QUARK MODEL PREDICTIONS FOR THE MEASURABLE GENERALIZED STATISTICAL TENSORS

By P. Kielanowski

Institute of Theoretical Physics, Warsaw University, Warsaw*

(*Received February 26, 1970*)

A set of linear relations for the measurable generalized statistical tensors completely equivalent to the quark model predictions for the transition amplitudes for the reaction $0^-+\frac{1}{2}^+\rightarrow 1^-+\frac{3}{2}^+$ is given. The given relations can be tested directly by means of experimental data for joint decay distributions of the final particles.

1. *Introduction*

Let us consider the reaction:

$$0^-+\frac{1}{2}^+\rightarrow 1^-+\frac{3}{2}^+, \tag{1}$$

where $\frac{1}{2}^+$, $\frac{3}{2}^+$, 0^- and 1^- denote baryons from the $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet, mesons from 0^- and 1^- octets, respectively.

In the following we shall find the relations between the measurable generalized statistical tensors (MGST) [1] for the reaction (1) which are completely equivalent to the relations between the amplitudes predicted by the quark model. A positive experimental check of these relations between MGST will mean that the relations between the amplitudes are satisfied and any theoretical model must comply with them. However, it will not necessarily mean that the quark model is verified because only a small number of the quark model assumptions is used [2]. It will only mean that the quark model is compatible with experiment.

Some of our quark model predictions for MGST have been already found and tested by Białas, Kotański and Zalewski [3]; however, it is not clear from their method whether their relations give a complete check of all quark model relations for the amplitudes.

Our relations, which are all linear, can be easily tested. Moreover, this test will give a full answer whether the relations between the amplitudes are exactly such as predicted by the quark model. It will also indicate the spin quantization frame in which the additivity assumption holds.

* Address: Instytut Fizyki Teoretycznej UW, Warszawa, Hoża 69, Polska.

2. *The basic results*

It is well known [2] that the quark model equipped with the additivity assumption gives some relations among the amplitudes of the reaction (1).

For simplicity we denote:

$$\begin{aligned} &f_1 = f_{1\frac{3}{2}0\frac{1}{2}}, \quad f_5 = f_{1\frac{1}{2}0-\frac{1}{2}}, \quad f_9 = f_{0-\frac{1}{2}0-\frac{1}{2}}, \\ &f_2 = f_{-1\frac{3}{2}0\frac{1}{2}}, \quad f_6 = f_{-1\frac{1}{2}0-\frac{1}{2}}, \quad f_{10} = f_{0-\frac{3}{2}0\frac{1}{2}}, \\ &f_3 = f_{0\frac{3}{2}0-\frac{1}{2}}, \quad f_7 = f_{1-\frac{1}{2}0\frac{1}{2}}, \quad f_{11} = f_{1-\frac{3}{2}0-\frac{1}{2}}, \\ &f_4 = f_{0\frac{1}{2}0\frac{1}{2}}, \quad f_8 = f_{-1-\frac{1}{2}0\frac{1}{2}}, \quad f_{12} = f_{-1-\frac{3}{2}0-\frac{1}{2}}. \end{aligned} \tag{2}$$

All other transversity amplitudes vanish by virtue of parity conservation.

The quark model predictions for the amplitudes of the reaction (1) read [2]:

class-a relations

$$\begin{aligned} &f_1 = \sqrt{3}\, e^{-i\psi} f_5, \quad f_4 = e^{-i\psi} f_9, \quad f_{11} = \sqrt{3}\, e^{i\varphi} f_7, \\ &f_2 = \sqrt{3}\, e^{-i\psi} f_6, \qquad\qquad f_{12} = \sqrt{3}\, e^{i\psi} f_8, \\ &\qquad f_3 = 0, \ f_{10} = 0, \end{aligned} \tag{3}$$

class-b relations

$$f_2 = e^{2i\varphi - 3i\psi} f_{11}, \tag{4}$$

$$f_6 = e^{2i\varphi - i\psi} f_7, \tag{5}$$

class-c relations

$$f_1 = e^{-2i\varphi - 3i\psi - 4i\alpha} f_{12}, \tag{6}$$

$$f_5 = e^{-2i\varphi - i\psi - 4i\alpha} f_8. \tag{7}$$

The phenomenological parameters α, φ and ψ are interpreted as the angles between the conventional transversity frames for the particles 1, 3 and 4 and the transversity frames in which the additivity assumption is supposed to hold. An extensive discussion of the relations (3)–(7) and an interpretation of the angles α, φ and ψ is given in [2]. Let us only notice that the relations (4) and (5), (6) and (7) are equivalent if the relations (3) hold, so in the following we shall give tests for the relations (3), (5) and (7) only.

The final particles in the reaction (1) are all unstable. The angular distribution of momenta of their decay products allows measurement of the following MGST:

$$\begin{aligned} &A^{000}_{000}, A^{002}_{000}, A^{020}_{000}, A^{022}_{000}, A^{100}_{000}, A^{102}_{000}, A^{120}_{000}, A^{122}_{000}, \\ &A^{002}_{002}, A^{022}_{002}, A^{102}_{002}, A^{122}_{002}, A^{020}_{020}, A^{022}_{020}, A^{120}_{020}, A^{122}_{020}, \\ &A^{022}_{022}, A^{122}_{101}, A^{022}_{02-2}, A^{122}_{02-2}, A^{022}_{011}, A^{122}_{011}, A^{022}_{01-1}, A^{022}_{01-1}, \\ &A^{122}_{121}, A^{122}_{112}, A^{122}_{12-1}, A^{122}_{1-12}, A^{122}_{1-21}, A^{122}_{11-2}, A^{122}_{1-2-1}, A^{122}_{1-1-2}. \end{aligned} \tag{8}$$

The MGST (8), as functions of amplitudes, are given in Appendix A.

It is clear that the Eqs. (3)–(7) give many relations between the MGST (8). We do not intend to present all of them, but only choose those which are sufficient for an experimental check of the relations (3)–(7). We now write down some groups of relations for the amplitudes separately and present the corresponding equivalent relations for the MGST.

I $f_3 = 0, f_{10} = 0$

$$A^{000}_{000}+A^{002}_{000}-\sqrt{2}\,A^{020}_{000}-\sqrt{2}\,A^{022}_{000} = 0. \tag{9}$$

II $f_3 = 0$

$$A^{000}_{000}+A^{002}_{000}-\sqrt{2}\,A^{020}_{000}-\sqrt{2}\,A^{022}_{000}- \tag{10}$$

$$-A^{100}_{000}-A^{102}_{000}+\sqrt{2}\,A^{120}_{000}+\sqrt{2}\,A^{122}_{000} = 0.$$

III $f_{10} = 0$

$$A^{000}_{000}+A^{002}_{000}-\sqrt{2}\,A^{020}_{000}-\sqrt{2}\,A^{022}_{000}+ \tag{11}$$

$$+A^{100}_{000}+A^{102}_{000}-\sqrt{2}\,A^{120}_{000}-\sqrt{2}\,A^{122}_{000} = 0.$$

IV $f_3 = e^{-i\varphi} f_9$

$$A^{100}_{000}-A^{102}_{000}-\sqrt{2}\,A^{120}_{000}+\sqrt{2}\,A^{122}_{000} = 0. \tag{12}$$

and

$$e^{-i\varphi}\sqrt{2}\,A^{122}_{011} = A^{122}_{112}, \tag{13}$$

or

$$e^{-i\varphi}\sqrt{2}\,A^{122}_{0-11} = A^{122}_{1-12}. \tag{14}$$

The relations (13) and (14) can be used for the determination of the angle φ only in the case when the relation (9) is fulfilled. The relation (13) or (14) cannot be used if either A^{122}_{112} or A^{122}_{1-12} vanishes.

V

$$f_1 = \sqrt{3}\,e^{-i\varphi} f_5 \qquad f_2 = \sqrt{3}\,e^{-i\varphi} f_6$$

$$f_{11} = \sqrt{3}\,e^{i\varphi} f_7 \qquad f_{12} = \sqrt{3}\,e^{i\varphi} f_8$$

$$-2A^{002}_{000}+A^{020}_{000}-A^{100}_{000}+A^{102}_{000}-\sqrt{2}\,A^{120}_{000} = 0, \tag{15}$$

$$-2A^{002}_{000}+A^{020}_{000}-A^{100}_{000}-A^{102}_{000}+\sqrt{2}\,A^{120}_{000} = 0, \tag{16}$$

$$A^{020}_{020}-2A^{022}_{020}-2A^{120}_{020}+A^{122}_{020} = 0, \tag{17}$$

$$A^{020}_{020}-2A^{022}_{020}+2A^{120}_{020}-A^{122}_{020} = 0 \tag{18}$$

and

$$A^{100}_{000}+A^{102}_{000} = 3\sqrt{3}\,e^{i\varphi}\,A^{102}_{101}. \tag{19}$$

Equations (15)–(19) are equivalent to relations **V** if the additional conditions hold:

$$A^{020}_{020}\pm A^{022}_{020}\pm A^{120}_{020}+A^{122}_{020} \neq 0 \tag{20}$$

and

$$A^{102}_{101} \neq 0. \tag{21}$$

The angle ψ evaluated from Eq. (19) must be also equal to the angle ψ evaluated from Eq. (13) or (14).

VI $f_6 = e^{2i\varphi - i\psi} f_7$

$$A^{022}_{0-11} + A^{122}_{0-11} = e^{2i(\varphi-\psi)}(A^{022}_{01-1} - A^{122}_{01-1}). \tag{22}$$

VII $f_5 = e^{-2i\varphi - i\psi - 4i\alpha} f_8$

$$A^{022}_{011} + A^{122}_{011} = e^{-2i(\varphi+\psi+2\alpha)}(A^{022}_{0-1-1} - A^{122}_{0-1-1}). \tag{23}$$

The proof that the relations (15)–(19) between MGST are equivalent to the given relations for the amplitudes is given in Appendix B. The proofs for other relations are very simple.

3. *Discussion*

It should be stressed that if the relations (9)–(23) are experimentally confirmed, then the relations (3)–(7) for the amplitudes must be true and an experimental check of any other relations among the MGST does not introduce anything essentially new. It should be also pointed out that for an experimental check of our relations no measurements of polarization of decay products are required.

The author is very thankful to Professor J. Werle and to M. Kupczyński for stimulating discussions and reading the manuscript.

APPENDIX A

The measurable generalized statistical tensors as the functions of the reaction amplitudes:

$M = 000$

L	$\lvert f_1\rvert^2$	$\lvert f_2\rvert^2$	$\lvert f_3\rvert^2$	$\lvert f_4\rvert^2$	$\lvert f_5\rvert^2$	$\lvert f_6\rvert^2$	$\lvert f_7\rvert^2$	$\lvert f_8\rvert^2$	$\lvert f_9\rvert^2$	$\lvert f_{10}\rvert^2$	$\lvert f_{11}\rvert^2$	$\lvert f_{12}\rvert^2$	Factor
000	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	$\times \frac{1}{2\sqrt{6}}$
002	+1	+1	+1	−1	−1	−1	−1	−1	−1	+1	+1	+1	$\times \frac{1}{2\sqrt{6}}$
020	+1	+1	−2	−2	+1	+1	+1	+1	−2	−2	+1	+1	$\times \frac{1}{2\sqrt{12}}$
022	+1	+1	−2	+2	−1	−1	−1	−1	+2	−2	+1	+1	$\times \frac{1}{2\sqrt{12}}$
100	+1	+1	−1	+1	−1	−1	+1	+1	−1	+1	−1	−1	$\times \frac{1}{2\sqrt{6}}$
102	+1	+1	−1	−1	+1	+1	−1	−1	+1	+1	−1	−1	$\times \frac{1}{2\sqrt{6}}$
120	+1	+1	+2	−2	−1	−1	+1	+1	+2	−2	−1	−1	$\times \frac{1}{2\sqrt{12}}$
122	+1	+1	+2	+2	+1	+1	−1	−1	−2	−2	−1	−1	$\times \frac{1}{2\sqrt{12}}$

$M = 002$

L	$f_1 f_7^*$	$f_2 f_8^*$	$f_3 f_9^*$	$f_4 f_{10}$	$f_5 f_{11}^*$	$f_6 f_{12}^*$	Factor
002	+1	+1	+1	+1	+1	+1	$\times \frac{1}{2\sqrt{3}}$
022	+1	+1	−2	−2	+1	+1	$\times \frac{1}{2\sqrt{6}}$
102	+1	+1	−1	+1	−1	−1	$\times \frac{1}{2\sqrt{3}}$
122	+1	+1	+2	−2	−1	−1	$\times \frac{1}{2\sqrt{6}}$

$M = 02-2$

L	$f_7 f_2^*$	$f_{11} f_6^*$	Factor
022	+1	+1	$\times \frac{1}{2}$
122	+1	−1	$\times \frac{1}{2}$

$M = 020$

L	$f_1 f_2^*$	$f_5 f_6^*$	$f_7 f_8^*$	$f_{11} f_{12}^*$	Factor
020	+1	+1	+1	+1	$\times \frac{1}{2\sqrt{2}}$
022	+1	−1	−1	+1	$\times \frac{1}{2\sqrt{2}}$
120	+1	−1	+1	−1	$\times \frac{1}{2\sqrt{2}}$
122	+1	+1	−1	−1	$\times \frac{1}{2\sqrt{2}}$

$M = 022$

L	$f_1 f_8^*$	$f_5 f_{12}^*$	Factor
022	+1	−1	$\times \frac{1}{2}$
122	+1	+1	$\times \frac{1}{2}$

$M = 011$

L	$f_1 f_4^*$	$f_3 f_6^*$	$f_7 f_{10}^*$	$f_9 f_{12}^*$	Factor
022	+1	−1	−1	+1	$\times \frac{1}{2\sqrt{2}}$
122	+1	−1	−1	−1	$\times \frac{1}{2\sqrt{2}}$

$M = 01-1$

L	$f_4 f_2^*$	$f_5 f_3^*$	$f_{10} f_8^*$	$f_{11} f_9^*$	Factor
022	+1	−1	−1	+1	$\times \frac{1}{2\sqrt{2}}$
122	+1	+1	−1	−1	$\times \frac{1}{2\sqrt{2}}$

$M = 101$

L	$f_1 f_5^*$	$f_2 f_6^*$	$f_7 f_{11}^*$	$f_8 f_{12}^*$	Factor
102	+1	+1	−1	−1	$\times \frac{1}{\sqrt{6}}$
122	+1	+1	−1	−1	$\times \frac{1}{2\sqrt{3}}$

$M = 110$

L	$f_1 f_3^*$	$f_4 f_6^*$	$f_7 f_9^*$	$f_{10} f_{12}^*$	Factor
120	+1	−1	+1	−1	$\times \frac{1}{2\sqrt{2}}$
122	+1	+1	−1	−1	$\times \frac{1}{2\sqrt{2}}$

$M = 1-10$

L	$f_2 f_3^*$	$f_4 f_5^*$	$f_8 f_9^*$	$f_{10} f_{11}^*$	Factor
120	+1	−1	+1	−1	$\times \frac{1}{2\sqrt{2}}$
122	+1	+1	−1	−1	$\times \frac{1}{2\sqrt{2}}$

$M = 10-1$

L	$f_4 f_3^*$	$f_{10} f_9^*$	Factor
102	−1	+1	$\times \frac{1}{\sqrt{6}}$
122	+1	−1	$\times \frac{1}{\sqrt{3}}$

$$A^{122}_{121} = \frac{1}{\sqrt{2}}(f_1 f_6^* - f_7 f_{12}^*) \qquad A^{122}_{1-21} = \frac{1}{\sqrt{2}}(f_2 f_5^* - f_8 f_{11}^*)$$

$$A^{122}_{112} = \tfrac{1}{2}(f_1 f_9^* - f_4 f_{12}^*) \qquad A^{122}_{1-22} = \tfrac{1}{2}(f_2 f_9^* - f_4 f_{11}^*)$$

$$A^{122}_{11-2} = \tfrac{1}{2}(f_7 f_3^* - f_{10} f_6^*) \qquad A^{122}_{1-1-2} = \tfrac{1}{2}(f_8 f_3^* - f_{10} f_5^*)$$

$$A^{122}_{12-1} = 0 \qquad A^{122}_{1-2-1} = 0$$

APPENDIX B

To prove that the relations (15)–(19) yield the relations:

$$f_1 = \sqrt{3}\, e^{-i\varphi} f_5, \quad f_2 = \sqrt{3}\, e^{-i\varphi} f_6, \qquad \text{(B1)}$$
$$f_{11} = \sqrt{3}\, e^{i\varphi} f_7, \quad f_{12} = \sqrt{3}\, e^{i\varphi} f_8$$

we write the relations (15)–(19) in terms of the amplitudes:

$$|f_1|^2 + |f_2|^2 = 3(|f_5|^2 + |f_6|^2), \qquad \text{(B2)}$$

$$|f_{11}|^2 + |f_{12}|^2 = 3(|f_7|^2 + |f_8|^2), \qquad \text{(B3)}$$

$$f_1 f_2^* = 3 f_5 f_6^*, \qquad \text{(B4)}$$

$$f_{11} f_{12}^* = 3 f_7 f_8^* \qquad \text{(B5)}$$

and

$$f_1 f_5^* + f_2 f_6^* - f_7 f_{11}^* - f_8 f_{12}^* = \frac{1}{\sqrt{3}} e^{-i\varphi}(|f_1|^2 + |f_2|^2) - \frac{1}{\sqrt{3}} e^{-i\varphi}(|f_7|^2 + |f_8|^2). \qquad \text{(B6)}$$

If $f_1 f_2^*$ does not vanish, then multiplying Eq. (B4) by $e^{i\gamma}$ and the complex conjugate of Eq. (B4) by $e^{-i\gamma}$, and adding the products to (B2) we obtain the following equality equivalent to Eqs (B2) and (B4), which has to hold for an arbitrary γ:

$$|f_1 + e^{i\gamma} f_2|^2 = 3|f_5 + e^{i\gamma} f_6|^2, \qquad \text{(B7)}$$

whereupon:

$$f_1 = \sqrt{3}\, e^{-i\beta} f_5, \qquad \text{(B8)}$$
$$f_2 = \sqrt{3}\, e^{-i\beta} f_6.$$

If $f_{11} f_{12}^*$ does not vanish, the relations (B3) and (B5) yield

$$f_{11} = \sqrt{3}\, e^{-i\nu} f_7, \qquad f_{12} = \sqrt{3}\, e^{-i\nu} f_8. \tag{B9}$$

where β and ν are real parameters.

Finally, if $|f_1|^2+|f_2|^2 \neq |f_7|^2+|f_8|^2$ the relation (B6) gives:

$$\beta = \nu = \psi \tag{B10}$$

and thus the relations (15)–(19) yield the relations (B1) for the amplitudes and also allow to determine the phenomenological angle ψ.

REFERENCES

[1] P. Kielanowski, M. Kupczyński, Warsaw University Preprint IFT 2/70.

[2] M. Kupczyński, *Acta Phys. Polon.* **B1**, 301 (1970).

[3] A. Białas, K. Zalewski, *Nuclear Phys.*, **B6**, 465 (1968); A. Kotański, K. Zalewski, *Nuclear Phys.*, **B13**, 119 (1969); A. Kotański, K. Zalewski, *Phys. Letters*, **29B**, 596 (1969); A, Kotański, K. Zalewski, *Nuclear Phys.*, **B15**, 242 (1970).