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REVIEW OF EXPERIMENTAL DATA ON HADRON-NUCLEUS COLLISIONS AT HIGH ENERGIES*,**

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In this review an attempt is made to summarize briefly all that is presently known experimentally about hadron-nucleus collisions at high energies. Comparisons with theoretical models are kept to a minimum. However, an outline of some theoretical ideas that have been put forward in interpreting the data is included.

1. *Introduction*

The nucleus is the only tool available which allows, in a direct way, the experimental study of such intriguing topics as: the space-time development of particle production; the interaction of resonances with nucleons; and perhaps even the interactions of almost free quarks. It is thus not surprising that hadron-nucleus collisions at high energies have attracted in the last few years increasing interest, both experimental [1] and theoretical [2].

On the experimental side many interesting features have been observed, the most significant of which, in my opinion, are: the almost complete lack of cascading in hadron-nucleus collisions; the apparent near transparency of nuclear matter to coherently produced multiparticle states; the A-dependence of muon pair production by hadrons; and the A-dependence of the production of hadrons with large transverse momenta. On the theoretical side, although considerable progress has been made in understanding various qualitative features of hadron-nucleus collisions in terms of models of hadron-hadron interactions, no clear picture has so far emerged. However, all analyses, based on such intrinsically different theoretical ideas as hydrodynamics [3], cumulative effects [4], par-

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ton [5] or multiperipheral [6] models, suggest that hadron-nucleus collisions may indeed give us a better understanding of hadrons and their interactions.

In this review an attempt is made to summarize briefly all that is presently known experimentally about hadron-nucleus collisions at high energies. Comparisons with theoretical models will be kept to a minimum. When included they will only be used to help focus on what may be the more important aspects of the data. An outline of some theoretical ideas that have been put forward in interpreting the data is given in Appendix I. Neither in the text nor in the references has any attempt been made to be complete. In the literature, particularly that which covers emulsion work [7], there is considerable duplication of results. When this occurs, data for this review have been selected on an almost random basis (with some preference given to our results!). I apologize for all omissions.

2. *Hadron-nucleus cross sections*

Total and inelastic hadron-nucleus cross sections at high energies show no surprises. The mesured values, summarized in Fig. 1 can, to within experimental and theoretical uncertainties, be derived from hadron-nucleon cross sections using the Glauber Model. That is, they can be derived from the mean free path of hadrons in nuclear matter, as calculated from the elementary cross sections. It should be pointed out, however, that these seemingly obvious results are not, in general, a necessary consequence of the elementary cross sections. For example, in the context of the multiperipheral model and soft field

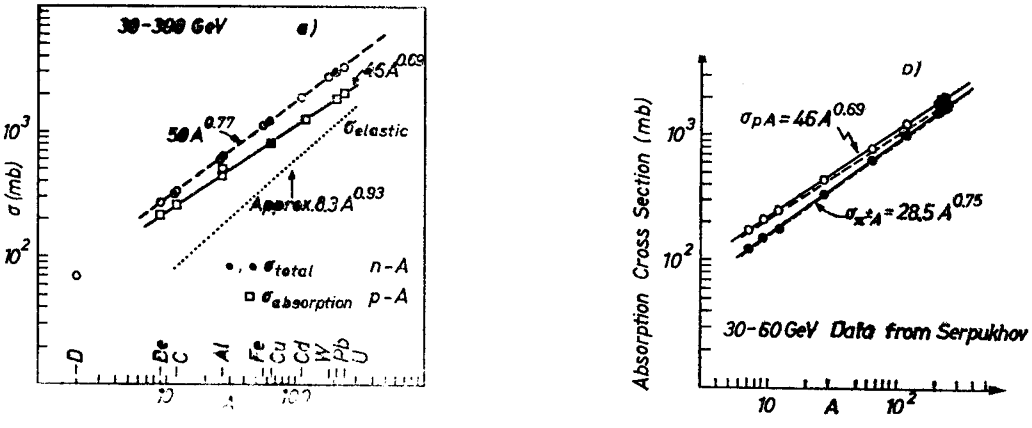


Fig. 1a). Total and inelastic cross sections in nucleon-nucleus collisions. The total cross sections are from Jones et al. [31], and from Biel et al. [32], averaged over the range 30–300 GeV. The inelastic cross sections are from Denisov et al. [33] averaged over the range 20–60 GeV. The elastic cross sections were obtained by subtraction. b). Comparison of measured inelastic cross sections with cross sections calculated using mean free path of hadron in nuclear matter. The data are from Ref. [33]. The broken curves are the calculated values based on $\sigma_{pp}^{\text{inelastic}} = 32.3$ mb, $\sigma_{\pi p}^{\text{inelastic}} = 21.2$ mb, and a Woods-Saxon distribution of nucleons $\varrho(r) = \varrho_0\,(1+\exp{(r-c)/a})^{-1}$ where $a = 0.54$, $c = (0.978+0.0206A^{1/3})\,A^{1/3}$, $\varrho_0 = \dfrac{3A}{4\pi c^3(1+\pi^2/c^2)}$ as given by Negele [34]

theory [8], there is no compelling reason why the Glauber Model should work so well and heavy nulcei appear so black at high energies. Therefore these results impose strong constraints on the model.

3. Cross sections of multiparticle states produced inside nuclear matter

If coherent production of multiparticle states is analyzed in terms of the Glauber Model, one observes an apparent transparency of nuclear matter to newly produced states. To be more specific:

Consider reactions of the type $\alpha + A \rightarrow \beta + A$ (e. g., $\pi + A \rightarrow 3\pi + A$) where α is the incident particle, A the target nucleus and β some particular multiparticle state produced coherently. Assuming the state β is produced on a single nucleon at position $\vec{x} = (\vec{b}, z)$ one can write the nuclear production amplitude as [9]:

$$F(t) = f(0) \int d^3x e^{i\vec{q}\cdot\vec{b} + iq_L z} n(\vec{x}) \exp\left\{ -\frac{\sigma_i'}{2} \int_{-\infty}^{z} dz' n(\vec{b}, z') \frac{\sigma_f'}{2} \int_{z}^{\infty} dz' n(\vec{b}, z') \right\}, \tag{1}$$

where $f(0)$ is the forward production amplitude on a nucleon, q_L is the longitudinal momentum transfer, $n(\vec{x})$ is the nucleon density at $\vec{x}$, $\sigma_i' = \sigma_i(1 - i\alpha_i)$ is the total cross section for the incident particle α on a nucleon, $\sigma_f' = \sigma_f(1 - i\alpha_f)$ is the corresponding quantity for the outgoing multihadron state β.

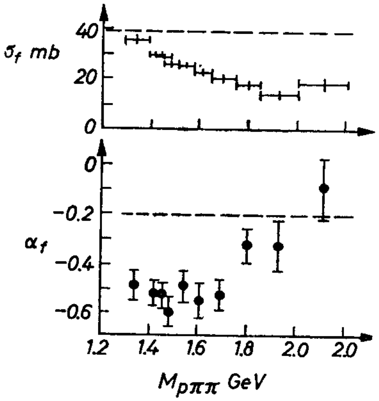


Fig. 2. Example of apparent transparency of nuclear matter to a coherently produced multihadron state. σ_f is the extracted total $(p\pi^+\pi^-)$-nucleon cross section from a Glauber analysis of $p+A \rightarrow p\pi^+\pi^- + A$ data at 22.5 GeV. α_f is the extracted value of the ratio of the real to the imaginary parts of the forward scattering amplitude. The data are from Ref. [10]

If one fits equation (1) to the measured differential cross section and obtains σ_f', one finds values of σ_f' which are comparable to, or smaller than, hadron-nucleon cross sections. They also tend to decrease as the mass of β increases.

For example, an analysis of $pA \rightarrow p\pi^+\pi^- + A$ data [10] yields a $(p\pi^+\pi^-)$-nucleon cross section which varies between 40 and 15 mb for $(p\pi^+\pi^-)$ masses in the range 1.2 to 2.2. GeV (see Fig. 2). Similar results have been extracted from $\pi \to 3\pi$, $\pi \to 5\pi$, $K \to K\pi\pi$, and $n \to p\pi^-$ data [9].

Clearly, this apparent transparency of nuclear matter to newly formed multiparticle states, is a reflection of the inadequacy of an analysis which ignores the interaction time and the evolution of the final asymptotic state as it propagates through the nucleus. For discussions of possible interpretations of the data see the recent review by Fäldt [9].

4. *Multiparticle production*

The most striking feature of multiparticle production in hadron-nucleus collisions is its weak A-dependence. There is little, if any, evidence of the buildup of an intranuclear cascade. Figure 3 illustrates how the pseudo-rapidity distribution varies with $\bar{\nu}$, the average thickness of the target nucleus. ($\bar{\nu}$ and other parameters used in describing the data are all defined in Appendix II).

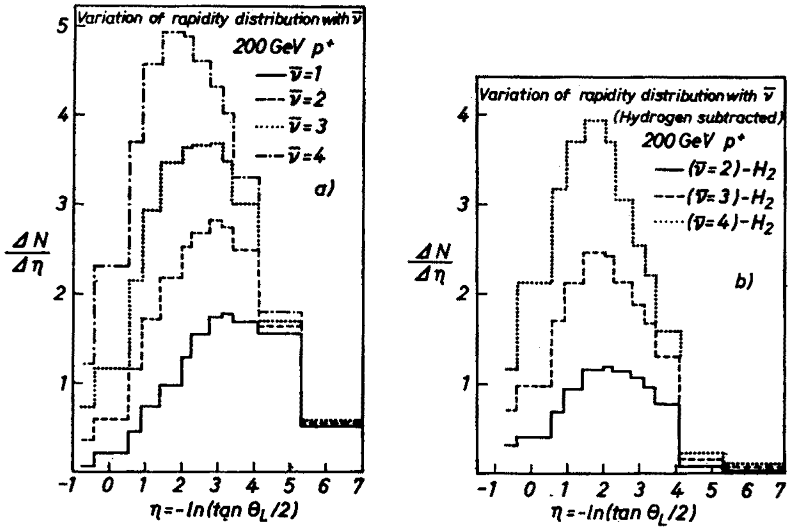


Fig. 3a, b). Example of the variation of pseudo-rapidity distributions with size of nuclear target. Data are from Busza et al. [35] $\bar{\nu}$ is a measure of the average thickness of the nucleus in units of the mean free path of protons in nuclear matter. N is the average number of charged relativistic particles produced in an inelastic collision. It includes both pions and fast recoil protons. For definitions see Appendix II

It is primarily this weak A-dependence and characteristic dependence of the pseudo-rapidity distributions on nuclear size that have, in recent years, stimulated theoretical work [2] on hadron-nucleus collisions; the hope being that such strong features in the data would prove powerful tests of theoretical models. Unfortunately, many classes of models qualitatively predict these results (see Appendix I) and the only definite conclusion that can be drawn is that in hadron-hadron interactions the asymptotic final state is not produced instantaneously. To learn more it is necessary to look in greater detail at the data. In particular:

(*i*) For a given range of rapidity how does the multiplicity vary wiht A? Are there, for example, indications that in the central plateau, for heavy nuclei, $R_A \sim 3$ or 4 independent of $\bar{\nu}$? If so, it would be strong evidence in favour of a multiperipheral or parton description of the data, in which such a plateau is a necessary consequence of the observed ratio of inelastic to total cross sections [8].

(*ii*) How do the rapidity distributions and R_A depend on energy? Is there evidence that the A-dependent region of rapidity expands with energy, indicative of some long range order? At very high energies does $R_A \rightarrow 1$ as predicted by Stodolsky's formation zone argument [11] and by models which assume that all particles are produced in clusters [12], most of which decay outside of the nucleus?

(*iii*) Is the leading particle spectrum strongly dependent on the target as predicted by models of a hydrodynamical nature [3]? Although there are still no definitive answers to these questions, a considerable amount of detailed information does exist.

The typical A-dependence of various regions of the pseudorapidity distribution are shown in Fig. 4, where $dN/d\eta$, normalized to $\bar{\nu} = 1$ (hydrogen) data, is plotted as a function of $\bar{\nu}$. It should be pointed out that there is still some controversy over the detailed A-dependence in the forward few units of pseudo-rapidity. This point is discussed in more detail later.

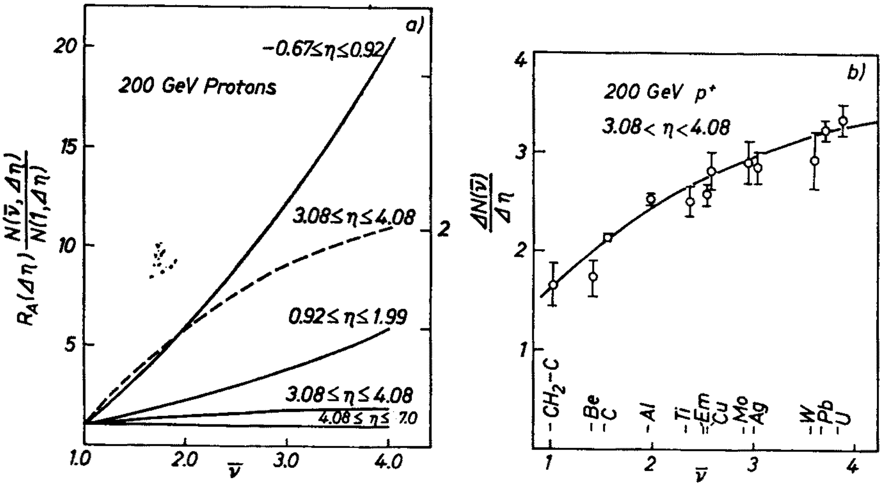


Fig. 4a). For various regions of the pseudorapidity distribution, the variation of $\Delta N/\Delta\eta$ as a function of $\bar{\nu}$. The scale for the broken curve is on the right. Data are from Busza et al. [35] These results show how little, if any, cascading occurs in the nucleus. b) Example of data which were used in obtaining Fig. 4a

Qualitatively the data suggest that there is some cascading in the target fragmentation region and no multiplication in the projectile fragmentation region. In the region of thec entral plateau, which at 200 GeV is still very small, there is a weak indication that R saturates as $\bar{\nu}$ increases.

The overall multiplication of particles in nuclei, as measured by $R_A = \langle n \rangle_A / \langle n \rangle_p$, depends only on $\bar{\nu}$ and is remarkably independent of incident energy and particle type.

See Fig. 5. For incident energies $\gtrsim$ 50 GeV, $R_A \simeq 1/2+1/2\bar{\nu}$. A more detailed comparison of the pseudorapidity distributions for incident protons and pions at the same energy (see Fig. 6) indicates that, to a large extent, the fact that R_A depends only on $\bar{\nu}$, and not on particle type, is fortuitous.

The only existing data covering a large energy range are from emulsions [7]. In Fig. 7 the value of R_A for emulsions is shown over the energy range 10–10^4 GeV. For $E > 300$ GeV the data are from cosmic rays and should be used with caution; each point is averaged

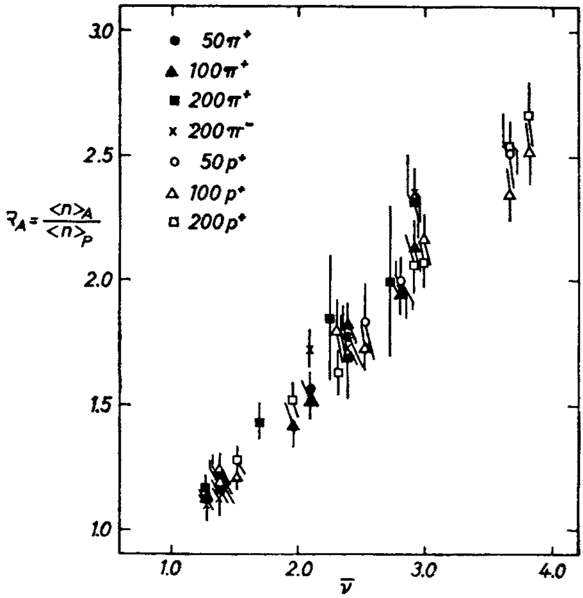


Fig. 5. Variation of R_A with $\bar{\nu}$. R_A is a measure of the multiplication of hadrons in the nucleus and $\bar{\nu}$ of the average thickness of the nucleus. For definitions see Appendix II. Data are from Busza et al. [35] As can be seen $R_A \sim 1/2+1/2\ \bar{\nu}$

over a wide energy range and the corresponding $\langle n \rangle_p$ obtained from an extrapolation of accelerator data using $\langle n \rangle_{ch} = 1.768 \ln S - 2.8$ and $\langle n \rangle_p = \langle n \rangle_{ch} - 0.5$. As can be seen, for $E \gtrsim 50$ GeV the data are consistent with R_{Em} being independent of energy. The same independence is seen over the range 50–200 GeV in p-nucleus and π-nucleus collisions (see Fig. 7).

In Fig. 8, a comparison of pseudo-rapidity distributions at various energies shows that in hadron-nucleus collisions, it is only the very backward target fragmentation region that is energy-independent. In addition, there are indications that even up to energies of a few TeV it is only the forward 2 or 3 units of rapidity that show no increase in multiplicity compared to hydrogen [13].

There are conflicting data as to the A-dependence of particles produced in the forward direction. For $p/p_{max} \gtrsim 0.5$ ($\eta \gtrsim 6.5$ at 200 GeV) the only precise data available are from beam surveys. These consistently show a decrease with A of particles produced per interacting proton. See Figs 9 and 10. Emulsion experiments (Fig. 11a) which use the number of observed nuclear fragments (N_h) as a measure of the size of the target nucleus, see in addition a decrease over the forward 2 or 3 units of pseudorapidity. On the other hand, to date all experiments, emulsion or counter, which have used pure nuclei as targets see no significant variation in that region of pseudorapidity. See Figs 3, 11b and 11c.

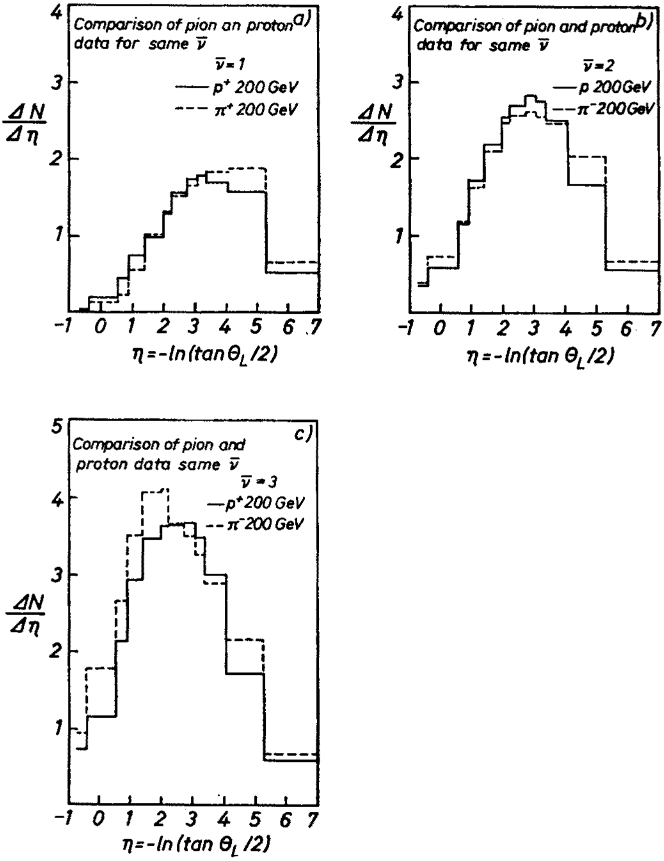


Fig. 6. Comparison of the pseudorapidity distributions for incident pions and protons. In each figure the π^- and p^+ distributions correspond to nucleus with the same average thickness $(\bar{\nu})$. The data are taken from Busza et al. [35]. These distributions were obtained by extrapolating to integer values of $\bar{\nu}$ data similar to those shown in Fig. 4b

As mentioned earlier, the behaviour of the forward region is important from a theoretical point of view and this experimental conflict should be resolved or understood.

There is overwhelming evidence that in hadron-nucleus collisions many target nucleons participate, and that the resultant multiplicity depends on the number of these nucleons. This is most apparent in Fig. 12, where the number of observed relativistic particles is

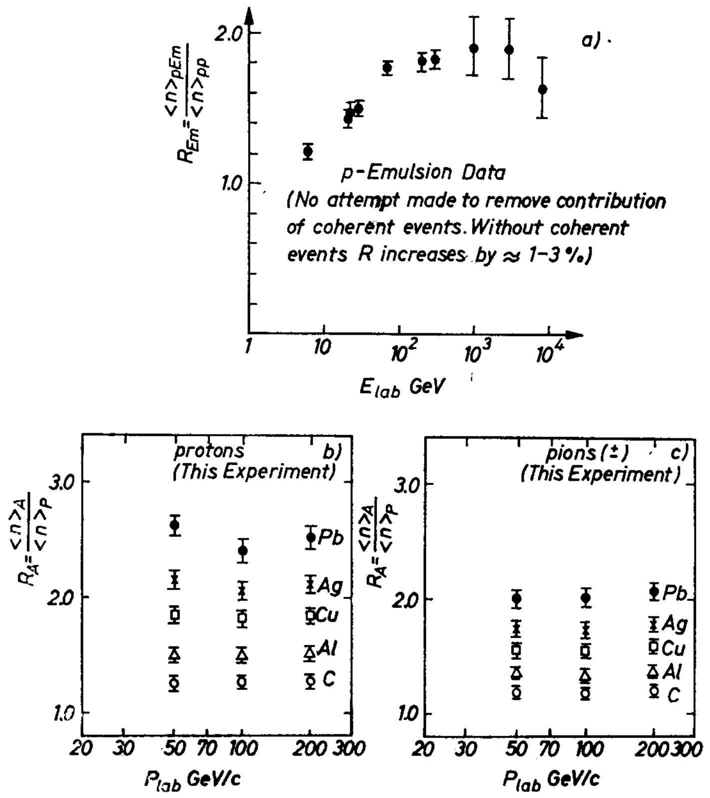


Fig. 7

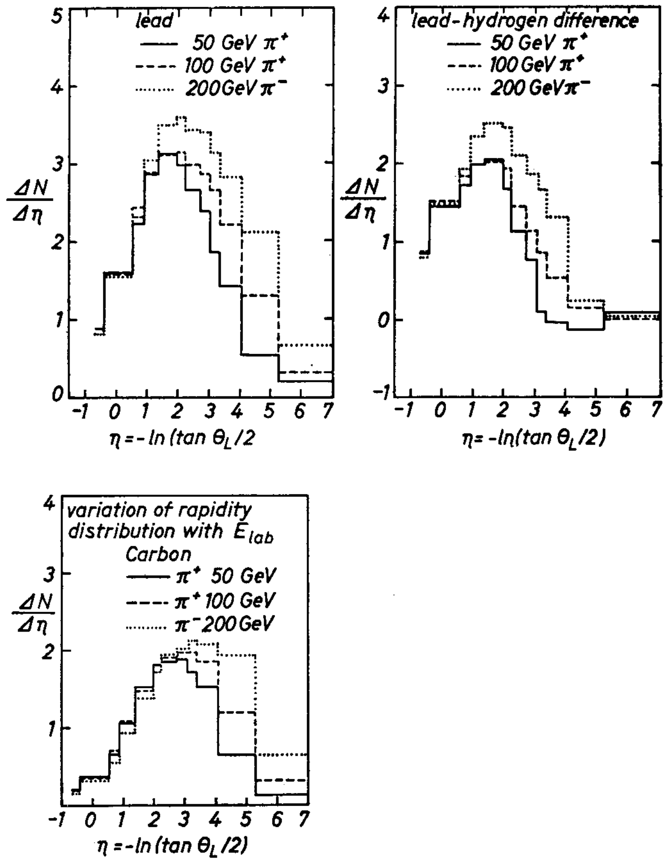


Fig. 8

Fig. 7. Energy dependence of the hadron multiplication factor, R_A in nuclei. For definitions of $\langle n \rangle_A$ and $\langle n \rangle_p$ see Appendix II. a) The emulsion data were obtained from Ref. [36]. See text for discussion of the cosmic ray data. b) and c) The data for these figures are from Ref. [35]

Fig. 8. Variation of pseudorapidity distributions with energy for Lead and Carbon target nuclei. In b) data are the same as in a), except that the corresponding hydrogen distribution has been removed. The growth of the A-dependent region in rapidity can be seen explicitly. Data are from Busza et al. [35]

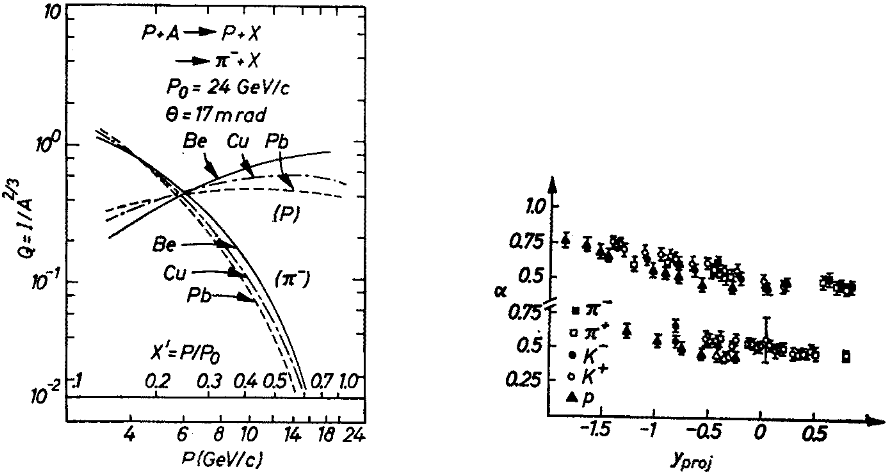


Fig. 9. A-dependence of the inclusive production of particles in the forward region for 19.2 and 24 GeV incident protons. The data are from Allaby et al. [37] at 19.2 GeV and Eichten et al. [38] at 24 GeV. Figure a) is taken from the compilation by Voyvodic [39] and b) from Bruno et al. [40]. In b) the A-dependence of the cross section is parametrised as $d\sigma/dp^3 \propto A^{\alpha}$

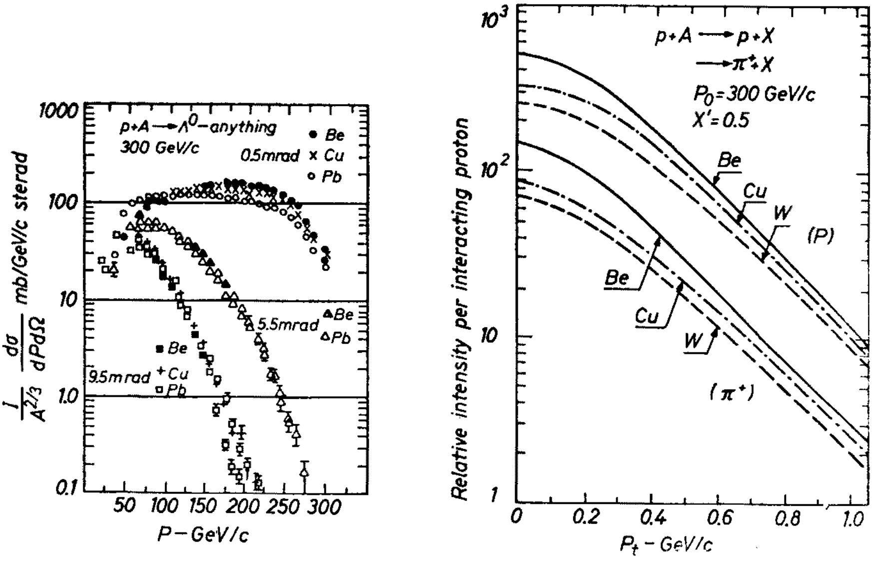


Fig. 10. Beam survey results at FNAL which illustrate the decrease with A of particles in the forward direction. The plots were taken from a compilation by Voyvodic [39]

plotted as a function of the number of nuclear fragments and fast recoil protons respectively.

Another interesting observation is the fact that in hadron-nucleus collisions the multiplicity distribution scales in the same way as in hadron-nucleon interactions; if one excludes the diffractive part and plots $\langle n\rangle\sigma_n/\sigma_{\text{inel}}$ as a function of $n/\langle n\rangle$, data from hadron-nucleon and hadron-nucleus collisions fall on the same universal curve (KNO scaling) [14]. This scaling has been directly observed in π^-C data [15], p-Emulsion data [16], and π^- Neon data [17] (see Fig. 13), but its validity for other nuclei can be inferred from the linear dependence of the dispersion D on $\langle n\rangle$ shown in Fig. 13. It should be remembered that for a Poisson distribution $D \propto \langle n\rangle^{1/2}$. At first sight this scaling may appear to be of fundamental importance. Several authors [18], however, have pointed out that it could simply follow from the way a nucleus averages data from collisions at different impact parameters.

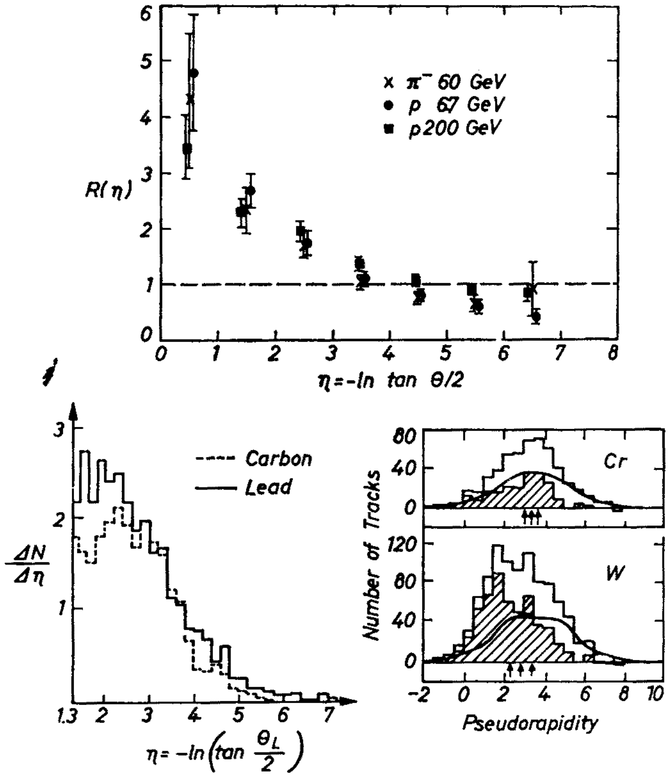


Fig. 11. Data on the A-dependence of leading particle spectrum. a) Emulsion data. Ratio $R(\eta)$ of number of particles produced in heavy nuclei to that in hydrogen. Identification of target nucleus through parameter N_h discussed in text. Data compiled by Wolter [41]. b) Spark chamber data of Abrosimov et al. [42] at 40 GeV. c) Data obtained by Florian et al. [43] using metal pellets in emulsion

Finally, since cumulative effects have been used as a possible interpretation of multiparticle production in hadron-nucleus collisions [4], a note on some results where these effects may have been seen directly.

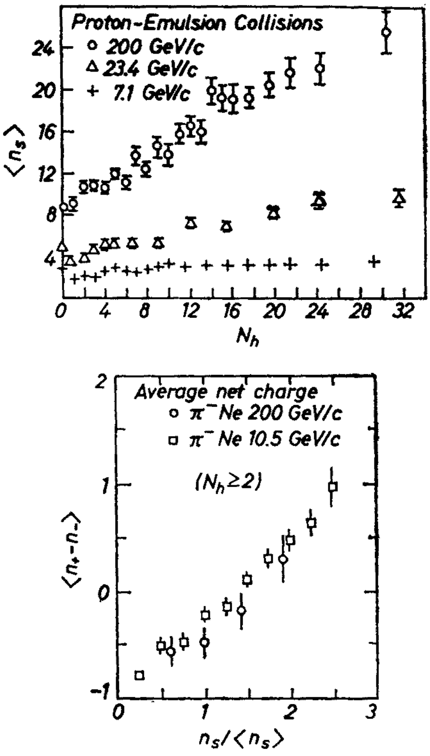


Fig. 12

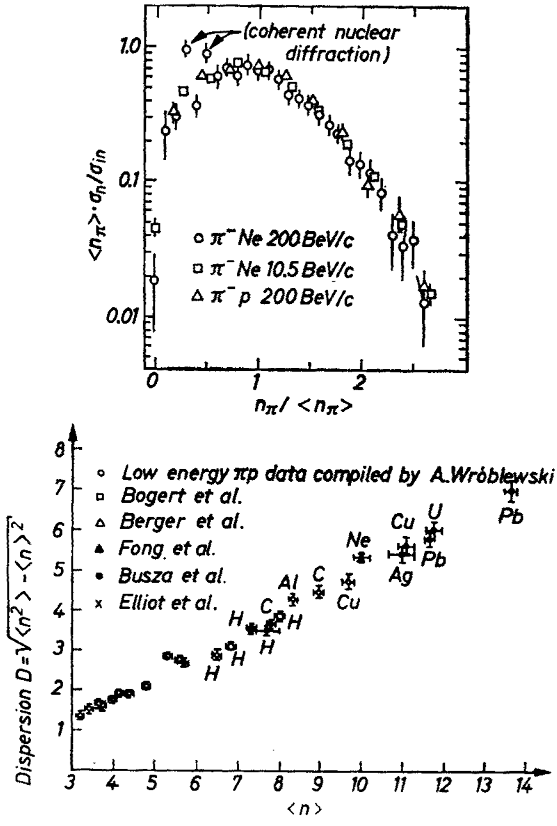


Fig. 13

Fig. 12. Evidence that many nucleons participate in the multiparticle production of particles in hadron-nucleus collisions. a) Correlation between number of relativistic particles produced (n_s) and number of visible nuclear fragments (N_h) in emulsion. Data from Ref. [44]. b) Correlation between the number of fast protons, as measured by the difference in the number of positive and negative particles, and the total number of fast particles. From Elliot et al. [17]

Fig. 13. Evidence for KNO scaling in hadron-nucleus collisions. a) Multiplicity distributions in π^--Ne collisions. Data from Elliott et al. [17] b) Dispersion as function of average multiplicity in π^--nuclear collisions. Data from Ref. [45]

In π^-Xe reactions at 9 GeV two interesting observations have been made [19]: (*i*) the average transverse momentum of π^0's decreases with the number of nuclear fragments (see Fig. 14); (*ii*) the angular distribution of π^0's is symmetric in the π^-p c. m. frame only if there are no nuclear fragments. For events with many nuclear fragments it is symmetric in the c. m. frame of a π^- and a target of mass equal to a few nucleons.

These data were obtained at a low energy and the results may be a trivial consequence of energy considerations. Nevertheless, it is interesting to speculate that they may be indicative of the importance of cumulative effects in nuclei [20].

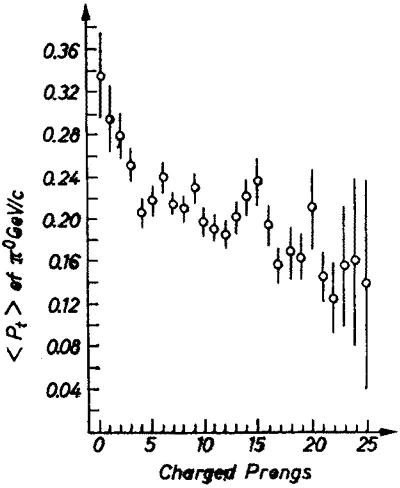


Fig. 14. Correlations between the average transverse momentum of π^0's and the total number of charged particles (primarily nuclear fragments) in π^-Xe collisions at 9 GeV/c. Data from Siemiarczuk [19]. The decrease of $\langle P_t \rangle$ with the number of prongs may be evidence that the π^0's are produced on a multi-nucleon target in the nucleus

5. *Production of particles with high transverse momentum*

Another totally unexpected result in hadron-nucleus collisions is the A-dependence of the inclusive particle production spectra at large transverse momenta. Examples of such spectra are shown in Fig. 15. For all produced particles, fits to the data of the form $E\dfrac{d\sigma}{dp^3} \propto A^{\alpha}$ yield values of $\alpha > 1$ for $p_\perp \gtrsim 2$ GeV/c. Furthermore, as shown in Fig. 16 over the entire range where measurements exist [21], (28.5 GeV — 400 GeV incident protons) the value of α depends only on $p_\perp$ and particle type.

It is possible that the interpretation of these data is uninteresting. Although most authors [22] claim that ordinary multiple scattering cannot explain such a large A-dependence, this possibility has not been conclusively ruled out [23]. It is unlikely that the A-dependence is a simple consequence of Fermi motion, cumulative effects or other mechanisms which raise the effective c. m. energy of the collision. As pointed out by Farrar [24], if this were the case, it would be difficult to understand why the p/π ratio increases with A and decreases with S as can be seen in Fig. 17.

Most of the experimental data have been taken at an angle which corresponds approximately to 90° in the pp c. m. system, and it has been suggested [25] that perhaps it is the c. m. total momentum (p^*) and not $p_\perp$ which is the relevant parameter for describing

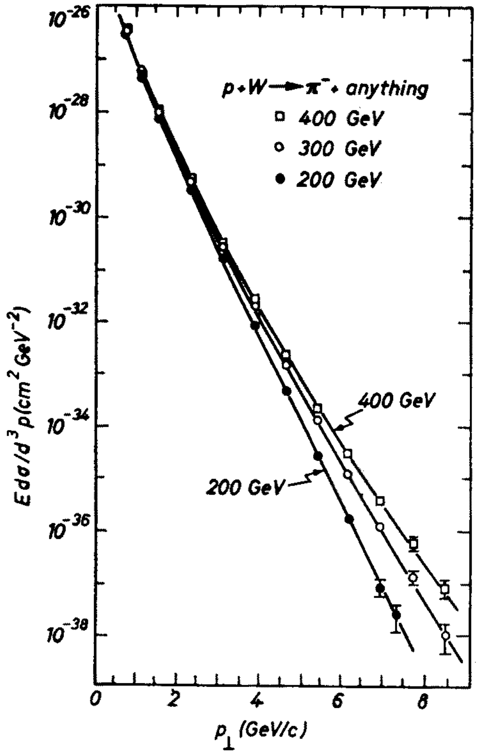


Fig. 15. Invariant cross section per W nucleus (divided by $\sigma_{pW}(\text{inel})/\sigma_{pp}(\text{total})$) for π^- mesons produced at 90° c. m. in p-W collisions. Data of Cronin et al. [21]

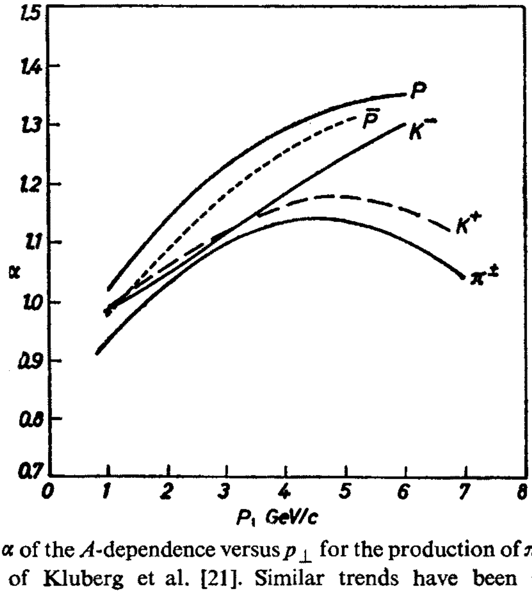


Fig. 16. Plots of the power α of the A-dependence versus $p_\perp$ for the production of $\pi^\pm, K^\pm, p^\pm$. Curves obtained from 200–400 GeV data of Kluberg et al. [21]. Similar trends have been seen at energies between 28.5 and 400 GeV. See Ref. [21]

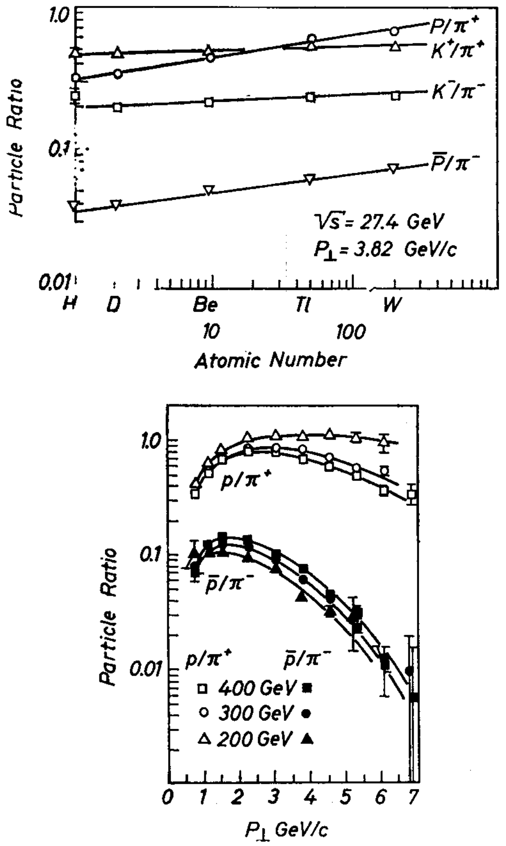


Fig. 17

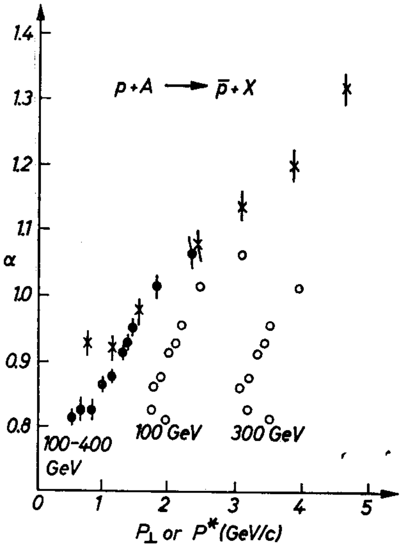


Fig. 18

Fig. 17a). Data showing that p/π^+ ratio increases with A. b) Data showing that p/π^+ ratio decreases with S. All data from Cronin et al. [21]

Fig. 18. Data which show that $\alpha(p_\perp)$ and not $\alpha(p^*)$ is independent of incident energy. α is the power of the A dependence, $P_\perp$ the perpendicular momentum of the $\bar{\text{p}}$, and p^* is the momentum of the $\bar{\text{p}}$ in the pp c. m. system; crosses: data of Cronin et al. [21] ($p_\perp = p^*$); closed circles: data of Gross et al. [46] plotted as function of $p_\perp$; open circles: same data plotted as function of p^*

large $p_\perp$ collisions in nuclei. For this reason in Fig. 18, the value of α obtained in p$A \to \bar{\text{p}}X$ is plotted first as a function of $p_\perp$ and then of p^*. As can clearly be seen, α is independent of incident energy for a given $p_\perp$ and not a given p^*.

For recent discussions of all these data, see Refs [23, 26].

6. *A-dependence of di-muon production by hadrons*

Several authors [5] have speculated that perhaps the large $p_\perp$ phenomena in nuclei are associated with the direct interaction of fast quarks in the projectile with those in the nucleus. If the fast quarks interact weakly, as is commonly assumed [27], an $A^{1.0}$ dependence follows immediately. To exceed an $A^{1.0}$ power further assumptions are necessary. An interesting consequence of such speculation is that the annihilation of fast quarks (Drell-Yan mechanism) should give an $A^{1.0}$ dependence for the production of di-muons with large masses [25].

Recent results from Fermilab on the A-dependence of di-muons produced by neutrons are shown in Fig. 19. Similar results have been seen at CERN and Serpukhov [28]. As can be seen the di-muon production goes as $A^{2/3}$ for low masses and approaches $A^{1.0}$ around 2-3 GeV. It is still an open question whether for $M_{\mu\mu} > 3$ GeV the power of A saturates at 1.0 or exceeds 1.0. The latter would once again be an unexpected result from hadron--nucleus collisions.

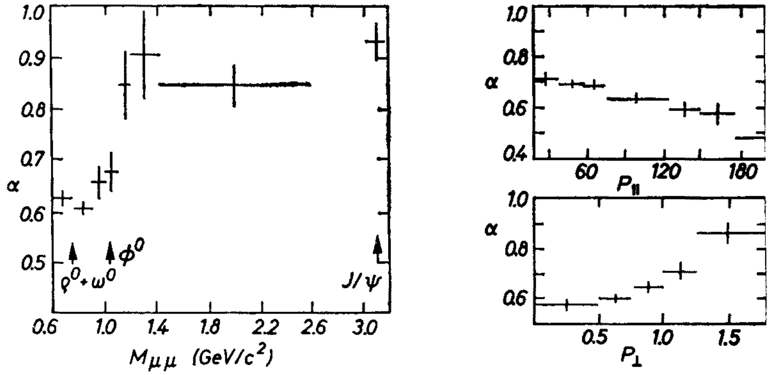


Fig. 19. A-dependence of dimuon production in neutron-nucleus collisions. α is the power of the A-dependence. p is the longitudinal momentum and $p_\perp$ the transverse momentum of the dimuon. Data from Binkley et al. [47]

7. Conclusions

Presently there is no model which in a convincing manner can account for all hadron--nucleus data in terms of our knowledge of hadron-nucleon interactions. At this stage, it is probably best for the data to speak for themselves. It is for this reason that in this review I have intentionally avoided direct comparisons with theoretical models.

It is possible that the interpretation of hadron-nucleus data is either trivial or too complicated to be of help in understanding hadron-nucleon interactions. Personally, I do not believe that this is the case. Is it likely that the apparent transparency of nuclear matter, which manifests itself in such a wide variety of phenomena as the coherent production of particles, the lack of intranuclear cascading, the large production of particles at large p_t, or the production of muon pairs, is not telling us something fundamental about the nature of hadronic interactions?

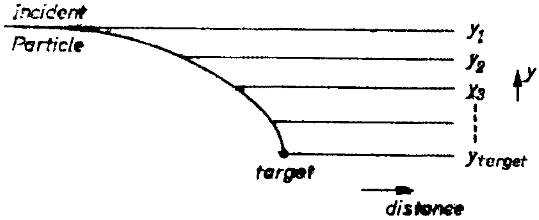
In collecting data for this review and attempting to understand it I have benefitted from numerous discussions with a large number of experimentalists and theorists. Rather than risk omitting some, I will not list them by name. I am grateful to them all. I wish to thank Dr. A. Białas and W. Czyż, and Drs. M. Benecke and L. Stodolsky for giving me the opportunity to lecture on this subject in Zakopane and Tutzing respectively. Finally, I wish to thank S. Anand, C. Tourtellotte, and G. Rowe for help in the preparation of this manuscript.

APPENDIX I

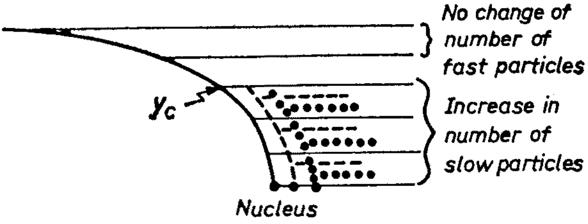
The following is a "hand-waving" description of some of the theoretical concepts that have been used in the interpretation of the experimental results. It is included for those not familiar with this field, with the intention of supplying a crude framework for the data.

I. Multiperipheral model [6]

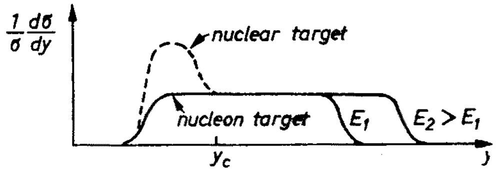
Multiparticle production in the **MPM** can be described as follows: an incident particle with rapidity y_1 emits, in some characteristic time T_0 (in the rest frame of the incident particle) a slower particle of rapidity y_2. The slower particle in time $\sim T_0$ (in its rest frame) emits an even slower particle of rapidity $y_3 (y_3 - y_2 \sim y_2 - y)$. This continues until the slowest particle, with a rapidity of the order of that of the target, interacts with the target. In the rest frame of the target, the process can be illustrated as follows:



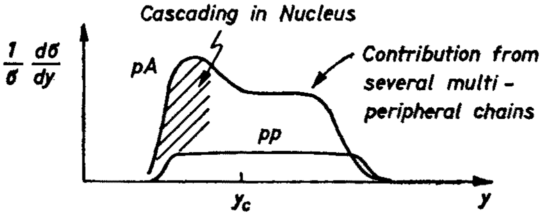
Consider now the interaction of a particle with a nucleus. The incident particle, or any one of the produced particles can interact with a second nucleon in the nucleus only if in time T_0 it has not passed the nucleus. At high energies only the slower particles in the multiperipheral chain satisfy this requirement. In fact only those with a laboratory rapidity less than $y_c \sim \ln\left(\frac{4R_0}{T_0}\right)$, where R_0 is the nuclear radius.



It follows that the rapidity distribution in hadron-nucleon and hadron-nucleus interactions will differ only for $y < y_c$.



These distributions predict for hadron-nucleus collisions a small rise in multiplicity, all in the target fragmentation region. They also predict that for $E \to \infty$, $R_A \to 1$. This picture also predicts $\frac{\sigma_{\text{inel}}(\pi A)}{\sigma_{\text{inel}}(pA)} = \frac{\sigma_{\text{inel}}(\pi p)}{\sigma_{\text{inel}}(\text{pp})}$. Both these results are in contradiction to the data. However agreement can be obtained if contributions from two or more multiperipheral chains (cuts) are included [8]. With several chains contributing, quantitatively the expected distribution is as follows:



A slight reduction of particles in the very forward direction could be expected from energy considerations.

II. Parton model [5]

If one assumes that only "wee" partons interact [27], qualitatively the picture of the interaction and predictions are identical to that of the MPM. The importance of more than one multiperipheral chain is equivalent in the parton picture to the assumption that the incident hadron is a superposition of more than one set of independent partons. Large P_t phenomena are associated with the rare hard collisions which immediately leads to a $A^{1.0}$ dependence. To exceed $A^{1.0}$ further assumptions are necessary.

III. Hydrodynamical models [3]

One can argue that in the initial stages of particle production it is meaningless to consider individual particle states. Collective variables may be the only appropriate ones for describing hadron-nucleus collisions. Such considerations lead to an interpretation of the data based on relativistic hydrodynamics. As an illustration of such considerations, consider Gottfried's Energy Flux Cascade Model [29]:

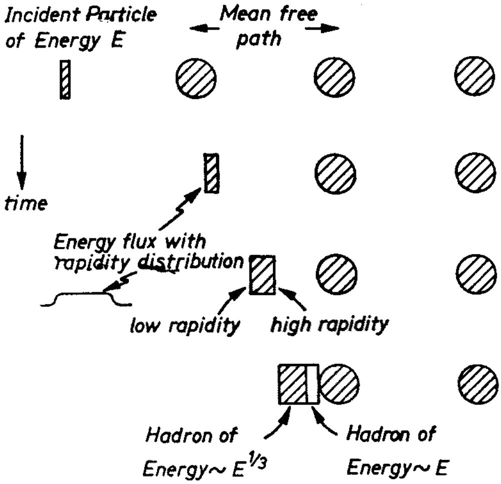
In this model the crucial assumptions are two-fold:

(*i*) following a hadron-nucleon interaction an excited hadronic state (energy flux) is formed. This energy flux has a rapidity distribution of matter within it which is the same as the observed asymptotic rapidity distribution of produced particles. Because of this distribution of rapidity, as time proceeds, the energy flux expands spatially, with faster components forming the head and slower components the tail.

(*ii*) any portion of the energy flux which has a size equal to that of a single hadron (in the rest frame of the average rapidity of that portion) behaves as a single hadron.

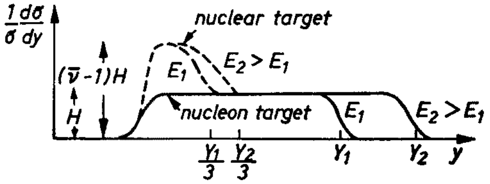
With these assumptions we can analyse qualitatively what happens in a hadron-nucleus collision.

After the first collision an energy flux is formed. It expands and after one mean free path in the nucleus it corresponds, on the average, to two particles; one with approximately the full energy E of the incident particle and one with energy $\sim E^{1/3}$.



The collision between the incident hadron and a nucleus is thus approximately equivalent to one hadron of energy E and $(\bar{\nu}-1)$ hadrons of energy $E^{1/3}$ colliding with one nucleon. The further multiplication of the products of the $(\bar{\nu}-1)$ collisions is negligible because of the low energy of the particles.

Since $\langle n\rangle_p \sim \ln s$, it immediately follows that in the EFC model the rapidity distribution is as follows:



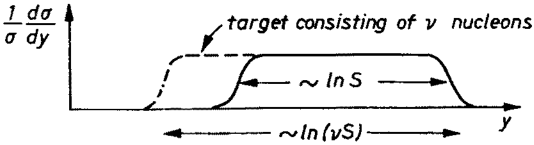
and $R_A \sim 1+1/3\,(\bar{\nu}-1)$, independent of energy. A more careful analysis gives $R_A \sim 1 + 0.38\,(\bar{\nu}-1)$ for $E \sim 200$ GeV.

IV. Coherent tube model [4]

Next, let us consider the predictions of another completely different possible mechanism. Suppose that the interaction time is so long that all the nucleons in the path of an incident particle act as a conglomerate, i. e., act cumulatively or coherently. Let ν be the number of nucleons and M_p be the mass of each. If E is the laboratory energy of the incident particle, the effective total c. m. energy squared for the interaction of the incident particle with the ν nucleons is:

$$S_\nu = 2\nu \mathrm{ME} = \nu S_1 .$$

If $\langle n(S)\rangle \sim \ln S$, in the laboratory frame the rapidity distribution will increase in length and simultaneously move backwards due to the change of the center of mass as follows:



Thus

$$R_A \sim \frac{\ln(\nu S)}{\ln S} \sim 1 + \frac{\ln \nu}{\ln S} \to 1 \text{ as } E \to \infty .$$

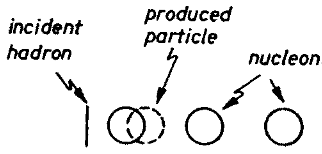
If $\langle n(S)\rangle \sim S^{1/4}$, R_A becomes independent of E and the A-dependent region covers the complete rapidity range.

To within a factor of two or so, considerations of this kind can also reproduce most of the observed A-dependence of production of particles at large P_t[26c].

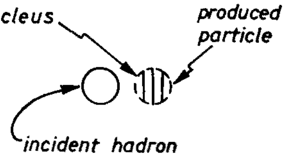
V. General considerations of interaction and formation times

It is also possible that some of the main features of hadron-nucleus interactions are consequences of some very general and simple considerations.

For example, consider hadron-nucleus collisions from the point of view of the produced particles [30].



in rest frame of particles produced with low rapidity



in rest frame of particles produced with high rapidity

Surely it is only meaningful to consider the re-interaction of the produced particles with subsequent nucleons only in the case of the very slow particles, in whose frame the nucleus extends beyond the produced particle.

Or consider particle production from the point of view of the "formation time" [11]. Suppose a particle of wavelength λ_1 produces in an interaction a particle of wavelength λ_2. The point of production is only defined to within a region where λ_1 and λ_2 separate significantly in phase. If this region (formation zone) is large compared to a nucleus, no multiplication can occur. The formation zone argument also automatically leads to the α in A^α of large P_t production to increase with P_t, as observed.

APPENDIX II

Definition of $\langle n\rangle_A$, $\bar{\nu}$ and η

It has become conventional to present data on hadron-nucleus multiplicities in terms of few parameters which unfortunately in the literature are not used very consistently. These parameters are now defined in a way I consider most useful for the interpretation of the data.

A measure of the average multiplication of particles inside a nucleus is given by

$$R_A = \frac{\langle n\rangle_A}{\langle n\rangle_p}$$

where $\langle n\rangle_A$ is the average number of charged relativistic particles ($\beta \gtrsim 0.7$) produced in an inelastic and incoherent interaction between a hadron and nucleus A. $\langle n\rangle_p$ is the average number of charged relativistic particles produced in an inelastic collision of the same hadron with a proton. It is approximately given by

$$\langle n\rangle_p = \langle n\rangle_{ch} - 0.5.$$

In emulsion experiments $\langle n\rangle_A$ is often refered to as $\langle n\rangle_s$, the "s" for "shower particles".

The average thickness, $\bar{\nu}$, of a nucleus seen by an incident hadron h is conventionally measured in units of the mean free path for absorption of h in the nucleus, i. e., $\bar{\nu}$ is the average number of inelastic collisions h would make with nucleons in the nucleus if, following each collision, it remained as a single hadron h. From the definition of $\bar{\nu}$ it follows that

$$\bar{\nu} = \frac{A\sigma_{\mathrm{hp(inelastic)}}}{\sigma_{\mathrm{h}A\mathrm{(inelastic)}}}.$$

For protons:

$$\bar{\nu} \approx \frac{A \cdot 32.3}{46A^{0.69}} \approx 0.7A^{0.31}.$$

For pions:

$$\bar{\nu} \approx \frac{A \cdot 21.2}{28.5A^{0.75}} \approx 0.74A^{0.25}.$$

A convenient parameter for describing the longitudinal motion of relativistic particles is the rapidity y, defined in any frame by

$$y = \tfrac{1}{2} \ln \frac{E+p_{\|}}{E-p_{\|}}$$

where E is the total energy of the particle in that frame and $p_{\|}$ its longitudinal momentum. Under Lorentz transformation along the beam axis rapidity transforms as

$$y^{\text{lab}} = y^{\text{cm}} + \xi \quad \text{where} \quad \xi \approx \ln \sqrt{\frac{S}{M_{\text{target}}}} .$$

In many experiments on multiparticle production only angles are measured. From these measurements it is still possible to obtain an approximate rapidity distribution through the use of the pseudorapidity variable η

$$y^{\text{lab}} \approx \eta^{\text{lab}} = -\ln \tan \frac{\Theta_{\text{lab}}}{2} .$$

The approximation is excellent provided that

$$\frac{p-p_{\|}}{E-p_{\|}} \approx 1.$$

Thus it is a good approximation for produced pions and terrible for nucleons.

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