QUARK MODEL PREDICTIONS FOR THE PHOTOPRODUCTION OF RESONANCES

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The quark model predictions for the decay distributions of resonances photoproduced in the reactions $\gamma B \rightarrow PB^*$, $\gamma B \rightarrow VB$ and $\gamma B \rightarrow VB^*$ are given. The numbers of relations predicted for the three reactions are: three, three and sixty respectively.

1. *Introduction*

High energy photoproduction processes are usually interpreted in terms of the Vector Meson Dominance Model (VDM). For a recent review see Ref. [1]. In the framework of the VDM a photoproduction amplitude is expressed by the amplitude for the strong process with a vector meson replacing the initial photon. This makes it natural to apply the quark model to high energy photoproduction. References to early work in this field may be found in Ref. [2].

In this paper we consider the processes

$$\gamma B \rightarrow PB^*, \tag{1}$$

$$\gamma B \rightarrow VB, \tag{2}$$

$$\gamma B \rightarrow VB^* \tag{3}$$

where γ, P, V, B and B^* denote respectively: a photon, a pseudoscalar meson, a vector meson, a $\frac{1}{2}^+$ baryon and a $\frac{3}{2}^+$ isobar. For these reactions the quark model gives definite predictions for the decay distributions of the resonances. Gorczyca and Hayashi [3], [4] derived some linear relations between the statistical tensors or equivalently between the elements of the spin density matrices. Here we give more general formulae, which may

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be used to test both the linear and the nonlinear (*cf.* Ref. [5]) relations predicted by the quark model.

For the incident photon a general polarization state was assumed. Thus in the photon helicity frame the density matrix reads

$$\varrho = \frac{1}{2}\begin{pmatrix} 1+\alpha & 0 & -\beta+i\gamma \\ 0 & 0 & 0 \\ -\beta-i\gamma & 0 & 1-\alpha \end{pmatrix}. \tag{4}$$

In order to get simpler formulae we work with the transversity basis, *i.e.* with the spin projections on the normal to the reaction plane, where Eq. (4) implies

$$\varrho = \frac{1}{4}\begin{pmatrix} 1+\beta & \sqrt{2}(\gamma-i\alpha) & -1-\beta \\ \sqrt{2}(\gamma+i\alpha) & 2(1-\beta) & -\sqrt{2}(\gamma+i\alpha) \\ -1-\beta & -\sqrt{2}(\gamma-i\alpha) & 1+\beta \end{pmatrix}. \tag{5}$$

We use the symbol ϱ^{mn} to denote the elements of this density matrix. Thus *e.g.* ϱ^{00} means $\frac{1}{2}(1-\beta)$. The interpretation of the Stokes parameters α, β, γ is easily found from formula (4). In particular for an unpolarized beam $\alpha = \beta = \gamma = 0$. Parameter α is a measure of the circular polarization. The probability of finding the right-handed polarization is $\frac{1}{2}(1+\alpha)$ and the probability of finding the left-handed polarization is $\frac{1}{2}(1-\alpha)$. The probability of finding the linear polarization along the x-axis is $\frac{1}{2}(1+\beta)$ and the linear polarization along the y-axis is $\frac{1}{2}(1-\beta)$. Similarly, γ describes the linear polarization along the two axes forming with the x-axis angles $\pm 45°$.

The target baryon is assumed unpolarized.

2. *The reaction $\gamma B \to PB^*$*

Using the formalism of the scalar amplitudes [7] it is easy to see that in the framework of the VDM the most general amplitude consistent with the quark model is

$$T = K\left[-\frac{a_1}{\sqrt{6}}\varepsilon^0\Sigma^0 + \sqrt{\frac{2}{3}}\,(a_5\varepsilon^-\Sigma^+ + a_6\varepsilon^+\Sigma^- + a_7\varepsilon^+\Sigma^+ + a_8\varepsilon^-\Sigma^-)\right]. \tag{6}$$

Here and everywhere in the following all the spin reference frames are assumed to have their spin quantization axes (z axes) normal to the reaction plane. Symbols $\vec{\varepsilon}$ and $\vec{\Sigma}$ denote the sets of matrices defined in Ref. [7]. Here they are written in the spherical basis, thus

$$\varepsilon^+ = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \varepsilon^0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \varepsilon^- = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{7}$$

$$\Sigma^+ = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{3} & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \Sigma^0 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \Sigma^- = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & \sqrt{3} \end{pmatrix}. \tag{8}$$

The coefficients a_i do not depend on the spin states of the interacting particles. They are closely related to the quark quark scattering amplitudes f_i used in previous papers (see *e.g.* Ref. [5]). The coefficient K is characteristic to the VDM approach. It ensures the transversality of the photon. It is zero if the photon is longitudinal, and one otherwise.

From (5) and (6) a straightforward calculation yields for the statistical tensors [8] expressions in terms of the parameters a_i, α, β and γ. We list the formulae for the statistical tensors, which can be determined from data about the decay distribution of the B^*:

$$T_2^2 = \frac{1}{\sqrt{6}} [\varrho^{11}(a_8a_5^* + a_6a_7^*) + \varrho^{1-1}a_8a_7^* + \varrho^{-11}a_6a_5^*), \tag{9}$$

$$T_1^2 = \frac{1}{4\sqrt{3}} [\varrho^{10}a_8a_1^* + \varrho^{-10}a_6a_1^* + \varrho^{01}a_1a_5^* + \varrho^{0-1}a_1a_7^*], \tag{10}$$

$$T_0^2 = \frac{1}{4} - \frac{1}{8}\varrho^{00}|a_1|^2, \tag{11}$$

with the normalization condition

$$1 = \frac{1}{6}\varrho^{00}|a_1|^2 + \frac{2}{3}\varrho^{11}|a|^2 + \frac{4}{3}\mathrm{Re}\,[\varrho^{1-1}(a_7a_5^* + a_8a_6^*)], \tag{12}$$

where

$$|a|^2 = |a_5|^2 + |a_6|^2 + |a_7|^2 + |a_8|^2. \tag{13}$$

Here and in the following the asterisk means complex conjugation.

By counting parameters it is easily seen that the formulae (9)–(12) imply three relations between measurable quantities. These relations are quite simple [3]. For photons polarized perpendicularly to the reaction plane

$$T_2^2 = 0, \quad T_0^2 = -\frac{1}{2}. \tag{14}$$

For photons polarized in the reaction plane

$$T_0^2 = \frac{1}{4}. \tag{15}$$

The prediction $T_1^2 = 0$ does not count, since in both cases it follows from parity conservation.

Usually two ambiguities have to be discussed when applying the VDM and the quark model to photoproduction. Firstly, formula (6) is valid only if the additivity frames, corresponding to the initial transversity frames, are used for the final particles. A discussion of this point and references to earlier work can be found in Ref. [6]. Secondly, in order to use the VDM it is necessary to know the fundamental frame, *i.e.* the frame where the amplitude changes little when the mass of the photon increases from zero to the mass of a physical vector meson (see *e.g.* Ref. [9]). The second point will not be discussed in this paper. It affects the numerical values of the amplitudes a_i, which here are considered to be unknown parameters anyway. The formulae (14) and (15) are invariant with respect to rotations of the spin reference frames around the z axes. Therefore for the reactions considered in this section the orientation of the additivity frame is irrelevant. It will be important for the reactions considered further.

3. *The reaction $\gamma B \to VB$*

For the reaction $\gamma B \to VB$ the general form of the amplitude is

$$T = K\left[-\frac{1}{\sqrt{6}}(b_1+b_2E^0+b_3\sigma^0+b_4E^0\sigma^0)-(b_5E^-\sigma^+-b_6E^+\sigma^-+b_7E^+\sigma^+-b_8E^-\sigma^-)\right]. \tag{16}$$

Here $\vec{\sigma}$ are the Pauli matrices. In the spherical basis

$$\sigma^+ = -\sqrt{2}\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \quad \sigma^0 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \quad \sigma^- = \sqrt{2}\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}. \tag{17}$$

Further $\vec{E}$ are matrices with rows and columns labelled by the spin projections of the vector meson and the photon

$$E^+ = \begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}, \quad E^0 = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}, \quad E^- = \begin{pmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}. \tag{18}$$

The measurable statistical tensors are

$$T_2^2 = \varrho^{00}(b_5b_7^*+b_8b_6^*)+\frac{1}{6}\varrho^{-11}[|b_{13}|^2-|b_{24}|^2+2i\,\mathrm{Im}\,(b_1b_2^*+b_3b_4^*)], \tag{19}$$

$$T_1^2 = \frac{1}{\sqrt{2}}(\varrho^{0-1}-\varrho^{10})(b_5b_7^*+b_8b_6^*)+\frac{1}{\sqrt{2}}(\varrho^{01}|b_{58}|^2-\varrho^{-10}|b_{67}|^2)+$$
$$+\frac{1}{6\sqrt{2}}[\varrho^{-10}\{|b_{13}|^2-(b_2b_1^*+b_4b_3^*)\}-\varrho^{01}\{|b_{13}|^2+(b_1b_2^*+b_3b_4^*)\}], \tag{20}$$

$$T_0^2 = \frac{1}{\sqrt{6}}(-2+3\varrho^{00}|b|^2+\varrho^{11}(|b_{13}|^2+|b_{24}|^2)). \tag{21}$$

The normalization condition is

$$1 = (\varrho^{00}+\varrho^{11})|b^2|+\frac{1}{3}\varrho^{11}(|b_{13}|^2+|b_{24}|^2)+\frac{1}{6}\varrho^{00}|b_{13}|^2+2\,\mathrm{Re}\,[\varrho^{1-1}(b_5b_7^*+b_8b_6^*)] \tag{22}$$

and

$$|b_{ij}|^2 = |b_i|^2+|b_j|^2,$$
$$|b|^2 = |b_5|^2+|b_6|^2+|b_7|^2+|b_8|^2. \tag{23}$$

Counting the parameters one finds that the formulae (19)–(22) imply four relations between statistical tensors. Putting $\alpha = 0$ we loose two measurable parameters and get rid of two free parameters. Thus from the point of view of testing the quark model nothing is lost by putting $\alpha = 0$. With $\alpha = 0$ we find two linear relations

$$\mathrm{Re}\, T_2^2 = \frac{\sqrt{6}}{2} T_0^2 \quad \text{for} \quad \beta = 0, \tag{24}$$

$$\gamma^{-1}\, \mathrm{Re}\, T_1^2 = \frac{1}{2} \mathrm{Re}\, T_2^2 + \frac{3}{8}\sqrt{\frac{3}{2}}\, T_0^2 + \frac{1}{16} \quad \text{for} \quad \beta = -1. \tag{25}$$

Relation (24) was already given by Gorczyca and Hayashi [4]. Relation (25) can be tested in experiments with photons polarized at an angle of 45° with respect to the reaction plane. When $\gamma = 0$, the left hand side in (25) is a zero over zero symbol and the relation cannot be tested.

In order to test the full set of four relations a more complicated analysis is necessary. Eliminating the normalization condition we can represent each of the statistical tensors (19)–(21) as a ratio of two functions, each linear in α, β and γ. *E.g.*

$$T_2^2 = \frac{A+B\beta}{C+D\beta} \tag{26}$$

where $C+D\beta = 1$ is the normalization condition. Here A, B, C, D are known functions of the a_i, but do not depend on the polarization state of the photon. A and B are complex, while C and D are real. Therefore measuring the complex number T_2^2 for various values of β, we can determine five real parameters, *e.g.* Re A/D, Im A/D, Re B/D, Im B/D and C/D. Similarly T_1^2 (for $\alpha = 0$) and T_0^2 yield two real parameters each. These nine parameters should be fitted with a suitable choice of the five real free parameters occuring in the right hand sides of Eqs (19)–(22). A similar analysis was reported in Refs [5] and [6].

The relations derived in this section are not covariant with respect to rotations of the spin reference frame of the vector meson. This may be checked, for instance, by applying the transformation rule

$$R(\varphi) T_M^J = e^{iM\varphi} T_M^J, \tag{27}$$

where $R(\varphi)$ denotes a rotation by an angle φ around the z axis, to relation (24). Our results are valid by definition in the additivity frame corresponding to the transversity frame used for the initial photon. Since it is not known which is the additivity frame, there is in practice one more parameter in the theory. We may use an arbitrary spin reference frame, *e.g.* the transversity frame or the Jackson transversity frame, but we have to rotate the tensors (19) and (21) from the additivity frame into the chosen frame according to (27). By direct inspection of the formulae (19)–(22) it is found that (27) is equivalent to an irrelevant redefinition of the cofficients a_i supplemented by the transformation

$$\varrho^{mn} \to e^{i(m-n)\varphi} \varrho^{mn}. \tag{28}$$

Thus (28) may be used instead of (27). Here φ is a free parameter, which, like the a_i, depends in general on s and t. This reduces the number of predicted relations to three.

4. *The reaction $\gamma B \to VB^*$*

For the reaction $\gamma B \to VB^*$ the general expression for the amplitude reads

$$T = K\left[-\frac{1}{\sqrt{6}}(c_3\Sigma^0 + c_4E^0\Sigma^0) + \sqrt{\frac{2}{3}}(c_5E^-\Sigma^+ + c_6E^+\Sigma^- + c_7E^+\Sigma^+ + c_8E^-\Sigma^-)\right], \tag{29}$$

where $\vec{E}$ and $\vec{\Sigma}$ are defined by the formulae (18) and (8) and the coefficients c_i and K are discussed in the text under relation (8). It is remarkable that this formula is simpler than the corresponding formula (16) referring to the reaction $\gamma B \to VB$.

The formulae for the measurable statistical tensors are

$$T^{22}_{22} = \frac{1}{\sqrt{6}}\varrho^{00}c_8c_7^*, \tag{30}$$

$$T^{22}_{20} = \frac{1}{6}\varrho^{00}(c_5c_7^* + c_8c_6^*) - \frac{1}{12}\varrho^{-11}(c_3 - c_4)(c_3^* + c_4^*), \tag{31}$$

$$T^{22}_{2-2} = \frac{1}{\sqrt{6}}\varrho^{00}c_5c_6^*, \tag{32}$$

$$T^{22}_{11} = \frac{1}{4\sqrt{6}}[-\varrho^{00}(c_3c_7^* + c_8c_3^*) + (c_3^* + c_4^*)(\varrho^{11}c_8 + \varrho^{-11}c_6) - (c_3 - c_4)(\varrho^{11}c_7^* + \varrho^{-11}c_5^*)], \tag{33}$$

$$T^{22}_{1-1} = \frac{-1}{4\sqrt{6}}[\varrho^{00}(c_3c_6^* + c_5c_3^*) + (c_4 - c_3)(\varrho^{11}c_6^* + \varrho^{-11}c_8^*) - (c_3^* + c_4^*)(\varrho^{11}c_5 + \varrho^{-11}c_7)], \tag{34}$$

$$T^{22}_{00} = \frac{1}{6\sqrt{6}}[(\varrho^{00} - 2\varrho^{11})|c|^2 + (\varrho^{00} - \varrho^{11})|c_3|^2 - \varrho^{11}|c_4|^2 - 4\,\mathrm{Re}\,(\varrho^{1-1}(c_8c_6^* + c_5c_7^*))], \tag{35}$$

$$T^{22}_{02} = \frac{1}{6}[(2\varrho^{11} - \varrho^{00})(c_6c_7^* + c_8c_5^*) + 2(\varrho^{-11}c_6c_5^* + \varrho^{1-1}c_8c_7^*)], \tag{36}$$

$$T^2_2(V) = \frac{2}{3}\left[\varrho^{00}(c_5c_7^* + c_8c_6^*) + \frac{1}{4}\varrho^{-11}(c_3 - c_4)(c_3^* + c_4^*)\right], \tag{37}$$

$$T^2_0(V) = \frac{2}{3\sqrt{6}}\left[(\varrho^{00} - 2\varrho^{11})|c|^2 - \frac{1}{2}(\varrho^{00} - \varrho^{11})|c_3|^2 + \right.$$

$$\left. + \frac{1}{2}\varrho^{11}|c_4|^2 - 4\,\mathrm{Re}\,(\varrho^{1-1}(c_5c_7^* + c_8c_7^*))\right], \tag{38}$$

$$T^2_2(B^*) = \frac{1}{\sqrt{6}}[(\varrho^{00} + \varrho^{11})(c_6c_7^* + c_8c_5^*) + \varrho^{-11}c_6c_5^* + \varrho^{1-1}c_8c_7^*], \tag{39}$$

$$T^2_0(B^*) = \frac{1}{6}[(\varrho^{00} + \varrho^{11})|c|^2 - \frac{1}{2}\varrho^{00}|c_3|^2 - \varrho^{11}|c_{34}|^2 + 2\,\mathrm{Re}\,(\varrho^{1-1}(c_8c_6^* + c_5c_7^*))], \tag{40}$$

$$T^{22}_{21} = \frac{1}{4\sqrt{3}} [-\varrho^{-10}(c_3-c_4)c_7^* + \varrho^{01}c_8(c_3^*+c_4^*)], \tag{41}$$

$$T^{22}_{2-1} = \frac{1}{4\sqrt{3}} [\varrho^{-10}(c_3-c_4)c_6^* - \varrho^{01}c_5(c_3^*+c_4^*)], \tag{42}$$

$$\begin{aligned} T^{22}_{10} = \frac{1}{6\sqrt{2}} \Big[\varrho^{01} \left(|c_{58}|^2 + \frac{1}{2} c_3 (c_3^*+c_4^*) \right) + \\ + (\varrho^{0-1} - \varrho^{10})(c_5c_7^* + c_8c_6^*) - \varrho^{-10} \left(|c_{67}|^2 + \frac{1}{2} (c_3-c_4)c_3^* \right) \Big], \end{aligned} \tag{43}$$

$$T^{22}_{12} = \frac{1}{2\sqrt{3}} [c_8(\varrho^{01}c_5^* + \varrho^{1-1}c_7^*) - c_7^*(\varrho^{-10}c_6 + \varrho^{10}c_8)], \tag{44}$$

$$\begin{aligned} T^{22}_{01} = \frac{1}{6\sqrt{2}} \Big[\varrho^{10} \left(c_8c_5^* + \frac{1}{2} c_7^*(c_3+c_4) \right) - \varrho^{01} \left(c_3c_5^* + \frac{1}{2} c_6 (c_3^*+c_4^*) \right) + \\ + \varrho^{-10} \left(c_6c_3^* + \frac{1}{2} c_5^* (c_3-c_4) \right) - \varrho^{0-1} \left(c_3c_7^* + \frac{1}{2} c_8 (c_3^*-c_4^*) \right) \Big], \end{aligned} \tag{45}$$

$$T^{22}_{1-2} = \frac{1}{2\sqrt{3}} [c_5(\varrho^{0-1}c_6^* + \varrho^{01}c_8^*) - c_6^*(\varrho^{-10}c_7 - \varrho^{10}c_5)], \tag{46}$$

$$\begin{aligned} T^2_1(V) = \frac{1}{3\sqrt{2}} \Big[\varrho^{01} \left(2|c_{58}|^2 - \frac{1}{2} c_3 (c_3^*+c_4^*) \right) + \\ + 2(\varrho^{0-1} - \varrho^{10})(c_5c_7^* + c_8c_6^*) + \varrho^{-10} \left(-2|c_{67}|^2 + \frac{1}{2} c_3^*(c_3-c_4) \right) \Big], \end{aligned} \tag{47}$$

$$\begin{aligned} T^2_1(B^*) = \frac{1}{4\sqrt{3}} [\varrho^{10}((c_3+c_4)c_7^* - c_8c_3^*) + \varrho^{01}(c_3c_5^* - c_6(c_3^*+c_4^*)) + \\ + \varrho^{-10}(c_5^*(c_3-c_4) - c_6c_3^*) - \varrho^{0-1}(c_8(c_3^*-c_4^*) + c_3c_7^*)] \end{aligned} \tag{48}$$

with the normalization fixed by

$$1 = \frac{2}{3}(\varrho^{00} + \varrho^{11})|c|^2 + \frac{1}{6}\varrho^{00}|c_3|^2 + \frac{1}{3}\varrho^{11}|c_{34}|^2 + \frac{4}{3}\,\mathrm{Re}\,[\varrho^{1-1}(c_5c_7^* + c_8c_6^*)]. \tag{49}$$

Relations (30)–(49) involve 71 measurable quantities and, after the elimination of the overall phase and the overall normalization, ten parameters. Thus the formulae yield 61 predictions. Some of them are simple and linear [3], [4], a complete test, however, must be done as described in the preceding section: by using the formulae as they stand. The uncertainty in the choice of the spin reference frame introduces one more parameter. It may be included by modifying the spin density matrix of the initial photon according to (28). It can be checked that this is equivalent to the transformation of the statistical tensors according to the usual rules followed by a redefinition of the parameters a_i.

5. *Conclusions*

We considered the decay distributions of the resonances produced in the processes $\gamma B \to PB^*$, $\gamma B \to VB$ and $\gamma B \to VB^*$. For each reaction all the information about the decay distribution is contained in the statistical tensors. The statistical tensors depend on the initial spin states of the photon and of the baryon. For unpolarized initial baryons we found that the numbers of independent measurable parameters for the three reactions are 11, 11 and 71, respectively. These numbers exceed the numbers of parameters necessary to describe the corresponding processes in the framework of the quark model. Consequently the quark model predicts relations between the decay parameters.

For the process $\gamma B \to PB^*$ there are three relations given by the formulae (14) and (15). These formulae are valid whatever is the orientation of the additivity frames.

For the process $\gamma B \to VB$ there are also three relations. There would be one more if the orientation of the additivity frame for the vector meson were known. The relations may be tested by substituting (28) into the relations (19)–(22) and fitting simultaneously all the measurable quantities by one choice of the parameters a_i and φ. In principle the fit should be done for each energy and momentum transfer (s and t) separately, but experience with hadronic processes [5] suggests that for the parameters averaged over t a fit should also be possible.

For the process $\gamma B \to VB^*$ there are 60 predictions. The test is similar to that in the previous case.

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