ENERGY-MOMENTUM OF A CLOSED STATIC SYSTEM IN GENERAL RELATIVITY E = 0?

BY V. N. FOLOMESHKIN

Institute for High Energy Physics, Serpukhov*

(Received July 9, 1969)

It is shown that the standard expression for the total mass M of a closed static system in the form of the volume integral of canonical energy-momentum complex using the covariant quantity R as Lagrangian leads to the equality $M = M_s/2$, where M_s — Schwarzschild mass of the system. It follows that either the expression for the complex is inconsistent with the field equations or the "total" mass of the system is different from Schwarzschid mass, or $M = 0$. There are some ideas given here that favour the possibility of $M = 0$.

For an arbitrary Lagragian $L(\varphi^A, \varphi_i^A, \varphi_{ik}^A)$ where $\varphi_i^A = \delta_i\varphi^A$, $\varphi_{ik}^A = \partial_i\partial_k\varphi^A$, which describes a closed system the known expression for a canonical energy-momentum tensor

$$T_i^k = \left[\frac{\partial L}{\partial\varphi_k^A} - \left(\frac{\partial L}{\partial\varphi_{kl}^A}\right)\right]\varphi_i^A + \frac{\partial L}{\partial\varphi_{kl}^A}\varphi_{il}^A - L\delta_i^k \tag{1}$$

can be obtained by the standard method.

In this we have the expression $M = \int T_0^0 dV$ for the energy-momentum in Cartesian coordinates of Euclidian space.

In the case of a geometrized gravitational field, expression (1) preserves its form for the canonical energy-momentum complex of a system [1, 2], and usually it is accepted that the total energy of a matter plus gravitational field system are expressed by the standard form

$$M = \int (T_0^0 + t_0^0)\sqrt{-g}\, dV \tag{2}$$

where T_0^0, t_0^0 are the parts of the complex that are related to the matter and gravitational field respectively.

It is known that in the case of a static gravitational field it is possible to obtain another expression for the mass of a closed system [3–5]

$$M = 2\int \left(T_0^0 - \frac{1}{2}T\right)\sqrt{-g}\, dV$$

directly from the field equations.

---

* Address: Institute for High Energy Physics, Serpukhov, USSR.

It is natural to believe that expressions (2), (3) will give a consistent result.

Let us show that this is not (always) so.

It is necessary to make the following remark regarding a gravitational Lagrangian.

It is known that the function R can be divided into two parts, $R = B + G$, where G is not a covariant quantity and depends only on the first derivatives of the metric tensor. The variation of the term $\sqrt{-g}\,B$ has the form of a total derivative and makes no contribution to the equation of motion. Therefore when we obtain only the equations of motion the Lagrangians R and G are equivalent and this justifies the usage of the expression G for this particular case.

In general, however, the Lagrangians R and G are not equivalent. They are not equivalent when the expression for the energy-momentum complex is obtained since it is essential in the canonical procedure for the divergent terms and in particular the term $\delta\sqrt{-g}\,B$ to make a contribution to the expression for the energy-momentum complex.

Therefore bearing in mind the necessity of fulfilling the covariance condition it is necessary to use the covariant expression when obtaining the energy-momentum complex in the frames work of cononical formalism.

The equations of the gravitational field with the total Lagranian $L = R/16\pi + L_M$ is written in the form

$$R_i^k - \frac{1}{2}\,\delta_i^k R = 8\pi T_i^k, \tag{4}$$

where T_i^k is a symmetric energy-momentum tensor of matter

$$T_i^k = \frac{2}{\sqrt{-g}}\,\delta\sqrt{-g}\,L_M/\delta g^{ik}.$$

For a static gravitation field and the component t_0^0 of the gravitational part of the complex the first two terms in formula (1) are equal to zero and we obtain that

$$t_0^0 = -L\delta_0^0 = R/16\pi = -T/2. \tag{5}$$

Substituting t_0^0 in (2) with (6), we obtain

$$M = \int\left(T_0^0 - \frac{1}{2}\,T\right)\sqrt{-g}\,dV. \tag{6}$$

A comparison of (3) and (7) leads us to the necessity of making a choice among the following three possibilities:

1) equations (1), (2) and (4) are inconsistent,

2) the "total" mass, determined by expression (2) has no relation to the Schwarshild mass of a body,

3) $M = 0$.

It is possible to make some considerations in favour of the last possibility.

For any physical system for example, electromagnetic and electron-positron fields the symmetrical and canonical energy-momentum tensors differ only by the antisymmetrical part which has the form of a total derivative and gives no contribution to the energy of a system

$$\int (T^0_0)_{\text{canon}} \sqrt{-g}\, dV = \int (T^0_0)_{\text{symm}} \sqrt{-g}\, dV. \tag{7}$$

If we do not try to isolate artificially the gravitational field from the construction Lagrangian scheme of all the other fields, then it should be expected that equality (7) is valid for a gravitational field as well.

This immediately leads to the equality $M = 0$.

It was proposed in the papers of Lorentz [6], Levi-Civita [7], Souriau [8], Mandelstam [9], Folomeshkin [10] independently of one another that the symmetric tensor of gravitational field identical with the field equations should be used as the energy-momentum tensor of a gravitational field.

In conclusion the author would like to thank B. A. Arbusov, S. S. Gerstein, A. M. Lesnov, O. A. Chrustalev for the discussion of the paper.

REFERENCES

[1] N. V. Mitzkevich, *Ann. Phys.*, **1,** 319 (1958)
[2] C. Møller, *Mat Fys. Medd. Dan. Vid, Selsk,* **31,** (1959); **32,** (1960).
[3] R. C. Tolman, *Phys. Rev.*, **35,** 875 (1930).
[4] A. Papapetrou, *Proc. Roy. Irish Academy, A* **51,** 191 (1947).
[5] A. Papapetrou, *Phys. Rev.*, **73,** 1105 (1948).
[6] H. Lorentz, *Amst. Versl.*, **25,** 468 (1916).
[7] T. Levi-Civita, *Rend Acc. Linc.*, **26,** 381 (1917).
[8] I. Souriau, *CR Acad. Sci. (France)*, **245,** 958 (1957).
[9] S. Mandelstam, *Ann. Phys.*, **19,** 25 (1962).
[10] W. Folomeshkin, *Yadernyja Energia*, **6,** 1080 (1967).