{\rm T} = \left(0, \frac{\boldsymbol{p}' \times (\boldsymbol{p}' \times \boldsymbol{k}_\pi)}{|\boldsymbol{p}' \times (\boldsymbol{p}' \times \boldsymbol{k}_\pi)|}\right), \quad \xi^\mu_{\rm N} = \left(0, \frac{\boldsymbol{p}' \times \boldsymbol{k}_\pi}{|\boldsymbol{p}' \times \boldsymbol{k}_\pi|}\right). \tag{A.1}$$

For the CCQE, $\boldsymbol{k}_\pi \to -\boldsymbol{k}$ in the upper formula.

For the final lepton, we have

$$\zeta^\mu_{\rm L} = \frac{1}{m}\left(|\boldsymbol{k}'|, \frac{E_{k'}\boldsymbol{k}'}{|\boldsymbol{k}'|}\right), \quad \zeta^\mu_{\rm T} = \left(0, \frac{\boldsymbol{k}' \times (\boldsymbol{k} \times \boldsymbol{q})}{|\boldsymbol{k}' \times (\boldsymbol{k} \times \boldsymbol{q})|}\right), \quad \zeta^\mu_{\rm N} = \left(0, \frac{\boldsymbol{k} \times \boldsymbol{q}}{|\boldsymbol{k} \times \boldsymbol{q}|}\right). \tag{A.2}$$

For the target nucleon, basis vectors read

$$\chi^\mu_{\rm L} = \frac{1}{E}(0, \boldsymbol{k}), \qquad \chi^\mu_{\rm T} = \left(0, \frac{\boldsymbol{k} \times (\boldsymbol{k} \times \boldsymbol{q})}{|\boldsymbol{k} \times (\boldsymbol{k} \times \boldsymbol{q})|}\right), \qquad \chi^\mu_{\rm N} = \left(0, \frac{\boldsymbol{k} \times \boldsymbol{q}}{|\boldsymbol{k} \times \boldsymbol{q}|}\right). \tag{A.3}$$

Notice that the longitudinal direction of the target spin is chosen along the momentum of the neutrino.

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