FIXED u CONTINUOUS MOMENT SUM RULES FOR Kp SCATTERING AND $\Lambda_\alpha-\Lambda_\gamma$ EXCHANGE-DEGENERACY

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The fixed u continuous moment sum rules for meson-nucleon scattering were derived and applied to the analysis of Kp scattering amplitude. It was found that sum rules are compatible with the hypothesis of $\Lambda_\alpha-\Lambda_\gamma$ exchange-degenerate trajectory dominance.

1. *Introduction*

Much effort has recently been devoted to the exploration of the Finite Energy Sum Rules (FESR) [1], *i.e.* the superconvergence relations for scattering amplitudes with subtracted asymptotic part of the Regge type. These sum rules, which are typically of the following form:

$$\int_{-N}^{N} \nu^n \operatorname{Im} F(\nu, t) d\nu = \sum_i \beta_i \frac{N^{\alpha_i+n+1}}{\alpha_i+n+1} \tag{1.1}$$

connect the Regge pole parameters β_i, α_i with the low energy part of the scattering amplitudes and in this way they play a useful role as the consistency conditions imposed on those parameters.

In the case of meson-nucleon scattering the FESR (1.1) have been used most extensively for the study of meson Regge pole parameters [1], which control the asymptotic behaviour of the scattering amplitudes at fixed momentum transfer t. It was realized, however, that FESR may also be applicable in the fixed u case, when the respective fermion Regge poles determine the asymptotic behaviour of scattering amplitudes. One obtains

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in this case the following fixed u finite energy sum rules [2]:

$$\frac{1}{4\pi^2}\int_{-N}^{N} s^n\,[\mathrm{Im}\,A_u(s,u)+(\sqrt{u}-M)\,\mathrm{Im}\,B_u(s,u)]ds$$
$$=\sum_i \frac{2\beta_i}{\pi}\frac{N^{\alpha_i+n+1/2}}{\alpha_i+n+{}^1/_2}\,]\tau_i-(-1)^n] \tag{1.2}$$

which connect the respective fermion Regge pole parameters with signature τ_i with the average over the t-channel (negative s) and s channel absorptive parts of scattering amplitudes.

One of the interesting problems which should be confronted with these sum rules for KN scattering is the problem of $\Lambda_\alpha-\Lambda_\gamma$ exchange degeneracy. It has recently been proposed that the strange baryon resonances which couple predominantly to the $\bar{K}N$ channel such as Λ_α (even signature) and Λ_γ (odd signature) recurrences should be exchange-degenerate due to the small "exchange forces" which come, in this case, from the KN channel [3]. This means that the respective resonances with alternating signature should lie on the same Regge trajectory and should couple to the $\bar{K}N$ channel through the same residuum function. In the language of sum rules (1.2), this means that the even and odd moment sum rules should be compatible with the same trajectory and residuum function on the r.h.s. of (1.2).

In studying the fixed u sum rules we need the t-channel imaginary part of the scattering amplitudes which is usually approximated by the respective meson exchange contributions. The only contributions which may be in practice estimated are the vector meson (ϱ and ω) contributions where the respective coupling constants may be deduced from universality, vector meson dominance etc. In the case of KN scattering amplitudes we can also take advantage of the fact that $(K^+N)_u$ amplitudes are superconvergent at fixed u and consider amplitudes which are pure $C=-1$ in the t-channel rather than amplitudes with fixed u-channel quantum numbers. In the former case the approximation of the t-channel contribution by ϱ and ω is of course more reliable than in the latter. Nevertheless, it is doubtful whether this approximation is of any use for higher moment sum rules which are needed if we wish to confront the even and odd signature Regge pole parameters.

The purpose of this paper is to generalize the fixed u integer moment sum rules (1.2) into the continuous moment ones (CMSR) appropriate to the fixed u case and to apply them to the KN scattering amplitude in order to see whether the $\Lambda_\alpha-\Lambda_\gamma$ exchange degeneracy is compatible with them. The CMSR which we derive, are the FESR for the auxiliary amplitudes being the product of the genuine scattering amplitudes (A_u and B_u) and the simple analytic function of the variable s which has the right hand cut singularity only. At this point our sum rules differ from fixed t CMSR, where owing to the crossing symmetry, the left and right hand cuts are treated symmetrically [4]. The sum rules which are obtained in the fixed u case have the following form (formula (2.8) of the Section 2):

$$\frac{1}{4\pi^2}\int_{2s_0-\bar{s}}^{\bar{s}} \mathrm{Im}\,G(s,u;\lambda)ds$$

$$\equiv \frac{1}{4\pi^2} \int\limits_{2s_0-\bar{s}}^{\bar{s}} \{\mathrm{Im}\,[(s_0-s)^\lambda A_u(s,u)] + (\sqrt{u}-M)\cdot \mathrm{Im}\,[(s_0-s)^\lambda \cdot B_u(s,u)]\} ds$$

$$= \frac{2}{\pi} \sum_i \frac{\beta_i(\sqrt{u})}{\alpha_i(\sqrt{u})+\lambda+{}^1/_2} \left[-1 - \frac{\sin \pi\lambda}{\cos \pi\alpha_i} + \tau_i \frac{\cos \pi(\lambda+\alpha_i)}{\cos \pi\alpha_i}\right] \cdot (\bar{s}-s_0)^{\lambda+\alpha_i+1/2} \qquad (1.3)$$

where λ is a continuous parameter. The auxiliary function G is chosen in such a way that the real parts of the scattering amplitudes A and B appear for $s \geqslant s_0$, where they may be constructed whenever phase shift analysis is available. On the other hand, in the t-channel contributions there appear still only imaginary parts, and they may be subsequently aproximated by the vector-meson contributions. The pleasant feature of the CMSR (1.3) is the simple form of their r.h.s. in the case of single exchange-degenerate trajectory dominance

$$\text{r.h.s.} = \frac{4\beta(\sqrt{u})}{\pi(\alpha+\lambda+{}^1/_2)} \left[-1 - \frac{\sin \pi\lambda}{\cos \pi\alpha}\right] (\bar{s}-s_0)^{\lambda+\alpha+1/2}. \qquad (1.4)$$

Thus the CMSR offer the possibility of testing the exchange-degeneracy hypothesis using lower moment sum rules where the approximation of the t-channel contributions through ϱ and ω may still be reliable.

The presentation of the paper is as follows. In Section 2 the CMSR (1.3) are derived. In Section 3 we analyse them in the case of the KN scattering amplitude, taking the $T_{(K^+p)_s} - T_{(K^-p)_s}$ amplitude in the analysis. This amplitude is pure $C = -1$ in the t-channel and is believed [16, 17] to be dominated by the single $\Lambda_\alpha - \Lambda_\gamma$ exchange-degenerate trajectory. We also compare our sum rule results with the r.h.s. of Eq. (1.3) where the appropriate Regge pole parameters are put in the form deduced from high energy fits of backward K^+p scattering. Finally in Section 4 a brief summary of results is given.

2. *Fixed u continuous sum rules for meson-nucleon scattering*

In this section we derive CMSR appropriate for the analysis of fermion Regge pole parameters.

Let us introduce the respective u-channel invariant amplitudes A_u, B_u. The asymptotic behaviour of these amplitudes as the functions of s for a fixed value of u is controlled by the u-channel fermion Regge poles [5] which contribute in the following way to the asymptotic part of the amplitudes A_u and B_u [2, 5]:

$$f_1^{AS}(\sqrt{u}, s) = \frac{1}{8\pi} [A_u^{AS} + (\sqrt{u}-M)B_u^{AS}] = \sum_i \frac{\beta_i(\sqrt{u})}{\cos \pi\alpha_i(\sqrt{u})} [1+e^{-i\pi(\alpha-1/2)} \cdot \tau_i] s^{\alpha_i(\sqrt{u})-1/2} \qquad (2.1)$$

where α_i and β_i denote the trajectory and residuum functions respectively of the given Regge poles with signature τ_i and $\tau_i P = -1$. Regge pole parameters with signature τ_i and $\tau_i P = 1$ are related to those with $\tau_i P = -1$ through MacDowell symmetry [5]:

$$\alpha_{\tau P=1}(\sqrt{u}) = \alpha_{\tau P=-1}(-\sqrt{u})$$
$$\beta_{\tau P=1}(\sqrt{u}) = -\beta_{\tau P=-1}(-\sqrt{u}) \qquad (2.2)$$

If the asymptotic formula (2.1) is combined with the fixed u dispersion relations for the amplitude $f_1(s, \sqrt{u})$:

$$f_1(s, \sqrt{u}) = \frac{1}{8\pi^2} \int_{t_0}^{\infty} \frac{\operatorname{Im} A_u(t', u) + (\sqrt{u} - M) \operatorname{Im} B(t', u)}{t' - t(s, u)} dt' +$$

$$+ \frac{1}{8\pi^2} \int_{0}^{\infty} \frac{\operatorname{Im} A_u(s', u) + (\sqrt{u} - M) \operatorname{Im} B_u(s', u)}{s' - s} ds'$$

one obtains the familiar fixed u Finite Energy Sum Rules [2]

$$\frac{1}{4\pi^2} \int_{-N}^{N} [\operatorname{Im} A_u(s', u) + (\sqrt{u} - M) \operatorname{Im} B(s', u)] s'^n ds'$$

$$= \frac{2}{\pi} \sum_i \beta_i(\sqrt{u}) \frac{N^{\alpha_i(\sqrt{u}) + n + 1/2}}{\alpha_i(\sqrt{u}) + n + 1/2} [\tau_i - (-1)^n] \tag{2.3}$$

which have been discussed for πN [2] and for KN [6] scattering. We now generalize the integer moment sum rules (2.3) into the continuous moment ones. With this aim let us introduce an auxiliary function $G\ (s, u; \lambda)$ which is defined as

$$G(s, u; \lambda) = (s_0 - s)^{\lambda} f_1(s, u)$$

$$s_0 \gtrsim (M + \mu)^2 \tag{2.4}$$

where $(M+\mu)^2$ is the elastic threshold in the s-channel. Let us assume for convenience that the asymptotic behaviour of the amplitude $f_1(s, u)$ is given by the following expression

$$f_1^{AS} = \sum_i \beta_i \frac{(s - s_0)^{\alpha_i - 1/2} + \tau_i (s_0 - s)^{\alpha_i - 1/2}}{\cos \pi \alpha_i} . \tag{2.5}$$

(This formula is equivalent to (2.1) up to the leading powers in s.) From (2.4) and (2.5) we obtain the following expression for the asymptotic part of the function $G\ (s, u; \lambda)$

$$G^{AS}(s, u; \lambda) = (s_0 - s)^{\lambda} \sum_i \frac{\beta_i [(s - s_0)^{\alpha_i - 1/2} + \tau (s_0 - s)^{\alpha_i - 1/2}]}{\cos \pi \alpha_i} . \tag{2.6}$$

Let us now consider an integral along the contour as in Fig. 1. From the Cauchy theorem we obtain:

$$\int_{2s_0 - \bar{s}}^{\bar{s}} G(s' + i\varepsilon, u; \lambda) ds = - \int_{SC} G(s, u; \lambda) ds$$

$$\cong - \int_{SC} G^{AS}(s, u; \lambda) ds = \sum_i \frac{1}{\alpha_i + \lambda + 1/2} \frac{(\bar{s} - s_0)^{\alpha_i + \lambda + 1/2}}{\cos \pi \alpha_i} \beta_i \times$$

$$\times \{(e^{-i\pi\lambda} - e^{-i\pi(\alpha + 1/2)}) + \tau_i (e^{-i\pi(\lambda + \alpha - 1/2)} - 1)\} \tag{2.7}$$

where SC denotes the semicircle.

Taking the imaginary part on both sides of Eq. (2.7) we obtain finally the following identity:

$$\frac{1}{4\pi^2}\int_{2s_0-\bar{s}}^{\bar{s}}\{\mathrm{Im}[(s_0-s)^\lambda A_u(s,u)]+(\sqrt{u}-M)\,\mathrm{Im}\,[(s_0-s)^\lambda B_u(s,u)]\}ds$$

$$\equiv\frac{1}{4\pi^2}\int_{2s_0-\bar{s}}^{s_0}(s_0-s)^\lambda\,[\mathrm{Im}\,A_u(s,u)+(\sqrt{u}-M)\,\mathrm{Im}\,B_u\,(s,u)]ds+$$

$$+\frac{1}{4\pi^2}\int_{s_0}^{\bar{s}}(s-s_0)^\lambda\{\cos\pi\lambda[\mathrm{Im}\,A_u(s,u)+(\sqrt{u}-M)\,\mathrm{Im}\,B_u(s,u)]-$$

$$-\sin\pi\lambda[\mathrm{Re}\,A_u(s,u)+(\sqrt{u}-M)\,\mathrm{Re}\,B_u(s,u)]\}ds$$

$$=\frac{2}{\pi}\sum_i\left\{\beta_i(\sqrt{u})\left[-1-\frac{\sin\pi\lambda}{\cos\pi\alpha_i}+\tau_i\frac{\cos\pi(\lambda+\alpha_i)}{\cos\pi\alpha_i}\right]\times\frac{(\bar{s}-s_0)^{\alpha_i+\lambda+1/2}}{\alpha_i+\lambda+{}^1/_2}\right\}\qquad(2.8)$$

which is our fixed u CMSR with the continuous parameter λ. Let us now discuss some features of the sum rules (2.8). It is seen from (2.8) that, apart from the t-channel and s-channel imaginary parts of the scattering amplitudes A and B, their real parts are also needed in general. Due to the fact that the function $(s_0-s)^\lambda$, which multiplies the genuine scattering amplitudes A and B, carries only the right hand cut singularity a knowledge of these real parts is required for $s\geqslant s_0$ only. Thus, in the t-channel contributions only imaginary parts appear and they may be subsequently approximated by the respective meson-exchange contributions.

Since the real parts of scattering amplitudes are required for s lying in the physical region of the s-channel, we may use a partial wave expansion and obtain the real parts whenever the corresponding phase-shift analysis is available. (The same, of course, refers to the imaginary parts of scattering amplitudes in the physical region of the s-channel.) We should bear in mind, however, that since we use scattering amplitudes at fixed u ($u\leqslant 0$) this means that we need, in general, to go to the unphysical region of the s-channel $|\cos\vartheta(s,u)|>1$, where the use of truncated partial wave series may be wrong, especially for real parts of scattering amplitudes. The region of s-variable, where the use of truncated partial wave expansion of real parts of scattering amplitudes (at fixed $u\leqslant 0$) would be most questionable is the low energy region, just above the s-channel threshold. In this region and for $u\leqslant 0$ the real parts of scattering amplitudes are strongly affected by the nearby t-channel contributions and it is rather doubtful whether the truncated partial wave series would be correct in this case. The situation is far more favourable for imaginary parts, since the contributions of higher waves, in this case, are strongly suppressed owing to unitarity. Thus, in practical applications of sum rules (2.8) it is safer to push the value s_0 somewhere above the s-channel elastic threshold in order to avoid the need of real parts very near the threshold. An unpleasant feature of the sum rules (2.8) is the rather complicated structure of their right hand side if several Regge poles contribute without any correlation between their parameters. This involved structure is also the consequence of the analytic properties of the function $(s_0 - s)^\lambda$, which multiplies the genuine scattering amplitudes A and B. The r.h.s. can, of course, be simplified if we use the "crossing symmetric" function of the type $(s_0^2 - s^2)^\lambda$ in the definition of the auxiliary amplitude $G(su; \lambda)$ (as in the fixed t case) but the resulting CMSR are not suitable in the fixed u case, since they require in general a knowledge of the real parts in the t-channel.

Thus we expect our sum rules (2.8) to be useful only in those cases when a single Regge pole is believed to dominate. In the case of single, exchange degenerate trajectory $\alpha_{\tau=1}(\sqrt{u}) = \alpha_{\tau=-1} = \alpha$, $\beta_{\tau=+1}(\sqrt{u}) = \beta_{\tau=-1}(\sqrt{u}) = \beta$ the r.h.s. of our sum rules reduces to the following simple form:

$$\text{r.h.s.} = \frac{4\beta(\sqrt{u})}{(\alpha+\lambda+{}^1/_2)\,\pi}\left[-1 - \frac{\sin \pi\lambda}{\cos \pi\alpha}\right](\bar{s}-s_0)^{\alpha+\lambda+1/2}.$$

Therefore we expect that sum rules (2.8) will be very useful in this case and, in particular, they offer the possibility of testing the exchange-degeneracy hypothesis with the use of rather low moment sum rules $\lambda \lesssim 1$ where the approximation of the t-channel contributions by the nearby meson-exchange contributions may be still reliable.

3. *Analysis of fixed u CMSR for Kp scattering*

In the preceding section we derived the fixed u CMSR and in this section we apply them to the analysis of Kp scattering amplitude at fixed u.

It has recently been suggested [3] that, owing to the absence of resonances in the KN channel (*i.e.* owing to the absence of the significant exchange forces, from the point of view of the $\bar{K}N$ channel), the strange baryon resonances which couple predominantly to the $\bar{K}N$ system should be exchange degenerate, *i.e.* the resonances with alternating signature should be described by the same Regge trajectory and should couple to $\bar{K}N$ channel through the same residuum function. The best candidate for these exchange degenerate Regge recurrences are the Λ_α ($1/2^+$, $5/2^+$...) and Λ_γ ($3/2^-$, $7/2^-$...) recurrences which couple predominantly to the $\bar{K}N$ channel and the fact that they are described by a single Regge-trajectory is impressively confirmed by the corresponding Chew-Frautschi plot [3].

In order to test this $\Lambda_\alpha - \Lambda_\gamma$ exchange degeneracy hypothesis using the FESR technique, we should consider the KN scattering amplitudes with the fixed u-channel quantum numbers $I_u = Y_u = 0$. This means, however, that in the t-channel imaginary part we need the $C = +1$ contributions (like A_2, f_0, S_0 etc. contributions in the meson-exchange approximation).

As we remarked in the Introduction, in the case of KN scattering sum rules we can eliminate these contributions, taking advantage of the fact that the $(KN)_u$ amplitudes are superconvergent at fixed u and consider only those amplitudes which are pure $C = -1$ in the t-channel. The fixed u superconvergence relations might be questionable if the cuts in the complex j-plane generated, for instance, by the simultaneus exchange of K^* and N Regge poles were important. The absence of the backward peaks for high energy $\bar{K}N$ [7]

scattering suggests that they are probably quite unimportant. Thus in the sum rules we might consider the following amplitude:

$$T = 2(T_{K^-p} - T_{K^+p}) - (T_{K^-n} - T_{K^+n})$$

which is pure $C = -1$ in the t-channel and is still dominated asymptotically by the $Y = I = 0$ Regge poles which couple to the $\bar{K}N$ channel. This amplitude, though it is pure $C = -1$ in the t-channel, is still not very suitable for CMSR analysis owing to the incomplete information about the s-channel K^+n amplitude.

Thus in what follows we consider the following combination of s-channel scattering amplitudes:

$$T_\Lambda = T_{(K^+p)_s} - T_{(K^-p)_s} \tag{3.1}$$

which is still believed to be dominated by the Λ type Regge poles; since $I = 1$ trajectories couple rather weakly to the $\bar{K}N$ channel.

This may be seen by comparing, for instance, the ΛKN and ΣKN coupling constants [8]. The weak coupling of those trajectories at $u = 0$ is also supported by the analysis of fixed u integer moment sum rules for KN scattering [6]. If the single exchange -degenerate $\Lambda_\alpha - \Lambda_\gamma$ Regge trajectory dominates in the asymptotic behaviour of the amplitude (3.1), then we obtain the following CMSR for the corresponding $f_1^\Lambda(\sqrt{u}, s)$ amplitudes (see Eq. (2.8) and (2.9) of the preceding section):

$$\frac{1}{4\pi^2} \int_{2s_0-\bar{s}}^{\bar{s}} \mathrm{Im}[(s_0-s)^\lambda A'_\Lambda(s, u)]ds + (M-\sqrt{u}) \int_{2s_0-\bar{s}}^{\bar{s}} \mathrm{Im}\,[(s_0-s)^\lambda B_\Lambda(s, u)]ds$$
$$= -\frac{4\beta_\Lambda}{\pi \cos \pi\alpha_\Lambda}\left[1 + \frac{\sin \pi\lambda}{\cos \pi\alpha_\Lambda}\right](s_0-\bar{s})^{\alpha+\lambda+1/2}. \tag{3.2}$$

If we assume that the α_Λ trajectory is an even function of $\sqrt{u}$ and if we decompose the residuum function $\beta(\sqrt{u})$ as below

$$-\beta_\Lambda(\sqrt{u}) = a_\Lambda(u) - \sqrt{u}\,b_\Lambda(u) \tag{3.3}$$

we obtain the following sum rules for $a_\Lambda(u)$ and $b_\Lambda(u)$

$$\frac{1}{4\pi^2} \int_{2s_0-\bar{s}}^{\bar{s}} \mathrm{Im}\,[(s_0-s)^\lambda A_\Lambda(s, u) + MB_\Lambda(s, u)]ds$$
$$= \frac{4}{\pi}\frac{a_\Lambda(u)}{\alpha+{}^1/_2+\lambda}\left[1 + \frac{\sin \pi\lambda}{\cos \pi\alpha}\right](\bar{s}-s_0)^{\alpha+1/2+\lambda} \tag{3.4a}$$

$$\frac{1}{4\pi^2} \int_{2s_0-\bar{s}}^{\bar{s}} \mathrm{Im}\,\{(s_0-s)^\lambda B_\Lambda(s, u)\}ds$$
$$= \frac{4}{\pi}\frac{b_\Lambda(u)}{\alpha+\lambda+{}^1/_2}\left[1 + \frac{\sin \pi\lambda}{\cos \pi\alpha}\right](\bar{s}-s_0)^{\alpha+\lambda+1/2} \tag{3.4b}$$

(Note that since our amplitudes are defined in the s-channel we have changed the sign of the B amplitude in the sum rules (3.4), (3.4b) as compared with the sumrules (2.8) where the corresponding amplitudes have been defined in the u-channel.) We now list the various contributions which have been included in the evaluation of the sum rule integrals on the left hand side of (3.4).

A. The t-channel contributions

They are approximated by the ϱ and ω which contribute in the following way to our sum rules [2]:

$$-\frac{1}{4\pi^2}\int_t \mathrm{Im}\,\{A_\Lambda^t(s,u)\,(s_0-s)^\lambda\}ds = -\frac{g_{\varrho K\bar{K}}g_{\varrho NN}}{4\pi}\mu_\varrho \times$$

$$\times\frac{2(M^2+M_K^2)-M_\varrho^2-2u}{2M}\times\{s_0-2(M^2+M_K^2)+M_\varrho^2+u\}^\lambda$$

$$-\frac{\varrho_{\omega K\bar{K}}\varrho_{\omega NN}}{4\pi}\mu_\omega\frac{2\,(M^2+M_K^2)-M_\omega^2-2u}{2M}\times\{s_0-2(M^2+M_K^2)+M_\omega^2+u\}^\lambda$$

$$-\frac{1}{4\pi^2}\int_t \mathrm{Im}\,\{B_\Lambda^t(s,u)(s_0-s)^\lambda\}ds = \frac{g_{\varrho K\bar{K}}g_{\varrho N\bar{N}}}{4\pi}(1+2\mu_\varrho)(s_0-2(M^2+M_K^2)+M_\varrho^2+u)^\lambda$$

$$+\frac{g_{\omega K\bar{K}}g_{\omega N\bar{N}}}{4\pi}(1+2\mu_\omega)(s_0-2(M^2+M_K^2)+M_\omega^2+u)^\lambda$$

where M is the nucleon mass, M_i denotes the mass of particle i. We have put the following values for the respective parameters [9]

$$g_{\varrho K\bar{K}}g_{\varrho N\bar{N}} = \frac{1}{3}g_{\omega K\bar{K}}\varrho_{\omega N\bar{N}} = \frac{g_\varrho^2}{4\pi} = 2.5.$$

(This correspond to the ϱ width $\Gamma = 125$ MeV)

$$\mu_\varrho = 1.85$$
$$\mu_\omega = 0.05$$

as suggested by universality and vector-meson dominance of electromagnetic form-factors [9].

B. The s-channel contributions of K^-p amplitude

This part of the left hand side contains the following contributions:

i. The Σ and Λ poles which have been estimated for two different sets of g_Λ^2 and g_Σ^2 coupling constants

$$\frac{g_\Lambda^2}{4\pi} = 5.7 \qquad \frac{g_\Sigma^2}{4\pi} = 1.7$$

(Zovko values [10])

$$\frac{g_\Lambda^2}{4\pi} = 13.5 \qquad \frac{g_\Sigma^2}{4\pi} = 0.3$$

(Kim values [11])

(For a review of the present state of knowledge about these coupling constants see Ref. [8].)

ii. Integral over the unphysical region of the K^-p amplitude $(M_\Lambda+m_\pi)^2 \leqslant s < (M_p+ +m_K)^2$ which has been approximated by the $Y_1^*(1385)$ pole with the Frye-Warnock [12] coupling constant $g^2_{Y_1^*}/4\pi = 1.9/M_p^2$ and by the s-waves in the scattering length approximation with the following values for the scattering lengths a_I [13]

$$a_0 = (-1.67+i\,0.72)\,fm$$

$$a_1 = (-0.003+i.0.69)\,fm.$$

iii. Integral over the low energy region $(M_p+M_K)^2 \lesssim s \lesssim s_0 = 2.49\ \mathrm{GeV}^2$ which has been evaluated by using the SL approximation for s-waves and by including resonances in higher waves.

iv. The integral over the region $s_0 < s \lesssim 4.07\ \mathrm{GeV}^2$ which has been evaluated using the resonance + background parametrization of Armenteros *et al.* [14].

v. The integral over the region $4.07 < s < 5.07\ \mathrm{GeV}^2$ which has been approximated by retaining the resonant partial waves only[1].

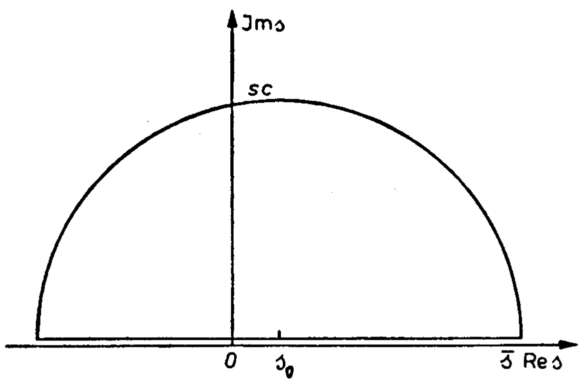

Fig. 1. Contour of integration

C. The s-channel contributions of the K^+p amplitude have been estimated using the most recent K^+p phase shift analysis [15] which covers the whole energy range $(M+ +M_K)^2 < s \lesssim 5.07\ \mathrm{GeV}^2$.

The resonant partial waves in the K^-p amplitudes have been parametrized in the same way as in Ref. [14], namely by the Breit-Wigner form with the energy-dependent widths.

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[1] We have continued the resonance + background parametriation of K^-p scattering amplitude up to $\bar{s} = 4.07\ \mathrm{GeV}^2$ ($p_{\mathrm{lab}} \sim 1.5$ GeV/c) despite the fact that it has only been done up to $p_{\mathrm{lab}} = 1.20$ GeV/c [14]. It has been argued in Ref. [4] (Dass, Michael) that this parametrization may be applicable up to $p_{\mathrm{lab}} \approx 1.5$ GeV/c. At any rate we have found that the nonresonant background of K^-p amplitude gives small contributions to our sum rules and that the dominant contributions from the s-channel physical region of this amplitude come from its resonant part.

We have extended our sum rule integrals over K^-p amplitudes up to $\bar{s} = 5.07\ \mathrm{GeV}^2$ ($p_{\mathrm{lab}} = 2$ GeV/c), retaining the resonant partial waves only in order to treat more carefully the Λ (2100) contribution, which is not fully taken into account with the integrals cut off at $\bar{s} \cong 4\ \mathrm{GeV}^2$.

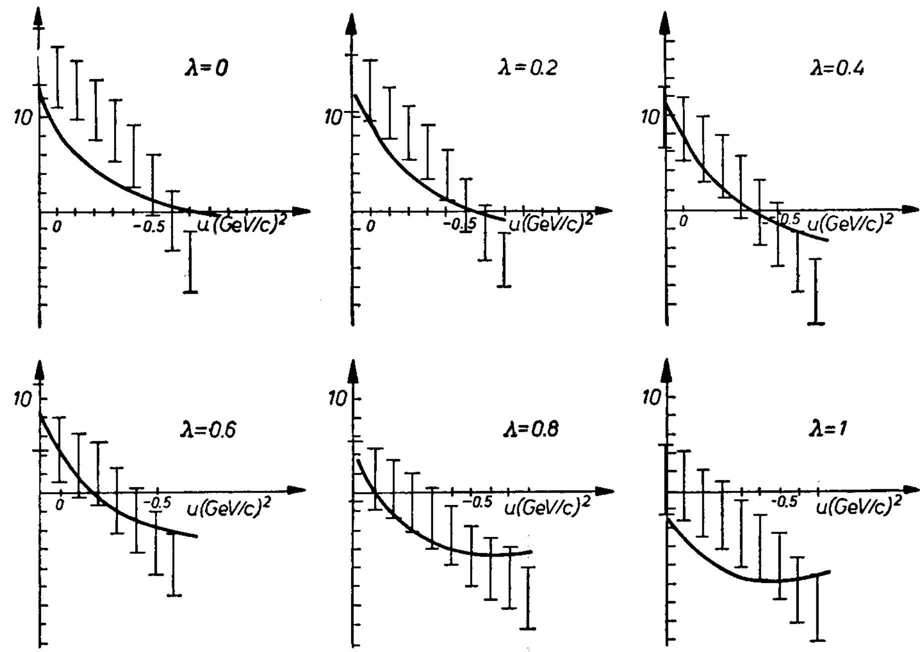

Fig. 2. The sum rule results for the B amplitude plotted as the functions of u for different values of the parameter λ. Upper and lower points correspond to the Zovko and Kim coupling constants respectively. The continuous curve is the r.h.s. of Eq. (3.4) with the $\Lambda_\alpha - \Lambda_\gamma$ trajectory parameters taken from Ref. [17] (see also formulas (3.6), (3.7))

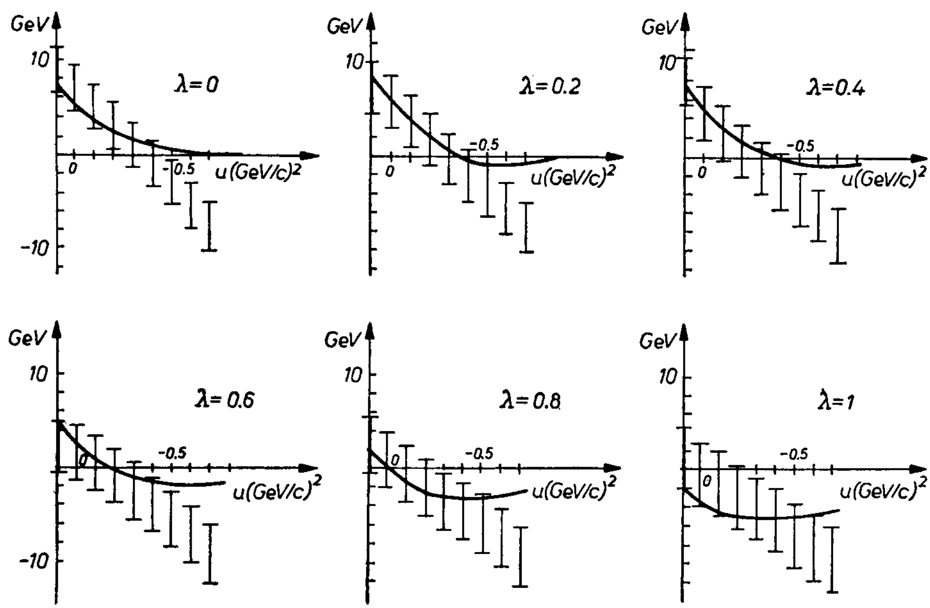

Fig. 3. The sum rule results for the $A+MB$ amplitude plotted as the functions of u for different values of the parameter λ. Upper and lower points correspond to the Zovko and Kim coupling constants respectively. The continuous curve is the r.h.s. of Eq. (3.4) with the $\Lambda_\alpha - \Lambda_\gamma$ parameters taken from Ref. [17]

We have included the following resonances in our sum rules:

$$\Lambda(1520),\ \Lambda(1690),\ \Lambda(1815),\ \Lambda(1830),\ \Lambda(2100)$$
$$\Sigma(1770),\ \Sigma(1910),\ \Sigma(2030)$$

with their parameters taken from the most recent Rosenfeld table. (They do not differ appreciably from those cited in Ref. [14].) The parameter s_0, which appears in the sum rules (3.4a, b), has been put as $s_0 = 2.49\ \text{GeV}^2$ whether the upper limit of integration $\bar{s}$ is equal $\bar{s} = 5.07\ \text{GeV}^2$. The reason for putting s_0 somewhere above the elastic threshold was discussed in the preceding section. Our sum rule results (the left hand sides of Eqs (3.4a) and (3.4b)) are plotted in Figs 2 and 3 as the functions of u for several values of

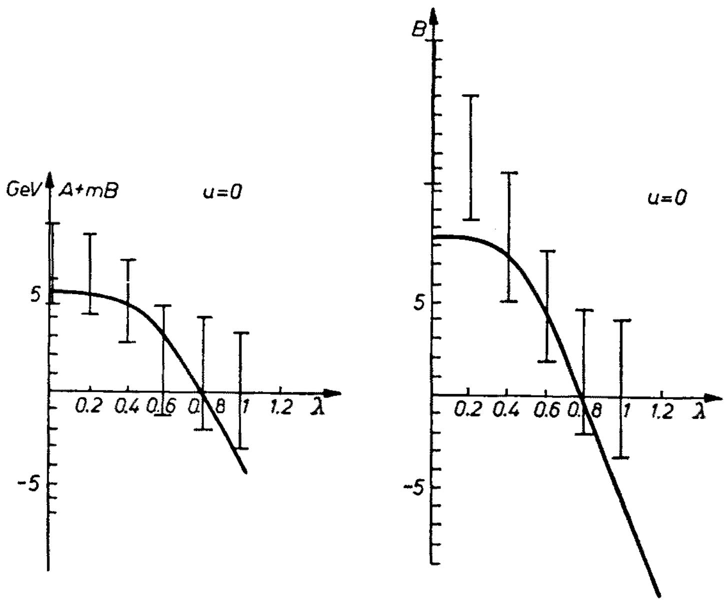

Fig. 4. The sum rule results for B and $A+MB$ amplitudes for $u = 0$

the parameters λ, and in Fig. 4 as functions of λ, for $u = 0$. We see that the sum rules are quite sensitive to the adopted values of $\varrho^2_{\Lambda KN}/4\pi$ coupling constants, as is usual in the case of KN sum rules, thus only qualitative predictions may be extracted from them. Thus we should like to point out that the sum rule results for both amplitudes change sign for certain $-u_0(\lambda)$ which has a tendency to shift to the left as λ increases. We tentatively interpret this fact as being the rough reflection of the factor $1+\dfrac{\sin\pi\lambda}{\cos\pi\alpha}$ in the r.h.s. of Eqs (3. 4a, 4b) which is an immediate consequence of $\Lambda_\alpha - \Lambda_\gamma$ exchange-degeneracy. In order to see to what extent the sum rules are compatible with the r.h.s., we plot in the same figures the functions

$$a_\Lambda(u)\,\frac{(\bar{s}-s_0)^{\alpha+\lambda+1/2}}{\alpha+\lambda+{}^1/_2}\left[1+\frac{\sin\pi\lambda}{\cos\pi\alpha}\right]$$

$$b_A(u)\,\frac{(s-s_0)^{\alpha+\lambda+1/2}}{\alpha+\lambda+1/2}\left[1+\frac{\sin\pi\lambda}{\cos\pi\alpha}\right] \tag{3.5}$$

with the following parametrization for the residuum function

$$-\beta_A=(\alpha+1/2)(\alpha+3/2)\beta_0(u)\left(1-\frac{\sqrt{u}}{M_A}\right)\left(1-\frac{\sqrt{u}}{M_{Y_0'}}\right) \tag{3.6}$$

which, for $\Lambda_\alpha-\Lambda_\gamma$, has been suggested in Ref. [16]. (The factors $\left(1-\frac{\sqrt{u}}{M_A}\right)$ and $(1-\sqrt{u}/M_Y)$ are responsible for the absence of $1/2^-$ and $3/2^+$ MacDowell partners of the Λ_α and Λ_γ. (Note that, according to Eq. (2.1), the $\beta(\sqrt{u})$ is the residuum of $\Lambda_\beta-\Lambda_\delta$ trajectory.)

In plotting the r.h.s. we use the following parametriation of the $\beta_0(u)$, which has been obtained from the recent fit of backward K^+p scattering [17]:

$$\beta_0=6.39\ \mathrm{GeV}^{-1}\exp(0.5078\,u)$$

$$\alpha_A(u)=-0.7+1.116\,u. \tag{3.7}$$

We see that our sum rules are in general agreement with the r.h.s. of the exchange-degenerate type, at least at the following qualitative points:

1. The CMSR results are, in general, small near those values of $u_0(\lambda)$ whenever the r.h.s. vanishes due to the factor $\left(1+\frac{\sin\pi\lambda}{\cos\pi\alpha}\right)$.

2. They predict the same relative sign of a_A and b_A as appears in the parametrization (3.6).

3. They agree in order of magnitude with the r.h.s. which is computed using the $\Lambda_\alpha-\Lambda_\gamma$ trajectory parameters obtained from high energy fits of backward K^+p scattering. In particular, the sum rules predict fairly substantial $\sqrt{u}$ dependence of the Λ residuum function, this being in agreement with the parametrization (3.6).

Nevertheless, we have noticed that the agreement begins to deteriorate for higher moment sum rules $\lambda \gtrsim 1.2$ (*cf.* Fig. 4.) but it is almost certain that this is due to the omission of distant t-channel contributions.

To sum up, we can say that though it is impossible to make definite quantitative predictions from fixed u sum rules for KN scattering, owing to the uncertainties in the input data and very crude estimation of the t-channel contributions, they are useful as consistency conditions on Regge pole parameters. In the case of the discussed K^+p scattering, they are quite compatible with the single, $\Lambda_\alpha-\Lambda_\gamma$ exchange-degenerate trajectory dominance.

4. *Summary of results*

We have developed a technique of fixed u continuous moment sum rules and applied them subsequently to the K^+p amplitude. We have found that sum rule results are compatible with the $\Lambda_\alpha-\Lambda_\gamma$ exchange-degeneracy hypothesis and agree qualitatively with the $\Lambda_\alpha-\Lambda_\gamma$ Regge pole parameters obtained from high energy fits of K^+p backward scattering.

Nevertheless, owing to the crude estimation of the t-channel contributions, the fixed u sum rules are less powerful than their fixed t counterpart and may therefore be used only as consistency conditions rather than as a source of quantitative information about the respective fermion Regge pole parameters.

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REFERENCES

[1] R. Dolen, D. Horn, C. Schmid, *Phys. Rev.*, **166**, 1768 (1968); Chan-Hong-Mo, *Talk given at Vienna Conference on High Energy Physics*, Vienna, August 1968 and references therein; K. Igi, S. Matsuda, *Phys. Rev. Letters*, **18**, 625 (1967); A. A. Logunov, L. D. Soloviev, A. N. Tavkhelidze, *Phys. Letters*, **24B**, 181 (1967).

[2] C. B. Chiu, M. DerSarkissian, *Nuovo Cimento*, **55A**, 396 (1968); A. Bartl, W. Widder, *Z. Phys.*, **217**, 217 (1968); see also S. D. Beder, J. Filkenstein, *Phys. Rev.*, **160**, 1363 (1967); D. Griffith W. Palmer, *Phys. Rev.*, **161**, 1606 (1967); R. Ramachandran, *Phys. Rev.*, **166**, 1528 (1968); G. V. Dass C. Michael, *Phys. Rev.*, **162**, 1403 (1967); V. Barger, C. Michael, R. J. N. Phillips, *Phys. Rev.*, **185**, 1852 (1969).

[3] C. Schmid, *CERN-preprint* TH-960, November 1968.

[4] G. V. Dass, C. Michael, *Phys. Rev. Letters*, **20**, 1066 (1968); V. Barger, R. J. N. Philips, *Phys. Letters*, **26B**, 730 (1968); *Phys. Rev. Letters*, **21**, 865 (1968); for further references see *e.g.* Chan-Hong-Mo, Ref. [1].

[5] V. Singh, *Phys. Rev.*, **129**, 1889 (1963).

[6] V. Barger, C. Michael, R. J. N. Phillips, *Phys. Rev.*, **185**, 1852 (1969); Z. Gołąb-Meyer, J. Kwieciński, *Acta Phys. Polon.*, **36**, 227 (1969).

[7] J. Banaigs, J, Berger, C. Bonnel, J. Duflo, L. Goldzahl, P. Plonin, W. F. Baker, P. J. Carlson, V. Chabaud, A. Lundby, *Nuclear Phys.*, **B9**, 640 (1969).

[8] N. M. Queen, M. Restignoli G. Violini, *Fortschr. Phys.*, **17**, 467 (1969).

[9] V. Barger, *Talk given at Topical Conference on High Energy Physics*, CERN, January 1968 and references therein.

[10] N. Zovko, *Phys. Letters*, **23**, 143 (1966).

[11] J. K. Kim, *Phys. Rev. Letters*, **19**, 1079 (1957).

[12] G. Frye, R. L. Warnock, *Phys. Rev.*, **138**, B947 (1967).

[13] J. K. Kim, *Phys. Rev. Letters*, **14**, 29 (1965).

[14] A. Armenteros *et al.*, *Nuclear Phys.*, **B8**, 195 (1968).

[15] B. R. Martin, *Phys. Rev. Letters*, **21**, 1286 (1968); see also A. T. Lea, B. R. Martin, G. C. Oades, *Phys. Rev.*, **165**, 177 (1968).

[16] V. Barger, *Phys. Rev.* **179**, 371 (1969).

[17] P. Gizbert-Studnicki and A. Golemo, private communication.

Note added in proof

After this work was completed we have been informed by Mr P. Gizbert-Studnicki and Mr A. Golemo that the backward K^+p scattering may be also fitted assuming partial $\Lambda_\alpha - \Lambda_\gamma$ exchange degeneracy. The respective fits and their comparison with our sum results are presented in the paper: P. Gizbert-Studnicki and A. Golemo, *Acta Phys. Polon.*, **A37**, 143 (1970).