SATURATION AND GEOMETRICAL SCALING IN SMALL SYSTEMS*

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Saturation and geometrical scaling (GS) of gluon distributions are a consequence of the non-linear evolution equations of QCD. We argue that in *pp*, GS holds for the inelastic cross section rather than for the multiplicity distributions. We also discuss possible fluctuations of the proton saturation scale in *pA* collisions at the LHC.

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At the eQCD meetings in 2013 and 2015 [1, 2], we discussed the emergence of geometrical scaling for $F_2(x)/Q^2$ [3] in deep inelastic scattering (DIS) [4], and for charged particle multiplicity distributions in proton–proton collisions [5], and in heavy-ion collisions (HI) [6]. Here, after a short reminder, we recall recent analysis [7] of ALICE *pp* data [8], and discuss a hypothesis that the saturation scale may fluctuate in the proton [9] on the example of the *pA* scattering as measured by ALICE [10] at the LHC.

The cross section for not too hard gluon production in *pp* collisions can be described in the k_{T}-factorization approach by the formula [11]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}y\mathrm{d}^2p_{\mathrm{T}}} = \frac{3\pi\alpha_{\mathrm{s}}}{2}\frac{Q_{\mathrm{s}}^2(x)}{p_{\mathrm{T}}^2}\int\frac{\mathrm{d}^2\vec{k}_{\mathrm{T}}}{Q_{\mathrm{s}}^2(x)}\,\varphi_p\left(\vec{k}_{\mathrm{T}}^2/Q_{\mathrm{s}}^2(x)\right)\varphi_p\left(\left(\vec{k}-\vec{p}\right)_{\mathrm{T}}^2/Q_{\mathrm{s}}^2(x)\right), \tag{1}$$

where φ_p denotes the unintegrated gluon distribution that, in principle, depends on two variables $\varphi_p = \varphi_p(k_{\mathrm{T}}^2, x)$. In Eq. (1), we have assumed that produced gluons are in the mid-rapidity region ($y \simeq 0$), hence both Bjorken xs of colliding glouns are equal to $x_1 \simeq x_2$ (denoted in the following as x). Note that unintegrated gluon densities have dimension of transverse area.

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This is best seen from the very simple parametrization proposed by Kharzeev and Levin [12] in the context of HI collisions

$$\varphi_p\left(k_{\rm T}^2\right) = S_\perp \begin{cases} 1 & \text{for} \quad k_{\rm T}^2 < Q_{\rm s}^2 \\ k_{\rm T}^2/Q_{\rm s}^2 & \text{for} \quad Q_{\rm s}^2 < k_{\rm T}^2 \end{cases} \tag{2}$$

or by Golec–Biernat and Wüsthoff in the context of DIS [13]

$$\varphi_p\left(k_{\rm T}^2\right) = S_\perp \frac{3}{4\pi^2}\frac{k_{\rm T}^2}{Q_{\rm s}^2}\exp\left(-k_{\rm T}^2/Q_{\rm s}^2\right) . \tag{3}$$

In the case of DIS, $S_\perp = \sigma_0$ is the dipole–proton cross section for large dipoles and in (2), $S_\perp$ is the transverse size of an overlap of two large nuclei for a given centrality class. In both cases, one can assume that $S_\perp$ is energy-independent (or weakly dependent). Another feature of (2) and (3) is that $\varphi_p(k_{\rm T}^2, x) = \varphi_p(k_{\rm T}^2/Q_{\rm s}^2(x))$, where $Q_{\rm s}^2(x)$ is the *saturation momentum* that takes the following form $Q_{\rm s}^2(x) = Q_0^2(x/x_0)^{-\lambda}$ motivated by the traveling wave solutions [14] of the non-linear Balitski–Kovchegov evolution equations [15]. In that case, ${\rm d}^2\vec{k}_{\rm T}$ integration in (1) leads to

$$\frac{{\rm d}\sigma}{{\rm d}y{\rm d}^2p_{\rm T}} = S_\perp^2 \mathcal{F}(\tau) , \tag{4}$$

where $\tau = p_{\rm T}^2/Q_{\rm s}^2(x)$ is a scaling variable and $\mathcal{F}(\tau)$ is a function related to the integral of φ_ps. We shall follow here the parton–hadron duality [16], assuming that the charged particle spectra are on the average identical to the gluon spectra. Equation (4) has the property of GS if $S_\perp$ is energy-independent. In this case, the entire energy dependence is taken care of by the energy dependence of τ.

In order to test relation (4), we shall use the fact that for mid-rapidity

$$\tau = p_{\rm T}^2/Q_{\rm s}^2(x) = p_{\rm T}^2/Q_0^2 \,\left(p_{\rm T}/(x_0 W)\right)^\lambda , \tag{5}$$

where $W = \sqrt{s}$, x_0 and Q_0^2 are constants that are irrelevant for the present analysis. We take $Q_0^2 = 1$ GeV$^2/c$, $x_0 = 10^{-3}$. The only relevant parameter is λ. In Fig. 1, we plot ALICE *pp* data [8] in terms of $p_{\rm T}$ (left panel) and in terms of scaling variable τ (right panel) for $\lambda = 0.32$. We see that three different curves from the left panel in Fig. 1 overlap over some region if plotted in terms of the scaling variable τ. The exponent for which this happens over the largest interval of τ is $\lambda = 0.32$ [7], which is the value compatible with our model-independent analysis of the DIS data [4].

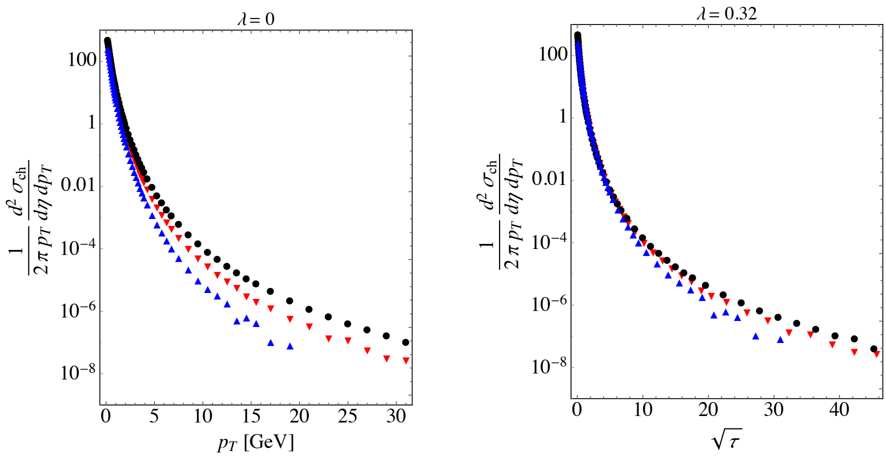

Fig. 1. (Color online) Data for pp scattering from ALICE [8] plotted in terms of p_{T} and $\sqrt{\tau}$. Full (black) circles correspond to $W = 7$ TeV, down (red) triangles to 2.76 TeV and up (blue) triangles to 0.9 TeV.

In order to illustrate the method of adjusting λ, we plot in Fig. 2 ratios of the cross sections at 7 TeV to 2.76 and 0.9 TeV. Approximate equality of both ratios close to unity for $\lambda = 0.32$ is the sign of GS for p_{T} up to 4.25 GeV/c [7].

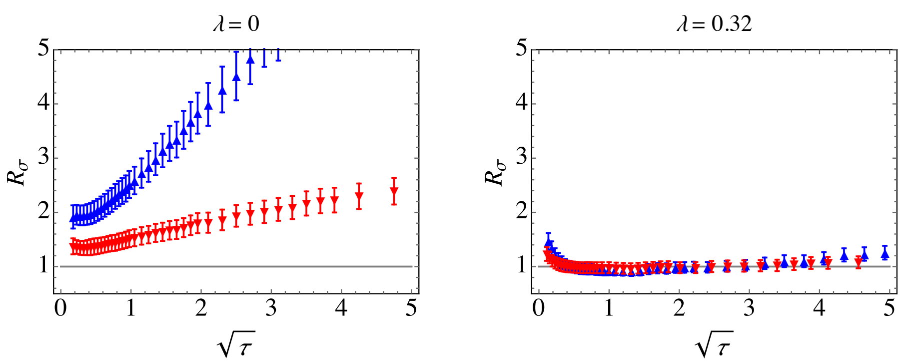

Fig. 2. (Color online) Ratios of the cross sections at 7/2.76 TeV — down (red) triangles and 7/0.9 TeV — up (blue) triangles, for $\lambda = 0$ (left) and 0.32 (right).

It has been argued previously that GS should hold for multiplicities, rather than for the cross sections. This would be true if the relation between the two was energy-independent. This may be the case in HI or pA collisions where we trigger on some $S_\perp$ by selecting the centrality classes with given number of participants, but it is not true in the case of the inelastic pp scattering

$$\frac{\mathrm{d}N}{\mathrm{d}y\mathrm{d}^2p_{\mathrm{T}}} = \frac{1}{\sigma^{\mathrm{MB}}(W)}\frac{\mathrm{d}\sigma}{\mathrm{d}y\mathrm{d}^2p_{\mathrm{T}}} = \frac{S_\perp^2}{\sigma^{\mathrm{MB}}(W)}\mathcal{F}(\tau)\,, \tag{6}$$

where the minimum bias cross section $\sigma^{\mathrm{MB}}(W) \neq S_\perp$ is energy-dependent. Repeating the procedure of constructing the ratios of the multiplicities rather than of the cross sections, we find the best scaling for $\lambda = 0.22 \div 0.24$ [7]. This is illustrated in Fig. 3 where the left panel is just an enlarged version of the right plot of Fig. 2, whereas the right panel corresponds to the ratios of the multiplicities for $\lambda = 0.22$. We see that, indeed, multiplicity scaling is achieved for smaller λ, but — at the same time — the scaling is of worse quality than for the cross sections and holds over a smaller interval of τ.

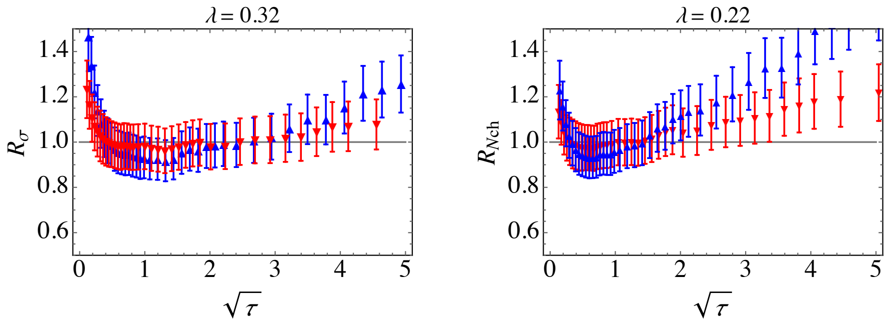

Fig. 3. Ratios of cross sections (left) for $\lambda = 0.32$ and multiplicities (right) for $\lambda = 0.22$. For the meaning of symbols, see Fig. 2.

In the case of two different systems, like in the pA scattering and/or $y \neq 0$, formula (1) contains two different distributions $\varphi_{p,A}$ characterized by two different saturation scales $Q_{p,A}(k_{\mathrm{T}}^2/s, \pm y)$. With simple parametrization (2) and assuming constant $S_\perp$corresponding to the definite centrality class, one arrives at a very simple formula for charged particle multiplicity [12]

$$\frac{\mathrm{d}N_{\mathrm{ch}}}{\mathrm{d}y} = S_\perp Q_p^2 \left(2 + \ln\frac{Q_A^2}{Q_p^2}\right)\,. \tag{7}$$

Formula (7) predicts both energy and rapidity dependence and also N_{part} dependence of multiplicities through the dependence of the saturation scales upon these quantities [12] (apart from $S_\perp$ dependence on N_{part})

$$\begin{aligned} Q_p^2(W,y) &= Q_0^2\left(\frac{W}{W_0}\right)^\lambda \exp(\lambda y)\,, \\ Q_A^2(W,y) &= Q_0^2 N_{\mathrm{part}}\left(\frac{W}{W_0}\right)^\lambda \exp(-\lambda y)\,, \end{aligned} \tag{8}$$

where we take $\lambda = 0.32$ as in DIS [4] and pp [7].

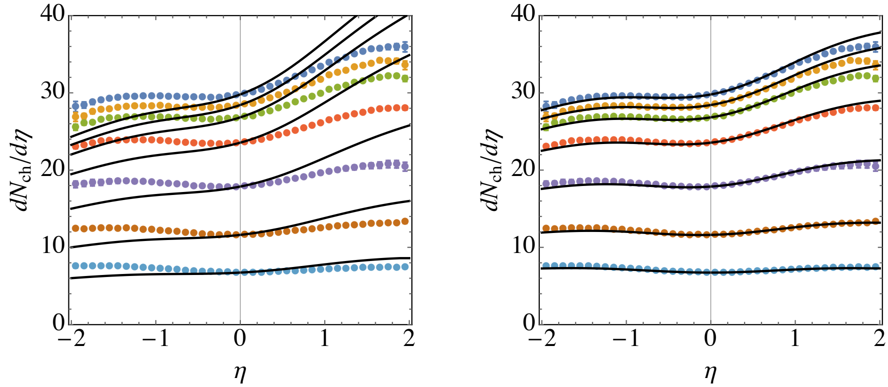

Fig. 4. Multiplicity spectra from Ref. [10] compared with the prediction of Eq. (7) without (left) and with fluctuations (right). For the meaning of symbols, see Ref. [9]. Normalization of theoretical predictions has been fitted and is given by Eq. (10).

It has been shown in Ref. [9] that these simple formulae fail to describe recent proton–Pb LHC data [10]. To resolve this issue, we have proposed to take into account possible fluctuations of the saturation scale in the proton according to the log-Gaussian distribution introduced in Ref. [17]

$$P(\rho) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(\ln Q_{\rm s}^2/Q_0^2 - \ln Q_p^2/Q_0^2\right)^2}{2\sigma^2}\right) . \tag{9}$$

Here, $Q_{\rm s}^2$ is the proton saturation momentum fluctuating around its logarithmic average denoted as $\ln Q_p^2$ (with Q_0^2 being an arbitrary momentum scale, which cancels out in (9)) and σ is the fluctuation width, which we assume to be y-independent (although it may, in principle, depend on W). Taking into account fluctuations (9) and the transformation from y to pseudorapidity η [9], we have been able to describe the multiplicity distributions adjusting the normalization in Eq. (7) for each centrality class. In Fig. 4, we show the results for the ALICE data for centrality class determination by the ZNA method ($N_{\rm part} = N_{\rm coll}^{\rm Pb-side} + 1$ from Table 7 in Ref. [10], whereas in Ref. [9] we have used V0A centrality determination). As in Ref. [9], we have to take rather large $\sigma \sim 1.55$ to describe the data. The normalization $S_\perp$ has been fitted to the data by means of the logarithmic parametrization

$$S_\perp = \left(0.88 + 0.47 \ln N_{\rm part}\right)^2 . \tag{10}$$

To summarize: We have presented new developments in the studies of GS for small systems, *i.e.* for pp and pA collisions. We have shown that a good quality scaling in pp is achieved for the inelastic cross sections rather than for the multiplicities. In the case of pA collisions, we have reported on a recent proposal to include the fluctuations of the saturation scale of the proton in order to describe recent data on multiplicity distributions $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}\eta$ for different centrality classes.

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