UNITARY MULTI-CHANNEL $\pi\pi$ SCATTERING AMPLITUDES OF f_2 AND ρ_3 MESONS*

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In a unitary multi-channel approach, precise determination of $\pi\pi$ scattering amplitudes for D and F waves has been presented. These scattering amplitudes are in the $I^G J^{PC} = 0^+2^{++}$ sector of the processes of $\pi\pi \to \pi\pi$, 4π, $K\bar{K}$ and $\eta\eta$, likewise in the $I^G J^{PC} = 1^+3^{--}$ sector of the processes of $\pi\pi \to \pi\pi$, 4π, $\omega\pi$ and $K\bar{K}$. The amplitudes were refined and re-fitted to the dispersion relations up to 1.1 GeV, and to the experimental data in the effective two pion mass from the threshold to 2.7 GeV and 1.9 GeV for D and F waves, respectively. Unsuitable behaviour of phase shift near threshold has been optimized for both waves.

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1. Introduction

In our previous work [1], we presented a general method of refining the $\pi\pi$ amplitudes by fitting them to the GKPY equations [2, 3] which are once-subtracted dispersion relations with imposed crossing symmetry condition. The method was demonstrated on refining of the unitary multi-channel S ($\pi\pi$, $K\bar{K}$ and $\eta\eta$) and P ($\pi\pi$, $\rho 2\pi$, and $\rho\sigma$) wave amplitudes from [4] with no change in their original mathematical structure mostly below 1.1 GeV. In this paper, we present analysis of the D- and F-wave amplitudes. The "original" ones for the D and F waves are taken from analysis [5] and those for the S and P from [1]. To perform this analysis, we use GKPY equations for the D and F waves constructed and presented in [3].

In presented analysis, the S-matrix formalism for N coupled channels was utilized similarly as in our previous dispersive analysis of S and P waves [1]. Due to the large number of opened channels in the D and F waves, the uniformizing variable could not be used and, therefore, the Jost matrix

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determinant was constructed using the multi-channel Breit–Wigner forms. We have used the same formalism for $D0$ and $F1$ waves as in Refs. [4,5] and updated the list of contributing resonant states for the $D0$ wave according to the latest issue of PDG [6]. To construct the structure of the $D0$ and $F1$ amplitudes, the matrix elements S_{ij} of the 4-channel S-matrix ($i,j = 1,2,\ldots 4$) are expressed via the Jost matrix determinant, $d(k_1,k_2,\ldots k_4)$ (k_i are the channel momenta), using the Le Couteur–Newton relations. The Jost determinant is considered in a separable form $d = d_{\mathrm{bgr}} d_{\mathrm{res}}$, where the resonance part d_{res} is described by the multi-channel Breit–Wigner form and the background part d_{bgr} represents mainly an influence of neglected channels and resonances (see Eqs. (1)–(13) from [7]).

2. Fits for the D and F waves

Fits have been performed into two steps. In the first step, the "original" amplitudes (old) from [5] were refined and fitted to the data only (new). In the second step, the new amplitudes were fitted to the data and dispersion relations simultaneously (re-fitted). In the isoscalar D wave, we have considered eleven states presented in the PDG summary table [6]: $f_2(1270)$, $f_2(1430)$, $f_2(1525)$, $f_2(1640)$, $f_2(1810)$, $f_2(1910)$, $f_2(1950)$, $f_2(2010)$, $f_2(2150)$, $f_2(2300)$, $f_2(2340)$. Free parameters for these fits are parameters of the Breit–Wigner form of $f_2(1270)$, $f_2(1430)$, $f_2(1525)$ and $f_2(1640)$ resonances *i.e.* masses and the partial widths f_{ri}. Moreover, the background parameters α_{11}, α_{12}, α_{10}, β_j, and γ_j from Eqs. (7)–(8) in [7] are free. Channels considered for the $D0$ partial waves are: 1 — $\pi\pi$, 2 — effective $(2\pi)(2\pi)$, 3 — $K\bar{K}$, and 4 — $\eta\eta$.

The experimental data for the $\pi\pi$ scattering are from analysis by Hyams *et al.* [8] and the data for inelastic scattering $\pi\pi \to K\bar{K}, \eta\eta$ from Ref. [9]. In successive fitting of the parameters to the data, we found a solution with $\chi^2/\mathrm{n.d.f.} = 239.28/(183-58) = 1.91$. The parameters of resonances of the new $D0$ amplitude are shown in Table I and a comparison with the old $D0$ amplitude [4] is given in Fig. 1. The background parameters are: $\alpha_{11} = 0.00096$, $\alpha_{13} = -0.04105$, $\alpha_{10} = -0.186$, $\beta_1 = -0.0531$, $\beta_3 = -1.99$, $\beta_4 = -1.47$, $\gamma_1 = 0.00128$, $\gamma_3 = 1.99$, and $\gamma_4 = 1.43$. This solution is a bit worse (the χ^2) than that in Ref. [4] but we have achieved the right behaviour of δ_{11} for energies below 800 MeV (see the detail in Fig. 1) which allows us to avoid a polynomial-like extension of the phase shift as in the case of the S and P amplitudes [1]. Please notice also that the set of resonances and their masses are in a good agreement with the PDG tables, see Table I.

In the isovector F wave, there is only one resonant state $\rho_3(1690)$ in summary tables of the PDG [6] which is relevant for the data description below 2 GeV. In [7], two resonance states $\rho_3(1690)$ and $\rho_3(1960)$ have been considered and it is clearly seen from Tables V and VI in [7], when parameters

TABLE I

Parameters of the Breit–Wigner form (in MeV) for the new $D0$ and $F1$ amplitudes.

State	PDG	M_r	f_{r1}	f_{r2}	f_{r3}	f_{r4}
$f_2(1270)$	1275.5 ± 0.8	1275.5	459.3	0.001	204.0	91.3
$f_2(1430)$	1430	1463.2	42.3	0.12	346.8	0.02
$f_2(1525)$	1525 ± 5	1570.7	0.01	207.5	128.4	96.3
$f_2(1640)$	1639 ± 6	1639.0	145.3	524.4	430.5	233.5
$f_2(1810)$	1815 ± 12	1815.0	163.5	279.2	497.2	590.3
$f_2(1910)$	1903 ± 9	1903.0	0.077	65.3	0.067	371.3
$f_2(1950)$	1944 ± 12	1944.0	5.01	59.5	625.7	97.9
$f_2(2010)$	2011 ± 62	2027.0	0.001	146.4	457.1	0.5
$f_2(2150)$	2157 ± 12	2157.0	0.015	445.8	148.1	354.6
$f_2(2300)$	2297 ± 28	2181.6	78.14	74.9	818.3	169.5
$f_2(2340)$	2345 ± 40	2383.3	46.20	7.1	633.2	163.8
$\rho_3(1690)$	1688.8 ± 2.1	1714.0	293.1	498.3	0.0	0.0

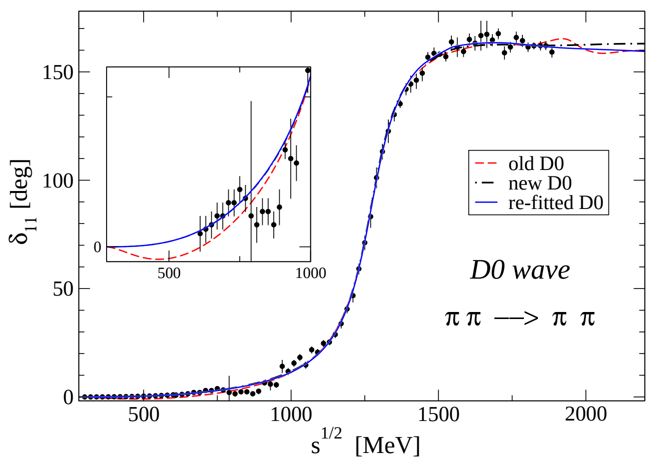

Fig. 1. Results of the new $D0$ (dash-dotted line), old $D0$ (dashed line) and re-fitted (solid line) $D0$ amplitudes for the phase shift in the $\pi\pi$ scattering are compared with experimental data from Ref. [8].

of the state $\rho_3(1960)$ are considered to be free, mass of the resonance state $\rho_3(1690)$ becomes almost infinity and results are not appropriate. Therefore, admissible results are from fits where the $\rho_3(1960)$ resonance state is omitted. We have, therefore, considered only $\rho_3(1690)$ in the analysis of the phase shift and inelasticity parameter in the $\pi\pi$ scattering [8]. In the analysis, we have included four channels: 1 — $\pi\pi$, 2 — effective $(2\pi)(2\pi)$, 3 — $\omega\pi$, and 4 — $K\bar{K}$.

In our analysis, the dispersion relations directly affect only the energy region below 1100 MeV but the higher energy region, where the $\rho_3(1690)$ resonance is clearly seen, is influenced indirectly. The $F1$ amplitude is, therefore, almost entirely determined by the experimental data. Since the resonant state is described by the Breit–Wigner form, which works well only in a vicinity of the resonance, we had to take a particular care of behaviour of the amplitude below about 1000 MeV. We have, therefore, chosen a simple modification of the phase shift by means of the background phase in the form of the cubic polynomial of energy s

$$d_{\mathrm{bgr}} = \exp\left[-i\left(\frac{2k_1}{\sqrt{s}}\right)^7\left(a_\alpha + \frac{4k_1^2}{4m_\pi^2}a_\beta + \left(\frac{4k_1^2}{4m_\pi^2}\right)^2 a_\gamma\right)\right], \tag{1}$$

where k_1 is equal to $\sqrt{s-4m_\pi^2}/2$ and a_α, a_β, and a_γ are background parameters.

In the fitting with the four-channel form, we found that only the channels 1 and 2 are important whereas the parameters f_{13} and f_{14} were almost zero. This corresponds quite well with the observed decay rates of the $\rho_3(1690)$ resonance: $67 \pm 22\%$, $23.6 \pm 1.3\%$, $16 \pm 6\%$, $3.8 \pm 1.2\%$, and $1.58 \pm 0.26\%$ into the $\pi^\pm\pi^+\pi^-\pi^0$, $\pi\pi$, $\omega\pi$, $K\bar{K}\pi$, and $K\bar{K}$ channels, respectively [6]. We did, therefore, a two-channel fit with $\chi^2/\mathrm{n.d.f.} = 80.84/(77-6) = 1.14$. The resonance parameters are shown in Table I and a comparison with the old $F1$ amplitude [4] is given in Fig. 2. The background parameters are $a_\alpha = 0.000008$, $a_\beta = -0.000998$, and $a_\gamma = 0.000016$. These parameters are quite small but they play an important role.

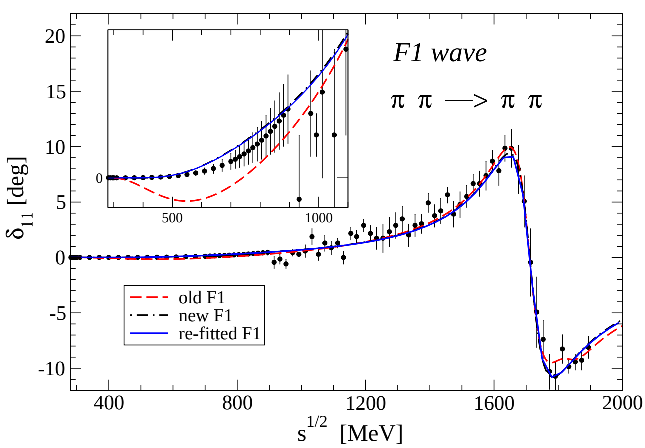

Fig. 2. Results of the new $F1$ (dash-dotted line), old $F1$ (dashed line) and refitted $F1$ (solid line) amplitude for the phase shift in the $\pi\pi$ scattering are compared with experimental data.

3. Results and conclusions

The method of analysis was generally the same as in our previous work [1]. As the starting amplitudes for the S and P waves, we have used just the "new extended $S0$ amplitude" and the "extended $P1$ amplitude" from [1]. Amplitudes of other partial waves *i.e.* $D0$ and $F1$ have been renewed and the $S2$ and $D2$ amplitudes have been taken from [2] and fixed.

First, the $D0$ and $F1$ amplitudes from [5] have been fitted to the corresponding experimental data separately. We call them "new $D0$" and "new $F1$". After the good fits for separate waves have been achieved, both the $D0$ and $F1$ together have been fitted to experimental data and GKPY from [3] and called "re-fitted $D0$" and "re-fitted $F0$". Definitions of the χ^2 functions were the same as in Eqs. (16)–(18) from [7] except of used number and kind of partial waves. We provide the re-fitted parameters of $D0$ and $F1$ partial waves in Table II. The background parameters of the new $D0$ amplitude after fitting are: $\alpha_{11} = 0.00110$, $\alpha_{13} = 0.03250$, $\alpha_{10} = -0.45986$, $\beta_1 = 0.12519$, $\beta_3 = -5.09539$, $\beta_4 = -3.68030$, $\gamma_1 = -0.36687$, $\gamma_3 = 7.27978$, and $\gamma_4 = 4.86869$. The background parameters of the new $F1$ amplitude after fitting are: $a_\alpha = 0.0000107$, $a_\beta = -0.001010$, and $a_\gamma = 0.0000158$. Table III shows values of the χ^2 for all wave amplitudes after fitting when all partial waves $S0$, $P1$, $D0$ and $F1$ are new in the fit.

TABLE II

Parameters of the Breit–Wigner form (in MeV) for the new $D0$ and $F1$ amplitudes after fitting. Initial value of the parameters are from Table I.

State	M_r	f_{r1}	f_{r2}	f_{r3}	f_{r4}
$f_2(1270)$	1278.2	457.2	0.001	154.0	87.4
$f_2(1430)$	1457.0	0.236	0.203	0.334	0.368
$f_2(1525)$	1577.8	0.001	302.1	189.9	71.1
$f_2(1640)$	1659.0	519.2	524.4	914.4	755.6
$\rho_3(1690)$	1713.8	292.6	498.7	0.0	0.0

TABLE III

Values of the χ^2 for all new wave amplitudes after fitting to the data and to the dispersion relations with n.d.f. equal to 844.

$\chi^2_{\mathrm{Data}}(S)$	$\chi^2_{\mathrm{Data}}(P)$	$\chi^2_{\mathrm{Data}}(D)$	$\chi^2_{\mathrm{Data}}(F)$	χ^2_{DR}	χ^2	χ^2/n.d.f.
288.6	302.4	219.1	80.5	126.3	1017.0	1.20

The work focuses particularly on providing precise determination of $D0$ and $F1$ partial wave amplitudes which satisfy the crossing symmetry condition and describe the experimental data very well. Amplitudes are constructed using resonance parameters described by the multi-channel Breit–Wigner form fitted to experimental data and GKPY equations without any other mathematical assumption. Finally and for illustration, we have plotted the results in Figs. 1 and 2 which show very good agreement of phase shifts of these two partial waves with experimentally observed values.

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