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BINDING ENERGY OF A Λ PARTICLE IN NUCLEAR MATTER WITH NONCENTRAL ΛN FORCES

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The binding energy of a Λ particle in nuclear matter, $B_\Lambda(\infty)$, is calculated in the frame of the self-consistent Brueckner $\mathcal{K}$ matrix theory. In the $\mathcal{K}$ matrix equations pure kinetic energies in the intermediate states are used. The expression for $B_\Lambda(\infty)$, which includes the rearrangement energy, is derived in the case of a ΛN interaction which is spin dependent, has a tensor and spin-orbit part and contains a hard core. Calculations have been performed with the modified Herndon-Tang potentials and with a one-boson-exchange potential. The tensor suppression effect is estimated to be about 2 MeV, which includes important contributions of the ^3P ΛN interaction. The role of the self-consistency requirement in the tensor suppression effect is discussed.

1. *Introduction*

In our recent paper [1] (hereafter referred to as I) we have presented, what we believe, a very accurate calculation of the binding energy of a Λ particle in nuclear matter, $B_\Lambda(\infty)$. In I we have used several pure central Λ-nucleon potentials, $v_{\Lambda N}$, adjusted to the known binding energies, B_Λ, of the light hypernuclei and to the measured cross-section for the elastic Λ-proton scattering. For each of the potentials $v_{\Lambda N}$ considered, we have calculated $B_\Lambda(\infty)$ according to the Brueckner theory. Recently, Bodmer and Rote [2], [3] have performed an almost identical calculation, and their results agree very well with those of I.

Let us summarize briefly the results of I. The calculated values of $B_\Lambda(\infty)$, in general, turn out to be larger than the empirically estimated value,

$$B_\Lambda(\infty) = 30 \pm 5 \text{ MeV}. \tag{1.1}$$

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The best ΛN potential H of Herndon and Tang [4], adjusted to the binding energies of the three- and four-body hypernuclei and to the Λp scattering data, leads to the calculated value of $B_\Lambda(\infty) = 39.4$ MeV. On the other hand, the ΛN potential F' of Herndon and Tang, with the charge-independent part adjusted to $B_\Lambda(^3H_\Lambda)$ and $B_\Lambda(^5He_\Lambda)$ leads to $B_\Lambda(\infty) = 32.5$ MeV. in agreement with the empirical value (1.1). Thus it appears possible to reconcile the properties of the two systems, $^5He_\Lambda$ and Λ+nuclear matter which are similar in the sense that both of them are spin saturated. However, the F' potential produces a total Λp scattering cross-section which is smaller than the measured cross-section by about 20%. Thus the problem obviously remains how to reconcile the ΛN interaction in an isolated ΛN system (in particular the Λp scattering) with properties of systems like $^5He_\Lambda$ and Λ+nuclear matter.

One of the possibilieties of solving this problem seems to be the inclusion of a tensor component into the ΛN interaction. One could expect that similarly as in the NN case, a tensor force effective in an isolated ΛN system might be less effective, *i. e.*, suppressed in the Λ+nuclear matter system, and similarly in ^{5}He. Namely, in the first order the contribution of a tensor force to $B_\Lambda(\infty)$ vanishes, being spin-averaged out. In the second and higher orders, the part of the intermediate states, for which nucleon momenta are smaller than k_F, the Fermi momentum of nuclear matter (in units of $\hbar$), is excluded by the Pauli principle. In consequence, one might expect the tensor suppression effect. Let us mention, however, a suggestion of Bodmer and Rote [2] who point out that this effect might be much smaller than in the NN case because of the expected short range of the ΛN tensor forces whose longest range is determined predominantly by the exchange of the pseudoscalar K and η mesons. Such short range ΛN forces excite nucleons predominantly to intermediate states above the Fermi surface. Consequently, the suppressing effect of the Pauli principle is expected to be less important.

In the present paper we calculate $B_\Lambda(\infty)$ with ΛN potentials containing a tensor component. In Section 2 the scheme of calculating $B_\Lambda(\infty)$ is presented. In the case of a pure central ΛN potential, the scheme — based on the Brueckner theory — has been explained in I. Thus, in Section 2, we emphasize the modifications of the scheme connected with the presence of the tensor component in $v_{\Lambda N}$. Some of the ΛN potentials applied in the present paper contain also a spin-orbit term. Consequently, the equations of Section 2 are written for the case when $v_{\Lambda N}$ contains a spin dependent central part, a tensor part, and a spin-orbit part. All of the parts contain a hard core repulsion and may differ in even and odd angular momentum states. The forms of the ΛN potentials applied in our calculation are given in Section 3. Our results obtained for $B_\Lambda(\infty)$ are presented and discussed in Section 4.

Some of the results of this paper have been presented in [5] and [6].

2. *The Scheme of calculating $B_\Lambda(\infty)$*

We assume that the ΛN potential has the following form:

$$v_{\Lambda N} = \{^3v_c^+(r) + v_t^+(r)\, S_{\Lambda N} + v_{LS}^+(r)\, \boldsymbol{LS}\}^3\Pi^+ + \{^3v_c^-(r) + $$

$$+ v_t^-(r)\, S_{\Lambda N} + v_{LS}^-(r)\, \boldsymbol{LS}\}^3\Pi^- + {}^1v_c^+(r)^1\Pi^+ + {}^1v_c^-(r)^1\Pi^-, \tag{2.1}$$

where the Π's are the proper projection operators, *e. g.*, ${}^3\Pi^-$ and ${}^1\Pi^+$ are respectively the projection operators onto the triplet odd and singlet even states of the relative ΛN motion. The tensor operator $S_{\Lambda N}$ is defined as usually:

$$S_{\Lambda N} = 3[(\boldsymbol{\sigma}_N \boldsymbol{r})(\boldsymbol{\sigma}_\Lambda \boldsymbol{r})/r^2] - \boldsymbol{\sigma}_\Lambda \boldsymbol{\sigma}_N, \tag{2.2}$$

where $\boldsymbol{\sigma}_N, \boldsymbol{\sigma}_\Lambda$ are the Pauli spin operators of the nucleon and the Λ particle respectively, and where $\boldsymbol{r}$ is the vector of the relative ΛN position. The relative ΛN angular momentum operator (in units of $\hbar$) is denoted by $\boldsymbol{L}$, and

$$\boldsymbol{S} = \tfrac{1}{2}(\boldsymbol{\sigma}_N + \boldsymbol{\sigma}_\Lambda) \tag{2.3}$$

is the total spin of Λ and N. We shall assume that all the radial functions $v(r)$ contain a hard core of radius r_c.

Our ΛN potential, $v_{\Lambda N}$, is assumed to be charge symmetric. Nuclear matter is assumed to have in each occupied momentum state four nucleons: two protons and two neutrons with spin up and down. In this system possible charge symmetry breaking components of $v_{\Lambda N}$ should have a negligible effect on $B_\Lambda(\infty)$.

Let us introduce the following notation. By $\boldsymbol{m}_N$ and $\boldsymbol{m}_\Lambda$ we denote the nucleon and Λ momenta (in units of $\hbar$) of the occupied states, by $\boldsymbol{k}_N, \boldsymbol{k}_\Lambda$ the momenta of the excited states, and by $\boldsymbol{p}_N, \boldsymbol{p}_\Lambda$ general momenta without any restrictions. In the ground state of the Λ + nuclear matter system,

$$\boldsymbol{m}_\Lambda = 0. \tag{2.4}$$

By s_N, s_Λ we denote the z component of the nucleon and particle spin, and by t_N the third component of the nucleon isotopic spin. For the total set $(\boldsymbol{m}_N, s_N, t_N)$ we use the notation $\tilde{m}_N$, and similarly $\tilde{m}_\Lambda = (\boldsymbol{m}_\Lambda, s_\Lambda)$. The notation: $\tilde{k}_N, \tilde{p}_N, \tilde{k}_\Lambda, \tilde{p}_\Lambda$, has an analogical meaning. Further-more, we denote by μ the reduced mass of the ΛN system:

$$\mu = \mathcal{M}_N \mathcal{M}_\Lambda / (\mathcal{M}_N + \mathcal{M}_\Lambda), \tag{2.5}$$

where $\mathcal{M}_N$, $\mathcal{M}_\Lambda$ are the nucleon and Λ masses divided by $\hbar^2$.

The binding energy of a Λ particle in nuclear matter, $B_\Lambda(\infty)$, is defined by:

$$-B_\Lambda(\infty) = E(A+1_\Lambda) - E(A), \tag{2.6}$$

where $E(A)$ and $E(A+1_\Lambda)$ are the potential energies of the ground state of nuclear matter and of the system: nuclear matter + Λ particle. For $E(A+1_\Lambda)$ and $E(A)$ we apply the Brueckner theory expressions [7]:

$$E(A+1_\Lambda) = \tfrac{1}{2} \sum \{\tilde{m}_N \tilde{m}'_N\} (\tilde{m}_N \tilde{m}'_N | K(A+1_\Lambda) | \tilde{m}_N \tilde{m}'_N - \tilde{m}'_N \tilde{m}_N) + \\ + \sum \{\tilde{m}_N\} (\tilde{m}_N \tilde{m}_\Lambda | \mathcal{K} | \tilde{m}_N \tilde{m}_\Lambda), \tag{2.7}$$

$$E(A) = \tfrac{1}{2} \sum \{\tilde{m}_N \tilde{m}'_N\} (\tilde{m}_N \tilde{m}'_N | K(A) | \tilde{m}_N \tilde{m}'_N - \tilde{m}'_N \tilde{m}_N), \tag{2.8}$$

where $K(A+1_\Lambda)$ and $\mathscr{K}$ are the reaction matrices for NN and ΛN interaction in the Λ+nuclear·matter system, and $K(A)$ is the reaction matrix for NN interaction in a pure nuclear matter. Subtracting the two expressions, (2.7), (2.8), we get:

$$-B_\Lambda(\infty) = V_\Lambda + V_R, \tag{2.9}$$

where the single Λ particle potential is:

$$V_\Lambda = \sum \{\tilde{m}_N\} (\tilde{m}_N\tilde{m}_\Lambda|\mathscr{K}|\tilde{m}_N\tilde{m}_\Lambda) = \tfrac{1}{2}\sum \{\tilde{m}_N s_\Lambda\} (\tilde{m}_N\tilde{m}_\Lambda|\mathscr{K}|\tilde{m}_N\tilde{m}_\Lambda), \tag{2.10}$$

and the rearrangement potential:

$$V_R = \tfrac{1}{2}\sum \{\tilde{m}_N\tilde{m}'_N\} (\tilde{m}_N\tilde{m}'_N|K(A+1_\Lambda)-K(A)|\tilde{m}_N\tilde{m}'_N - \tilde{m}'_N\tilde{m}_N). \tag{2.11}$$

It has been shown in I how to calculate V_Λ for a central ΛN interaction. The way of calculating V_R has been presented in [8] in the case of a central spin independent Serber NN interaction. Here, we shall present the way of calculating V_Λ and V_R in the case when both the ΛN and NN interactions are of a quite general form, and in particular contain tensor components. Otherwise, the way of calculating V_R, presented here, is simpler than that presented in [8] where nucleons in the intermediate states were assumed to have nonvanishing potential energies.

A. Calculation of V_Λ

First let us write the equation for the ΛN reaction matrix $\mathscr{K}$:

$$\begin{aligned}(\tilde{p}_N\tilde{p}_\Lambda|\mathscr{K}|\tilde{m}_N\tilde{m}_\Lambda) &= (\tilde{p}_N\tilde{p}_\Lambda|v_{\Lambda N}|\tilde{m}_N\tilde{m}_\Lambda) + \\ &+ \sum \{\tilde{k}_N\tilde{k}_\Lambda\} (\tilde{p}_N\tilde{p}_\Lambda|v_{\Lambda N}|\tilde{k}_N\tilde{k}_\Lambda)(\tilde{k}_N\tilde{k}_\Lambda|\mathscr{K}|\tilde{m}_N\tilde{m}_\Lambda)\times \\ &\times 1/[e_N(m_N)+V_\Lambda-\varepsilon_N(k_N)-\varepsilon_\Lambda(k_\Lambda)],\end{aligned} \tag{2.12}$$

where ε_N, ε_Λ are the N and Λ kinetic energies, and $e_N(m_N)$ is the single nucleon energy of the occupied state, $\tilde{m}_N$, in pure nuclear matter. Strictly speaking, instead of $e_N(m_N)$ one should write in Eq. (2.12) the single nucleon energy, $\bar{e}_N(m_N)$, of the occupied state in the Λ+nuclear matter system. However, the difference $\bar{e}_N - e_N$ is of the order $1/A$ and its effect on V_Λ vanishes in the limit $A \to \infty$. As we shall see in Section 2. B, this difference is the source of the rearrangement energy. Namely, in V_R, Eq. (2.11), there is an extra sum over $\tilde{m}_N$ which introduces an additional factor A.

Notice the appearance of pure Λ and N kinetic energies in the intermediate states in Eq. (2.12), explained in detail in I. As far as the single nucleon energies, $e_N(m_N)$ are concerned, we treat them as known quantities. Similarly as in I we use the form,

$$e_N(m_N) = m_N^2/2\mathscr{M}_N^* + A_0 \tag{2.13}$$

with the two constants, $\mathscr{M}_N^*$, A_0, fixed by the two conditions:

$$\tfrac{1}{2}\langle e_N+\varepsilon_N\rangle_{Av} = -\varepsilon_{vol}, \tag{2.14}$$

$$e_N(k_F) = -\varepsilon_{vol}, \tag{2.15}$$

where the subscript Av indicates the average value in the Fermi sea, and $\varepsilon_{\mathrm{vol}}$ is the binding energy per nucleon of nuclear matter.

When we change the s_{N}, s_{Λ} representation into the s, m_s representation, where s is the total spin of Λ and N, and m_s is its z component, Eq. (2.10) takes the form:

$$V_{\Lambda} = \sum \{\boldsymbol{m}_{\mathrm{N}} s m_s\} (\boldsymbol{m}_{\mathrm{N}} \boldsymbol{m}_{\Lambda} s m_s | \mathscr{K} | \boldsymbol{m}_{\mathrm{N}} \boldsymbol{m}_{\Lambda} s m_s). \tag{2.16}$$

For a charge symmetric ΛN interaction the $\mathscr{K}$ matrix does not depend on t_{N}, and the sum over t_{N} cancels the factor 1/2 in Eq. (2.10).

Because of the conservation of the total momentum and of the total spin s, we have

$$(\boldsymbol{p}_{\mathrm{N}} \boldsymbol{p}_{\Lambda} s' m_s' | \mathscr{K} | \boldsymbol{m}_{\mathrm{N}} \boldsymbol{m}_{\Lambda} s m_s) = \delta_{ss'} \delta_{\boldsymbol{PM}} (\boldsymbol{p} s m_s' | \mathscr{K} | \boldsymbol{m} s m_s), \tag{2.17}$$

where the center of mass momenta ,$\boldsymbol{P}$, $\boldsymbol{M}$, and the relative mementa, $\boldsymbol{p}$, $\boldsymbol{m}$, are:

$$\boldsymbol{P} = \boldsymbol{p}_{\mathrm{N}} + \boldsymbol{p}_{\Lambda}, \quad \boldsymbol{M} = \boldsymbol{m}_{\mathrm{N}} + \boldsymbol{m}_{\Lambda} = \boldsymbol{m}_{\mathrm{N}}, \tag{2.18}$$

$$\boldsymbol{p} = (\mathscr{M}_{\Lambda} \boldsymbol{p}_{\mathrm{N}} - \mathscr{M}_{\mathrm{N}} \boldsymbol{p}_{\Lambda}) / (\mathscr{M}_{\mathrm{N}} + \mathscr{M}_{\Lambda}),$$

$$\boldsymbol{m} = (\mathscr{M}_{\Lambda} \boldsymbol{m}_{\mathrm{N}} - \mathscr{M}_{\mathrm{N}} \boldsymbol{m}_{\Lambda}) / (\mathscr{M}_{\mathrm{N}} + \mathscr{M}_{\Lambda}) = \mu \boldsymbol{m}_{\mathrm{N}} / \mathscr{M}_{\mathrm{N}}. \tag{2.19}$$

The $\mathscr{K}$ matrix element on the right hand side of Eq. (2.17) depends on the center of mass momentum $\boldsymbol{M}$. Since, however, $\boldsymbol{m}_{\Lambda} = 0$ the center of mass momentum $\boldsymbol{M}$ is determined by the relative momentum $\boldsymbol{m}$:

$$\boldsymbol{M} = \mathscr{M}_{\mathrm{N}} \boldsymbol{m} / \mu. \tag{2.20}$$

Consequently, in our notation we do not indicate the dependence of $\mathscr{K}$ on $\boldsymbol{M}$.

So far all the states have been normalized in the periodicity box of volume Ω, *e.g.* $(\boldsymbol{r}|\boldsymbol{m}) = \exp(i\boldsymbol{r}\boldsymbol{m})/\sqrt{\Omega}$. Now it is more convenient to use the normalization $\langle \boldsymbol{r}|\boldsymbol{m}\rangle = \exp(i\boldsymbol{r}\boldsymbol{m})$. With the new normalization we have:

$$(\boldsymbol{p} s m_s' | \mathscr{K} | \boldsymbol{m} s m_s) = \frac{1}{\Omega} \langle \boldsymbol{p} s m_s' | \mathscr{K} | \boldsymbol{m} s m_s \rangle, \tag{2.21}$$

and Eq. (2.16) (with the help of Eq. (2.19)) may be rewritten as:

$$V_{\Lambda} = \left(\frac{1}{2\pi}\right)^3 \left(\frac{\mathscr{M}_{\mathrm{N}}}{\mu}\right)^3 \int_{m < \mu k_F / \mathscr{M}_{\mathrm{N}}} d\boldsymbol{m} \sum \{s\} \mathscr{K}(m, s), \tag{2.22}$$

where

$$\mathscr{K}(m, s) = \sum \{m_s\} \langle \boldsymbol{m} s m_s | \mathscr{K} | \boldsymbol{m} s m_s \rangle. \tag{2.23}$$

To determine the $\mathscr{K}$ matrix we follow the usual procedure [7], and introduce the ΛN wave function Ψ:

$$\mathscr{K} | \boldsymbol{m} s m_s \rangle = v_{\Lambda\mathrm{N}} | \Psi_{\boldsymbol{m} s m_s} \rangle. \tag{2.24}$$

Eq. (2.12) implies the following equation for $\Psi(\boldsymbol{r}) = \langle \boldsymbol{r} | \Psi \rangle$:

$$\Psi_{\boldsymbol{m} s m_s}(\boldsymbol{r}\xi) = e^{i\boldsymbol{m}\boldsymbol{r}} \zeta_{s m_s}(\xi) + \int d\boldsymbol{r}' G_{\boldsymbol{m}}(\boldsymbol{r}, \boldsymbol{r}') v_{\Lambda\mathrm{N}} \Psi_{\boldsymbol{m} s m_s}(\boldsymbol{r}'\xi), \tag{2.25}$$

where $\zeta_{sm_s}(\xi)$ is the spin function of the ΛN system. The Green function G_m, when expanded into spherical harmonics, takes the form:

$$G_m(\boldsymbol{r}, \boldsymbol{r}') = \sum \{l\} \left(\frac{4\pi}{2l+1}\right)^{1/2} G_m^{l}(r, \boldsymbol{r}')\, Y_{l0}(\hat{r}\hat{r}'), \tag{2.26}$$

with

$$G_m^l(r, r') = \frac{1}{2\pi^2} \int dp p^2 \bar{Q}(m, p) j_l(pr) j_l(pr')/[z(m) - p^2/2\mu]. \tag{2.27}$$

where

$$z(m) = e_{\rm N}(m_{\rm N}) + V_\Lambda - M^2/2(\mathscr{M}_{\rm N} + \mathscr{M}_\Lambda) = e_{\rm N}(m_{\rm N}) + V_\Lambda - \frac{\mathscr{M}_{\rm N}}{\mathscr{M}_\Lambda}\frac{m^2}{2\mu}, \tag{2.28}$$

and where the Pauli principle operator

$$\bar{Q}(m, p) = \begin{cases} 0 & \text{for} \quad p < k_F - \mathscr{M}_{\rm N} m/\mathscr{M}_\Lambda, \\ 1 & \text{for} \quad p > k_F + \mathscr{M}_{\rm N} m/\mathscr{M}_\Lambda, \\ \left[\left(p + \dfrac{\mathscr{M}_{\rm N} m}{\mathscr{M}_\Lambda}\right)^2 - k_F^2\right] \Big/ [4(\mathscr{M}_{\rm N}/\mathscr{M}_\Lambda) mp] & \text{otherwise.} \end{cases} \tag{2.29}$$

Actually, $\bar{Q}$ is an angle average of the exact Pauli principle operator, Q. The approximation $Q \cong \bar{Q}$ is necessary to make Eq. (2.25) rotationally invariant.

Now we expand Ψ into partial waves:

$$\Psi_{msm_s}(r\xi) = \sum \{Jll'\}[4\pi(2l+1)]^{1/2} i^l (lsom_s|Jm_s) u_{l'l}^{Js}(m, r)\; Y_{l's}^{Jm_s}(\hat{m}\hat{r}, \xi),$$

where $Y_{ls}^{Jm_J}$ are eigenfunctions of the angular momentum (l), spin (s), total spin (J) and its z component (m_J). Notice our notation for the radial functions $u_{l'l}^{Js}$ which differs in the order of the indices l', l from the notation of [7]. From Eq. (2.25) we get the following equations for the radial functions u:

$$u_{l'l}^{Js}(m, r) = j_l(mr)\delta_{l'l} + 4\pi \sum \{l''\} \int_0^\infty dr' r'^2 G_m^{l'}(r, r') v_{l'l''}^{Js}(r') u_{l''l}^{Js}(m, r'), \tag{2.31}$$

where

$$v_{l'l''}^{Js}(r) = \int d\hat{r} d\xi\, Y_{l's}^{Jm_J}(\hat{r}, \xi) v_{\Lambda \rm N} Y_{l''s}^{Jm_J}(\hat{r}, \xi). \tag{2.32}$$

If we replace the hard core of $v_{\Lambda\rm N}$ by a hard shell of the same radius r_c, we may get rid of the product vu in Eq. (2.31), which, inside of the hard core, takes the indeterminate form $\infty \times 0$ [7], [9]. Namely, instead of Eq. (2.31) we get:

$$u_{l'l}^{Js}(m, r) = s_l(m, r)\delta_{l'l} + 4\pi \sum \{l''\} \int_{r_c}^\infty dr' r'^2 F_m^l(r, r') v_{l'l''}^{Js}(r') u_{l''l}^{Js}(m, r'), \tag{2.33}$$

where

$$s_l(m, r) = j_l(mr) - j_l(mr_c) G_m^l(r, r_c)/G_m^l(r_c, r_c) \tag{2.34}$$

is the solution of Eq. (2.31) in the case of a pure hard shell interaction, and where the new Green functions,

$$F^l_m(r, r') = G^l_m(r, r') - G^l_m(r, r_c)G^l_m(r_c, r')/G^l_m(r_c, r_c). \tag{2.35}$$

When we insert the expansion (2.30) into Eq. (2.24) and calculate $\mathscr{K}(m, s)$. Eq, (2.23), we get:

$$\mathscr{K}(m, s) = \sum \{m_s\} \langle msm_s|v_{\Lambda N}|\Psi_{msm_s}\rangle$$

$$= \left(\frac{1}{2\pi}\right)^3 \sum \{sJll'\}(2J+1)4\pi \int_0^\infty drr^2 j_l(mr)\, v^{Js}_{ll'}(r) u^{Js}_{l'l}(m, r), \tag{2.36}$$

or finally:

$$\mathscr{K}(m, s) = \left(\frac{1}{2\pi}\right)^3 \sum \{sJl\}(2J+1)\Bigg[-\frac{j_l(mr_c)^2}{G^l_m(r_c, r_c)} + \\ + 4\pi \sum \{l'\} \int_{r_c}^\infty drr^2 s_l(m, r) v^{Js}_{ll'}(r) u^{Js}_{l'l}(m, r)\Bigg]. \tag{2.37}$$

To calculate V_Λ we must first solve equations (2.33) for the radial functions u, next calculate $\mathscr{K}(m, s)$, Eq. (2.37), and finally apply Eq. (2.22). Obviously, there is a self-consistency problem. Namely, the Green functions G^l_m, and consequently the modified Green functions F^l_m which appear in equations (2.33) depend on V_Λ, as is seen from Eqs (2.27–28). Thus we have to assume a certain input value of V_Λ which we denote by $-\Delta_\Lambda$. This quantity, Δ_Λ, determines the gap in the single Λ particle spectrum. The calculated value of V_Λ is a function of the assumed value of Δ_Λ. By repeating the whole calculation for a few values of Δ_Λ we may determine this function, $V_\Lambda = V_\Lambda(\Delta_\Lambda)$. The self-consistent value of V_Λ is obtained from the condition:

$$V_\Lambda(\Delta_\Lambda) = -\Delta_\Lambda. \tag{2.38}$$

For the sake of completeness let us write the expressions for $v^{Js}_{l'l}$ for our ΛN potential $v_{\Lambda N}$, Eq. (2.1). We have:

$$v^{J0}_{l'l}(r) = \delta_{l'l}\, {}^1v^\pm_c(r),$$

$$v^{J1}_{l'l}(r) = \delta_{l'l}\{{}^3v^\pm_c(r) + [\tfrac{1}{2}J(J+1) - \tfrac{1}{2}l(l+1) - 1]v^\pm_{LS}(r) + S^J_{l'l}v^\pm_t(r), \tag{2.39}$$

where the superscript + (—) has to be used for even (odd) values of l and l'. The only non-vanishing matrix elements, $S^J_{l'l}$, of the tensor operator are [10]:

$$S^J_{J-1,\,J-1} = -2(J-1)/(2J+1),$$

$$S^J_{J,\,J} = 2,$$

$$S^J_{J+1,\,J+1} = -2(J+2)/(2J+1),$$

$$S^J_{J+1,\,J-1} = S^J_{J-1,\,J+1} = 6[J(J+1)]^{1/2}/(2J+1). \tag{2.40}$$

B. Calculation of V_R.

To calculate the rearrangement potential according to Eq. (2.11) let us first write the equations for the NN reaction matrix $K(A)$:

$$K(A)|\tilde{m}_N\tilde{m}'_N) = v_{NN}|\tilde{m}_N\tilde{m}'_N) + \sum\{\tilde{k}_N\tilde{k}'_N\}v_{NN}|\tilde{k}_N\tilde{k}'_N)\times$$

$$\times\frac{1}{a}(\tilde{k}_N\tilde{k}'_N|K(A)|\tilde{m}_N\tilde{m}'_N), \tag{2.41}$$

where v_{NN} is the NN interaction, and

$$a = e_N(m_N)+e_N(m'_N)-\varepsilon_N(k_N)-\varepsilon_N(k'_N). \tag{2.42}$$

Notice that similarly as in Eq. (2.12) we use pure kinetic energies in the intermediate states. This considerably simplifies the calculation of V_R.

The equation for $K(A+1_\Lambda)$ looks exactly as Eq. (2.41) except that in place of a one should put

$$\bar{a} = \bar{e}_N(m_N)+\bar{e}_N(m'_N)-\varepsilon_N(k_N)-\varepsilon_N(k'_N), \tag{2.43}$$

where the single nucleon energy in the Λ+nuclear matter system, $\bar{e}_N(m_N)$, differs from $e_N(m_N)$ predominantly by:

$$\bar{e}_N(m_N)-e_N(m_N) \cong (\tilde{m}_N\tilde{m}_\Lambda|\mathscr{K}|\tilde{m}_N\tilde{m}_\Lambda). \tag{2.44}$$

From Eq. (2.41) and from an analogical equation for $K(A+1_\Lambda)$ we get:

$$[K(A+1_A)-K(A)]|\tilde{m}_N\tilde{m}'_N) = \sum\{\tilde{k}_N\tilde{k}'_N\}K(A)|\tilde{k}_N\tilde{k}'_N)\times$$

$$\times\left[\frac{1}{\bar{a}}-\frac{1}{a}\right](\tilde{k}_N\tilde{k}'_N|K(A+1_\Lambda)|\tilde{m}_N\tilde{m}'_N).$$

As explained in [8], we replace on the right hand side of Eq. (2.45) $K(A+1_\Lambda)$ by $K(A)$, make the approximation:

$$\frac{1}{\bar{a}}-\frac{1}{a} \cong -\left(\frac{1}{a}\right)^2\{[\bar{e}_N(m_N)-e_N(m_N)]+[\bar{e}_N(m'_N)-e_N(m'_N)]\}, \tag{2.46}$$

and apply the approximate equation (2.44). In this way we get:

$$(\tilde{m}_N\tilde{m}'_N|K(A+1_\Lambda)-K(A)|\tilde{m}_N\tilde{m}'_N-\tilde{m}'_N\tilde{m}_N) \cong -\sum\{\tilde{k}_N\tilde{k}'_N\}(\tilde{m}_N\tilde{m}'_N|K(A)|\tilde{k}_N\tilde{k}'_N)\times$$

$$\times\left(\frac{1}{a}\right)^2(\tilde{k}_N\tilde{k}'_N|K(A)|\tilde{m}_N\tilde{m}'_N-\tilde{m}'_N\tilde{m}_N)\,[(\tilde{m}_N\tilde{m}_\Lambda|\mathscr{K}|\tilde{m}_N\tilde{m}_\Lambda)+(\tilde{m}'_N\tilde{m}_\Lambda|\mathscr{K}|\tilde{m}'_N\tilde{m}_\Lambda)], \tag{2.47}$$

and with the help of Eq. (2.11) we obtain:

$$V_R = -\sum\{\tilde{m}_N\tilde{m}'_N\tilde{k}_N\tilde{k}'_N\}(\tilde{m}_N\tilde{m}'_N|K|\tilde{k}_N\tilde{k}'_N)\left(\frac{1}{a}\right)^2\times$$

$$\times(\tilde{k}_N\tilde{k}'_N|K|\tilde{m}_N\tilde{m}'_N-\tilde{m}'_N\tilde{m}_N)(\tilde{m}_N\tilde{m}_\Lambda|\mathscr{K}|\tilde{m}_N\tilde{m}_\Lambda), \tag{2.48}$$

where from now on we denote $K(A)$ simply as K.

If we introduce the total spin $\bar{s}$ of the two nucleons N, N′ and its z component $\bar{m}_s$, and also the total isotopic spin T of N and N′, we may transform Eq. (2.48) into

$$V_R = -2 \sum \{\boldsymbol{m}_N \boldsymbol{m}'_N \boldsymbol{k}_N \boldsymbol{k}'_N \bar{m}_s \bar{m}'_s T\bar{s}\} (2T+1)(1/a)^2 |(\boldsymbol{k}_N \boldsymbol{k}'_N \bar{s}\bar{m}'_s T|K|\boldsymbol{m}_N \boldsymbol{m}'_N \bar{s}\bar{m}_s T)|^2 \times$$
$$\times (1/4) \sum \{s_\Lambda s_N\} (\boldsymbol{m}_N \boldsymbol{m}_\Lambda s_N s_\Lambda | \mathscr{K} | \boldsymbol{m}_N \boldsymbol{m}_\Lambda s_N s_\Lambda). \tag{2.49}$$

The factor 2 in front arises from the exchange term (obviously all the two nucleon states must be antisymmetric under the exchange of their spatial, spin and isotopic spin coordinates). Notice that K does not depend on the third component of the total NN′ isotopic spin, similarly as $\mathscr{K}$ does not depend on t_N. Since V_R does not depend on s_Λ, we have introduced an additional summation $\frac{1}{2} \sum \{s_\Lambda\}$. Since $\sum \{s_\Lambda\}(|\mathscr{K}|)$ does not depend on s_N we have introduced one additional summation more, namely $\frac{1}{2} \sum \{s_N\}$.

Now, let us introduce the NN wave function Φ:

$$K|\boldsymbol{m}_N \boldsymbol{m}'_N \bar{s}\bar{m}_s T) = |\Phi^T_{\boldsymbol{m}_N \boldsymbol{m}'_N \bar{s}\bar{m}_s}), \tag{2.50}$$

and the NN difference function:

$$|\chi^T_{\boldsymbol{m}_N \boldsymbol{m}'_N \bar{s}\bar{m}_s}) = |\Phi^T_{\boldsymbol{m}_N \boldsymbol{m}'_N \bar{s}\bar{m}_s}) - |\boldsymbol{m}_N \boldsymbol{m}'_N \bar{s}\bar{m}_s T). \tag{2.51}$$

Now, Eq. (2.41) implies the relation:

$$\frac{1}{a} (\boldsymbol{k}_N \boldsymbol{k}'_N \bar{s}\, \bar{m}'_s T|K|\, \boldsymbol{m}_N \boldsymbol{m}'_N \bar{s}\, \bar{m}_s T) = (\boldsymbol{k}_N \boldsymbol{k}'_N \bar{s}\, \bar{m}'_s T|\, \chi^T_{\boldsymbol{m}_N \boldsymbol{m}'_N \bar{s}\bar{m}_s}), \tag{2.52}$$

which in turn leads to:

$$\sum \{\boldsymbol{k}_N \boldsymbol{k}'_N \bar{m}'_s\} \left(\frac{1}{a}\right)^2 |(\boldsymbol{k}_N \boldsymbol{k}'_N \bar{s}\, \bar{m}'_s T|K|\, \boldsymbol{m}_N \boldsymbol{m}'_N \bar{s}\, \bar{m}_s T)|^2$$
$$= \frac{1}{\Omega} \int d\boldsymbol{r} d\eta |\chi^T_{\boldsymbol{n}\bar{s}\bar{m}_s}(\boldsymbol{r}, \eta)|^2, \tag{2.53}$$

where we have introduced the relative momentum of the two nucleons,

$$\boldsymbol{n} = (\boldsymbol{m}_N - \boldsymbol{m}'_N)/2, \tag{2.54}$$

and where η denotes the spin variables of the two nucleons. As in Section 2. A we use the notation $\chi(\boldsymbol{r}) = \langle \boldsymbol{r}|\chi\rangle$ and similarly $\Phi(\boldsymbol{r}) = \langle \boldsymbol{r}|\Phi\rangle$. By $\boldsymbol{r}$ we denote here the relative position vector of the two nucleons.

With the help of Eq. (2.53) we may write the expression (2.49) for the rearrangement potential in the form:

$$V_R = -\frac{1}{2\Omega} \sum \{\boldsymbol{m}_N \boldsymbol{m}'_N T\bar{s}\}(2T+1)(2\bar{s}+1) \times$$
$$\times \int d\boldsymbol{r} d\eta |\chi^{T\bar{s}}_{\boldsymbol{n}}(\boldsymbol{r}\eta)|^2 \sum \{s_\Lambda s_N\}(\boldsymbol{m}_N \boldsymbol{m}_\Lambda s_N s_\Lambda | \mathscr{K} | \boldsymbol{m}_N \boldsymbol{m}_\Lambda s_N s_\Lambda), \tag{2.55}$$

where

$$|\chi^{T\bar{s}}_{\boldsymbol{n}}(\boldsymbol{r}, \eta)|^2 = \frac{1}{2\bar{s}+1} \sum \{\bar{m}_s\} |\chi^T_{\boldsymbol{n}\bar{s}\bar{m}_s}(\boldsymbol{r}, \eta)|^2. \tag{2.55'}$$

Now, we take advantage of the weak dependence of the wound integral, $\int dr d\eta |\chi|^2$, on the nucleon momenta, and approximate the wound integral by its average value in the Fermi sea:

$$\int dr d\eta |\chi_n^{T\bar{s}}(r, \eta)|^2 \cong \langle \int dr d\eta |\chi_n^{T\bar{s}}(r, \eta)|^2 \rangle_{Av}$$
$$= \int dm_N dm_N' \int dr d\eta |\chi_n^{T\bar{s}}(r, \eta)|^2 / \int dm_N dm_N'. \tag{2.56}$$

With the approximation (2.56) the summation over the nucleon momentum m_N in Eq. (2.55) concerns the $\mathscr{K}$ matrix elements only, and gives $\frac{1}{2}V_\Lambda$, according to Eq. (2.10) (the factor 1/2 comes from the summation over t_N), and the sum over m_N' produces simply the factor $A/4$. Thus we may write:

$$V_R = -\varkappa V_\Lambda, \tag{2.57}$$

where

$$\varkappa = \varrho \sum \{T\bar{s}\} \frac{(2T+1)(2\bar{s}+1)}{8} \left\langle \int dr d\eta |\chi_n^{T\bar{s}}(r, \eta)|^2 \right\rangle_{AV}, \tag{2.58}$$

where $\varrho = A/\Omega$ is the density of nuclear matter.

If we expand the NN wave function Φ into partial waves (similarly as we have expanded the ΛN function Ψ in Eq. (2.30)):

$$\Phi_{n\bar{s}\bar{m}_s}^T(r, \eta) = \sum \{Jll'\}[4\pi(2l+1)]^{1/2} i^l (l\bar{s}o\bar{m}_s|J\bar{m}_s) R_{l'l}^{J\bar{s}}(n, r) Y_{l'\bar{s}}^{J\bar{m}_s}(\hat{n}\hat{r}, \eta), \tag{2.59}$$

we get

$$\int dr d\eta |\chi_n^{T\bar{s}}(r, \eta)|^2 = \frac{4\pi}{2\bar{s}+1} \sum \{Jll'\}(2J+1)\times$$
$$\times \int dr r^2 [R_{l'l}^{J\bar{s}}(n, r) - \delta_{ll'} j_l(nr)]^2. \tag{2.60}$$

In the case of central NN forces,

$$R_{l'l}^{J\bar{s}}(n, r) = \delta_{l'l} R_l^{\bar{s}}(nr), \tag{2.61}$$

and we get:

$$\int dr d\eta |\chi_n^{T\bar{s}}(r, \eta)|^2 = 4\pi \sum \{l\}(2l+1) \int dr r^2 [R_l^{\bar{s}}(n, r) - j_l(nr)]^2. \tag{2.62}$$

The last equation may be put into another form. Namely, in the case of pure central NN forces,

$$\Phi_{n\bar{s}\bar{m}_s}^T(r, \eta) = \varphi_n^{T\bar{s}}(r) \zeta_{\bar{s}\bar{m}_s}(\eta), \tag{2.63}$$

$$\chi_{n\bar{s}\bar{m}_s}^T(r, \eta) = \bar{\chi}_n^{T\bar{s}}(r) \zeta_{\bar{s}\bar{m}_s}(\eta), \tag{2.64}$$

where ζ is the NN spin function, and

$$\bar{\chi}_n^{T\bar{s}}(r) = \varphi_n^{T\bar{s}}(r) - [e^{inr}]^{T\bar{s}}, \tag{2.65}$$

where the superscript $T\bar{s}$ at the plane wave denotes, *e.g.*, for $T = 1(0)$, $\bar{s} = 1$ only the odd (even) parity part of the plane wave. (Notice that the summations in Eqs (2.59–60, 62) run only over states antisymmetric under the exchange of all the coordinates of the two nucleons.) Consequently, we may write for $\varkappa$:

$$\varkappa = \varrho \left\{ \sum_{\{T\bar{s}\}} \frac{(2T+1)(2\bar{s}+1)}{8} \int d\boldsymbol{r} |\bar{\chi}_n^{T\bar{s}}(\boldsymbol{r})|^2 \right\}_{\mathrm{AV}} \tag{2.66}$$

where the subscript Av denotes the average value over the Fermi sea. This is the equation for $\varkappa$ quoted in I which is valid in the case of a pure central NN interaction only.

3. *The* ΛN *interaction*

At the moment we know very little about the tensor component of the ΛN interaction. One may either introduce it in a purely phenomenological way or deduce it from a field theoretical model. An example of the first approach is the modified Herndon and Tang (*MHT*) potential of Schrills and Darley [11], and of the second approach the one-boson-exchange potential (DP) of Downs and Phillips [12].

Schrills and Darley simply add to the potential F' of Herndon and Tang [4] a tensor interaction of varying strength and range. The range is always shorter than the range of the potential $\boldsymbol{F}'$ (see Section 1.) .In the notation of Eq. (2.1) the *MHT* potential is defined by:

$$ {}^1v_c^+(r) = \begin{cases} \infty & \text{for } r < r_c, \\ -U_{os} e^{-\lambda(r-r_c)} & \text{for } r > r_c, \end{cases}$$

$$ {}^1v_c^-(r) = y\,{}^1v_c^+(r),$$

$$ {}^3v_c^+(r) = \begin{cases} \infty & \text{for } r < r_c, \\ -U_{ot} e^{-\lambda(r-r_c)} & \text{for } r > r_c, \end{cases}$$

$$ v_t^+(r) = \begin{cases} \infty & \text{for } r < r_c, \\ -\delta U_{ot} e^{-\lambda_T(r-r_c)} & \text{for } r > r_c, \end{cases}$$

$$ {}^3v_c^-(r) = y\,{}^3v_c^+(r), \quad v_t^-(r) = y v_t^+(r),$$

$$ v_{LS}^+(r) = v_{LS}^-(r) = 0.$$

The values of the parameters are: $U_{os} = 921.6\,\mathrm{MeV}$, $U_{ot} = 828.5\,\mathrm{MeV}$, $\lambda = 4.427\,\mathrm{fm}^{-1}$, $r_c = 0.6\,\mathrm{fm}$. The parameter y measures the odd angular states suppression, and should be equal about 0.6 according to [4]. For the sake of comparison we have calculated $B_\Lambda(\infty)$ for $y = 0.6$ and also for $y = 1$ (no suppression). Schrills and Darley consider several values of λ_T and δ, and calculate the corresponding values of the Λp triplet S scattering parameters: the scattering length, a_t^p, and the effective range, r_t^p (see Table I). The singlet S Λp scattering parameters, a_s^p, r_s^p, of all the *MHT* potentials are the same as in the case of the F' potential. In calculating the Λp scattering parameters, the charge-symmetry breaking component of the potential F' has to be included into $v_{\Lambda N}$. In the present work we have calculated $B_\Lambda(\infty)$ for all the values of λ_T and δ considered in [11].

The DP potential has been derived by Downs and Phillips [12] under the assumption that the ΛN interaction may be described with the help of the exchange of the following bosons: η (pseudoscalar, $T = 0$), K (pseudoscalar, $T = 1/2$), ω (vector, $T = 0$), and a S

particle (scalar, $T = 0$). For the first four, the observed masses were used, for the S, a mass of three pion masses was used. A variety of cases, (a)—(j), have been considered in which the η, K, ω and K^* couplings were fixed within the ranges suggested by SU_3 symmetry and by comparison with the NN problem. The one remaining parameter, the coupling of S, was then adjusted in each case to optimize the fit to the ΛN triplet and siglet scattering lengths a_t, a_s. The early empirical values, $a_t = -0.5$ fm, $a_s = -3$ fm, have been assumed. The potentials (a)—(j) contain central, tensor, and spin-orbit terms. The charge-symmetry breaking component has been omitted. A feature of the potentials is the soft central core provided by the ω. Actually an additional spin independent hard core of radius $r_c = 0.3$ fm has been introduced to remove singularities in the noncentral potentials. The resulting Λp scattering cross-section was in a reasonable agreement with the early experimental data. No calculations of the hypernuclear binding energies have been performed with these potentials. The lengthy expressions for the DP potential are given in [12].

4. Results and discussion

The present calculations have been performed for $k_F = 1.35\ \text{fm}^{-1}$ which corresponds to the spacing parameter $r_0 = 1.12$ fm. In fixing the single nucleon spectrum, Section 2.A, we have used the value of $\varepsilon_{\text{vol}} = 15.8$ MeV for the coefficient of the volume term in the semi-empirical nuclear mass formula. This leads to the values: $\mathscr{M}_N^* = 0.393\,\mathscr{M}_N$, $A_0 = -112.0$ MeV, and Eq. (2.13) may be written as:

$$e_{\text{N}}(m_{\text{N}}) = 52.76\, m_{\text{N}}^2 - 112.0, \tag{4.1}$$

where e_{N} is given in MeV and m_{N} in fm^{-1}. The corresponding gap in the single nucleon spectrum is:

$$\Delta_{\text{N}} = \varepsilon_{\text{N}}(k_F) - e_{\text{N}}(k_F) = 53.6\ \text{MeV}. \tag{4.2}$$

As has been shown in I, relevant in our calculations is the average value of e_{N} in the Fermi sea,

$$\langle e_{\text{N}}\rangle_{\text{Av}} = -2\varepsilon_{\text{vol}} - (3/5)\varepsilon_{\text{N}}(k_F) = -54.3\ \text{MeV}, \tag{4.3}$$

for which the corresponding value of the gap is

$$\langle \Delta_{\text{N}}\rangle_{\text{Av}} = \varepsilon_{\text{N}}(k_F) - \langle e_{\text{N}}\rangle_{\text{Av}} = 92.1\ \text{MeV}. \tag{4.4}$$

The gap in the single Λ particle spectrum is:

$$\Delta_\Lambda = -V_\Lambda, \tag{4.5}$$

where V_Λ is the self-consistent Λ particle potential. The total gap, Δ, in the single nucleon and the single Λ particle spectrum is:

$$\Delta = \Delta_{\text{N}} + \Delta_\Lambda. \tag{4.6}$$

In calculating V_R, Eq. (2.57), we have used the value of $\varkappa = 0.1$, discussed in I. All the values of $B_\Lambda(\infty)$ obtained in the present paper contain the rearrangement correction, V_R.

The computational procedure and all the meshes of the present calculations have been the same as in I. All states with $l \leqslant 2$ have been included in our calculations, *i.e.* we have considered the following states of the ΛN system: ${}^1S_0$, ${}^1P_1$, ${}^1D_2$, ${}^3P_1$, ${}^3D_2$, ${}^3P_0$, ${}^3S_1+{}^3D_1$, ${}^3P_2(+{}^3F_2)$, ${}^3D_3(+{}^3G_3)$. The $l > 2$ states in the parentheses have been neglected. Consequently, the only really coupled state in our calculations is the ${}^3S_1+{}^3D_1$ state.

The results for $B_\Lambda(\infty)$ obtained with the *MHT* potentials are shown in Table I which also shows the contributions to $B_\Lambda(\infty)$ of the ΛN interaction in the singlet (B^s_Λ) and triplet

TABLE I

Results for $B_\Lambda(\infty)$, B^t_Λ, B^s_Λ, obtained with the ΛN potentials F', *MHT*, *H*, and the parameters of these potentials. All energies are given in MeV, lengths in fm, and λ_T is given in fm^{-1}

$v_{\Lambda N}$	δ λ_T	$-a^p_t$ r^p_t	$-a^p_s$ r^p_s	$y = 1$	$y = 0.6$
				B^t_Λ B^s_Λ $B_\Lambda(\infty)$	B^t_Λ B^s_Λ $B_\Lambda(\infty)$
F'	0	1.44 3.79		26.6 14.7 41.3	20.3 12.2 32.5
*MHT*1	0.12 4.76			29.3 14.5 43.8	22.9 12.0 34.9
*MHT*2	0.16 5.54	1.72 3.40		29.4 14.5 43.9	23.0 12.0 35.0
*MHT*3	0.20 6.22			29.5 14.5 44.0	23.0 12.0 35.0
*MHT*4	0.16 4.57			32.0 14.3 46.3	25.5 11.8 37.3
*MHT*5	0.20 5.15	2.08 3.08		32.2 14.3 46.5	25.6 11.8 37.4
*MHT*6	0.24 5.67			32.2 14.3 46.6	25.7 11.8 37.6
			2.29 3.05		
*MHT*7	−0.10 4.52			29.1 14.5 43.6	22.7 12.0 34.7
*MHT*8	−0.14 5.43	1.72 3.40		29.2 14.5 43.7	22.8 12.0 34.8
*MHT*9	−0.18 6.21			29.2 14.5 43.7	22.8 12.0 34.8
*MHT*10	−0.14 4.52			31.6 14.4 46.0	25.1 11.9 37.0
*MHT*11	−0.18 5.20	2.08 3.07		31.8 14.4 46.2	25.3 11.9 37.2
*MHT*12	−0.22 5.78			32.0 14.4 46.4	25.4 11.9 37.3
H	0	2.08 3.40	2.25 3.29	34.8 14.4 49.2	27.6 11.8 39.4

(B^s_Λ) states. For a few typical *MHT* potentials the contributions to $B_\Lambda(\infty)$ of all the partial waves are shown in Table II, together with the values of the self-consistent single Λ particle potential, V_Λ. Both Tables I and II contain also the results obtained with the charge-symmetric part of the best potential *H* of Herndon and Tang [4], fitted to $B_\Lambda({}^3H_\Lambda)$, $B_\Lambda({}^4H_\Lambda)$, $B_\Lambda({}^4He_\Lambda)$, and to the Λ*p* scattering. The hard core of the *H* potential, $r_c = 0.6$ fm, is the same as that of the F' and *MHT* potentials.

As may be seen from Table II, for fixed values of a^p_t and r^p_t the calculated values of $B_\Lambda(\infty)$ almost do not depend on the particular choice of the values of δ and λ_T. There is only a slight increase in $B_\Lambda(\infty)$ with increasing λ_T, *i.e.*, with decreasing range of the tensor potential. Furthermore, the resulting values of $B_\Lambda(\infty)$ for the corresponding positive and negative values of δ, *i.e.*, for attractive and repulsive tensor forces, are very close to each other. This is what one should expect because the contribution of a weak tensor ΛN force to $B_\Lambda(\infty)$ is essentially a second order effect.

A look at Table II shows that by suppressing the interaction in the odd angular momentum states (changing $y = 1$ into $y = 0.6$) we reduce $B_\Lambda(\infty)$ by a substantial amount. At the same time, however, the contributions to $B_\Lambda(\infty)$ of the even angular momentum states

TABLE II

Partial wave contributions to $B_\Lambda(\infty)$ and the self-consistent values of V_Λ, in MeV, for the ΛN potentials F', H, and some of the MHT potentials

$v_{\Lambda N}$	y	$^3S_1+^3D_1$	$^3P_2$	$^3D_3$	$^3P_0$	$^3P_1$	$^3D_2$	$^1S_0$	$^1P_1$	$^1D_2$	$B_\Lambda(\infty)$	$-V_\Lambda=\Delta_\Lambda$
F'	1.	13.5	6.9	0.4	1.4	4.1	0.3	9.4	5.0	0.3	41.3	45.9
	0.6	14.9	2.7	0.4	0.5	1.6	0.3	9.9	2.0	0.3	32.5	36.1
$MHT1$	1	15.9	6.5	0.4	0.5	5.7	0.4	9.3	5.0	0.3	43.8	48.7
	0.6	17.4	2.4	0.4	0.1	2.3	0.4	9.7	2.0	0.3	34.9	38.7
$MHT3$	1	16.1	6.6	0.4	0.7	5.4	0.3	9.3	5.0	0.3	44.0	48.8
	0.6	17.6	2.4	0.4	0.2	2.2	0.3	9.7	2.0	0.3	35.0	38.9
$MHT4$	1	18.4	6.2	0.4	0.2	6.5	0.4	9.1	5.0	0.3	46.3	51.5
	0.6	19.9	2.3	0.4	−0.1	2.6	0.4	9.6	2.0	0.3	37.3	41.4
$MHT6$	1	18.6	6.3	0.4	0.5	6.1	0.4	9.1	5.0	0.3	46.6	51.8
	0.6	20.2	2.3	0.4	0.0	2.4	0.4	9.6	2.0	0.3	37.6	41.7
$MHT10$	1	18.1	7.9	0.4	2.9	2.4	0.2	9.1	5.0	0.3	46.0	51.1
	0.6	19.7	2.9	0.4	1.2	0.7	0.2	9.6	2.0	0.3	37.0	41.2
$MHT12$	1	18.5	7.4	0.4	2.7	2.7	0.2	9.1	5.0	0.3	46.4	51.5
	0.6	20.0	2.9	0.4	1.1	0.9	0.2	9.6	2.0	0.3	37.3	41.5
H	1	18.7	8.5	0.5	1.7	5.1	0.4	8.6	5.5	0.4	49.2	54.7
	0.6	20.3	3.6	0.5	0.7	2.1	0.4	9.1	2.3	0.4	39.4	43.8

increase. The reason for this is the self-consistency imposed on our calculations. The $\mathcal{K}$ matrix for even states obeys the same equations for $y = 1$ and $y = 0.6$, except that the gap in the single Λ particle spectrum, Δ_Λ, and consequently the total gap, Δ, is different in the two cases. Typically, Δ decreases by about 10 MeV when we change $y = 1$ into $y = 0.6$. Now, a decrease in the gap Δ produces an increase in the effective ΛN interaction, the $\mathcal{K}$ matrix. This is the reason for the increase in the contributions to $B_\Lambda(\infty)$ of the even angular momentum states. It is a manifestation of a general feature namely, that the self-consistency stabilizes the resulting values of $B_\Lambda(\infty)$ against changes in $v_{\Lambda N}$.

When we compare the results obtained with the potential F' and the potentials $MHT4$–6 (and also with the potentials $MHT10$–12) we notice that by adding a seemingly small tensor interaction to the central potential F' we increase the value of $B_\Lambda(\infty)$ by about 5 MeV.

Because of the similarity of the $\Lambda+$ nuclear matter system and the $^5\text{He}_\Lambda$ system one would expect a similar increase in $B_\Lambda(^5\text{He}_\Lambda)$, contrary to the suggestion made in [11][1]. Compared to our results, the perturbational estimate of the contribution of the tensor force to $B_\Lambda(\infty)$ of Ranft [13] seems to be surprisingly small, though it has been obtained with a different ΛN potential.

Now let us discuss the tensor suppression effect. First let us notice (see Table I) that, except for small difference in the value of r_t^p, the potentials H and MHT4–6 (and also MHT10–12) are essentially equivalent in the problem of the low energy Λp scattering. The difference between the potentials H and MHT4–6 consists almost in entirely in the presence of the tensor component in the potentials MHT4–6. Thus, by comparing the values of $B_\Lambda(\infty)$ obtained with the potentials H and MHT4–6 we may estimate the tensor suppression effect. As it is seen from Table I, the value of $B_\Lambda(\infty)$ obtained with the potentials MHT6(4) is smaller than that obtained with the potential H by $D_T = 2.6(2.9)$ MeV for $y = 1$, and by $D_T = 1.8(2.1)$ MeV for $y = 0.6$. The quantity D_T may be considered to represent the magnitude of the tensor suppression effect. When we go from the potential MHT4 to the potential MHT6 (this corresponds to reducing the range of the tensor force), the value of D_T decreases by an amount of 0.3 MeV.

Notice that our values of $B_\Lambda(\infty)$ have been calculated self-consistently, *i.e.*, each value of $B_\Lambda(\infty)$ has been calculated for the corresponding self-consistent value of $-V_\Lambda = \Delta_\Lambda$. Within a linear approximation (certainly valid for 40 MeV $< \Delta_\Lambda <$ 55 MeV), we have for the potential H the following dependence for $B_\Lambda(\infty)$ as a function of Δ_Λ:

$$B_\Lambda(\infty; H, y = 1) = -0.21\, \Delta_\Lambda + 60.6, \tag{4.7}$$

where B_Λ and Δ_Λ are in MeV. For the self-consistent value of $\Delta_\Lambda = 54.7$ MeV, we get from Eg. (4.7) the value of $B_\Lambda(\infty)$ of Table I. Similarly, we have:

$$B_\Lambda(\infty; H, y = 0.6) = -0.20\, \Delta_\Lambda + 48.3. \tag{4.8}$$

To see what would be the magnitude of the tensor suppression effect, if we had not imposed the self-consistency requirement on our calculations, let us calculate $B_\Lambda(\infty; H, y)$ with the help of Eqs (4.7–8) for the self-consistent values of the potentials MHT6, MHT4, shown in Table II. Let us denote by D_T^0 the difference between these values of $B_\Lambda(\infty; H, y)$ and the corresponding $B_\Lambda(\infty)$ values for the MHT6 and MHT4 potentials, given in Table I. Thus, D_T^0 represents the magnitude of the tenscr suppression effect in the case when the self-consistency is disregarded (see Fig. 1 which shows an analogical situation in the case of the contribution of the $^3S_1 + {}^3D_1$ state to the tensor suppression effect). In this way one obtains for the potential MHT6(4): $D_T^0 = 2.8(3.1)$ MeV for $y = 1$, and $D_T^0 = 2.3(2.6)$ MeV for $y = 0.6$ (see Table III). A comparison with the corresponding values of D_T shows that the self-consistency requirement deminishes the tensor suppression effect by 0.2–0.5 MeV.

Notice, that $D_T(y = 1)$ is larger than $D_T(y = 0.6)$ by an amount of 0.8 MeV. This clearly indicates that the interaction in the ^3P states is important here, as may be seen directly

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[1] Actually, the increase in $B_\Lambda(^5\text{He}_\Lambda)$ might be even more important because $B_\Lambda(^5\text{He}_\Lambda)$ is determined predominantly by the ΛN interaction in the S state, for which the tensor superession effect is small.

from Table II. In this connection, it is worthwhile noticing that the expectation values of the tensor operator in the ^3P states are different from zero (in contradistinction to the case of the ^3S state), and we have:

$$\begin{aligned} {}^3P_0 &: v_{11}^{01} = {}^3v_c^- - 4v_t^-, \\ {}^3P_1 &: v_{11}^{11} = {}^3v_c^- + 2v_t^-, \\ {}^3P_2 &: v_{11}^{21} = {}^3v_c^- - (2/5)v_t^-. \end{aligned} \tag{4.9}$$

Thus for the separate ^3P states the effect of tensor forces is a first order effect[2]. The situation with the ^3D states is similar, except that the contribution of the D states to $B_\Lambda(\infty)$ is anyhow very small.

When we look at Table II we notice that the contribution of the coupled $^3S_1 + {}^3D_1$ state to D_T is surprisingly small, being equal 0.3—0.4 MeV for the $MHT4$ potential and 0.1 MeV for the $MHT6$ potential (notice that here the dependence on the range of the tensor force is more pronounced). Similarly as in the case of the total value of D_T, let us discuss the effect of self-consistency. To see what would happen with the $^3S_1 + {}^3D_1$ state contribution to D_T without the self-consistency requirement, let us write in a linear approximation (certainly valid for $40\ \mathrm{MeV} < \Delta_\Lambda < 55\ \mathrm{MeV}$) the dependence of the contribution of the $^3S_1 + {}^3D_1$ state to $B_\Lambda(\infty)$ on the value of Δ_Λ for the three potentials H, $MHT4$, $MHT6$:

$$\begin{aligned} B_\Lambda({}^3S_1 + {}^3D_1;\ H) &= 26.71 - 0.1464\ \Delta_\Lambda, \\ B_\Lambda({}^3S_1 + {}^3D_1;\ MHT4) &= 26.01 - 0.1485\ \Delta_\Lambda, \\ B_\Lambda({}^3S_1 + {}^3D_1;\ MHT6) &= 26.54 - 0.1528\ \Delta_\Lambda, \end{aligned} \tag{4.10}$$

where all quantities are in MeV. By subtracting the last two of these equations from the first one, we get for the contribution of the $^3S_1 + {}^3D_1$ state to D_T for the two potentials, $MHT4$ and $MHT6$,

$$\begin{aligned} D_T^0({}^3S_1 + {}^3D_1;\ MHT4) &= 0.71 + 0.0021\ \Delta_\Lambda, \\ D_T^0({}^3S_1 + {}^3D_1;\ MHT6) &= 0.17 + 0.0064\ \Delta_\Lambda, \end{aligned} \tag{4.11}$$

where the superscript 0 denotes that the difference has been calculated without the self-consistency requirement. If we insert into Eqs (4.11) the self-consistent Δ_Λ value for the H potential[3] we get: $D_T^0({}^3S_1 + {}^3D_1;\ MHT4) = 0.8$ MeV, $D_T^0({}^3S_1 + {}^3D_1;\ MHT6) = 0.5$ MeV. We then see that the magnitude of the $^3S_1 + {}^3D_1$ state contribution to the tensor suppression effect increases considerably if we disregard the self-consistency (but keeping a reasonable

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[2] However, when we calculate the total contribution to $B_\Lambda(\infty)$ of the ^3P states with the proper weighting factors (1/9 for the $^3P_0$, 3/9 for the $^3P_1$, and 5/9 for the $^3P_2$ state) we see that in this total contribution the tensor force does not contribute in first order. Thus the situation is similar to the ^3S state, except that the contribution to $B_\Lambda(\infty)$ of the tensor force in the $^3S_1$ state is realized through the coupling to the weak $^3D_1$ state (similarly as in the ΛN scatering).

[3] If we insert the self-consistent values of Δ_Λ for the $MHT4$ and for the $MHT6$ potentials, respectively we get practically the same result for D_T^0. Notice also that we get approximately the same result for D_T^0 whether we use the self-consistent Δ_Λ values for the H potential in the case of $y = 1$ or in the case of $y = 0.6$ (see Fig. 1).

TABLE III

The magnitude (in MeV) of the tensor suppression effect for the ΛN potentials *MHT*4, *MHT*6

$v_{\Lambda N}$	y	D_T		D_T^0	
		Total	$^3S_1+^3D_1$	Total	$^3S_1+^3D_1$
*MHT*4	1	2.9	0.3	3.6	0.8
	0.6	2.1	0.4	2.5	0.8
*MHT*6	1	2.6	0.1	3.3	0.5
	0.6	1.8	0.1	2.3	0.5

nonvanishing value of Δ_Λ). Our results for the tensor suppression effect are collected in Table III. The relation between $D_T(^3S_1+^3D_1;\ MHT4)$ and $D_T^0(^3S_1+^3D_1;\ MHT4)$ is shown in Fig. 1.

As one should expect (see [2]). $D_T^0(^3S_1+^3D_1)$ increases with Δ_Λ faster for a shorter range tensor force (*MHT*6) than for a longer range tensor force (*MHT*4). In the range of

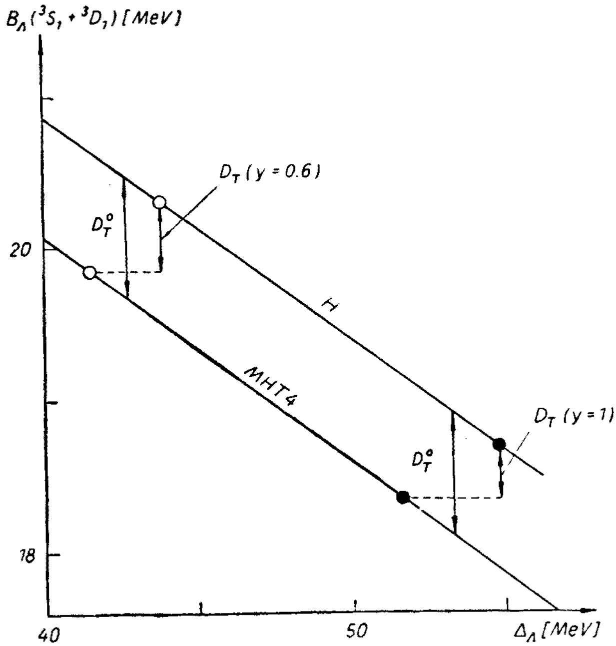

Fig. 1. $B_\Lambda(^3S_1+^3D_1)$ as a function of Δ_Λ for the ΛN potentials *H* and *MHT*4, and the values of D_T and D_T^0. The self-consistent points are: ● for $y = 1$, and ○ for $y = 0.6$

validity of the linear approximation (4.9) the tensor suppression, $D_T^0(^3S_1+^3D_1)$, is larger for the longer range tensor force (*MHT*4) than for the shorter range tensor force (*MHT*6).

Our discussion of the tensor suppression effect has been based on the assumption that the potentials *H* and *MHT*4, *MHT*6 are equivalent as far as the Λ*p* scattering is concerned. In fact, all the potentials have almost identical *S* wave Λ*p* scattering parameters (and they have the same hard core radius). The importance of the ^3P states in the tensor suppression effect which we have found in our calculations might be partly connected with the fact that the potentials H and $MHT4$, $MHT6$ may not be equivalent in the ^3P states of an isolated ΛN system. This point could be clarified by a comparison of the elastic Λp scattering cross--section obtained with the potentials H and $MHT4$, $MHT6$ for energies where the P wave becomes important.

Let us summarize our discussion. If we accept for the odd angular states suppression factor the value $y = 0.6$, adjusted in [4] to the Λp scattering, we get for the magnitude of the tensor suppression effect, $D_T \cong 2$ MeV. The most important contribution to D_T comes from the ΛN interaction in the ^3P states. The self-consistency condition reduces the value of D_T which would be larger by about 0.5 MeV without the self-consistency condition. The value of $D_T \cong 2$ MeV is important in reducing the calculated value of $B_\Lambda(\infty)$ towards its empirical value. It seems, however, to be not sufficient to bring into agreement the calculated and the empirically estimated values of $B_\Lambda(\infty)$.

Recently, Goodfellow and Nogami [14] have calculated the tensor suppression effect in nuclear matter to be less than 1 MeV. These authors put $\Delta_\Lambda = 0$ or 40 MeV in their $\mathscr{K}$ matrix equation, and use the triplet interaction of the tensor Yamaguchi form which acts in the $^3S_1+{}^3D_1$ state only. Consequently, the result of [14] should be compared with our values of $D_T^0(^3S_1+{}^3D_1)$ for $\Delta_\Lambda = 0$ and 40 MeV. If we extend the linear approximation of Eq. (4.11) to the value $\Delta_\Lambda = 0$ we get $D_T^0(^3S_1+{}^3D_1) \cong 0.2(0.7)$ MeV for the $MHT6(4)$ potential. For $\Delta_\Lambda \cong 40$ MeV we have (see Table III) the value of $D_T^0(^3S_1+{}^3D_1) = 0.5(0.8)$MeV for the potential $MHT6(4)$. Similarly as in the present work, an increase in D_T^0 has been noticed in [14] when $\Delta_\Lambda = 0$ has been replaced with $\Delta_\Lambda = 40$ MeV. Thus the results of [14] are consistent with ours.

In a recent paper by Law, Gunye and Bhaduri [15] the tensor suppression effect in $^5\mathrm{He}_\Lambda$ has been estimated to be small (about 0.5 MeV) for several ΛN potentials of the cut--off Yukawa type. In the calculation of the second order contribution of the tensor force, the value $\Delta_\Lambda = 0$ has been used. Furthermore, $B_\Lambda(^5\mathrm{He}_\Lambda)$ is determined predominantly by the S state ΛN interaction. Consequently, the result of [15] may be compared with our value of $D_T^0(^3S_1+{}^3D_1)$ for $\Delta_\Lambda = 0$, corrected for the difference in the average density of ^{4}He and of nuclear matter. In this sense the results of [15] seem to be consistent with ours.

TABLE IV

Results for $B_\Lambda(\infty)$, B_Λ^t, B_Λ^s, $B_\Lambda(^3S_1+{}^3D_1)$, in MeV, obtained with the one-boson-exchange ΛN potential DP(j), and the scattering lengths of this potential in fm

$v_{\Lambda N}$	$-a_t$	$-a_s$	$B_\Lambda(^3S_1+{}^3D_1)$	B_Λ^t	B_Λ^s	$B_\Lambda(\infty)$
$DP(j)$	~0.5	3.5	9.7	22.8	22.2	45.0

Now we would like to present our results obtained with the one-boson-exchange DP potentials. Actually we have calculated $B_\Lambda(\infty)$ only in the case of the DP potential (j) which corresponds to the weakest ω coupling. The results contained in Table IV show a comparatively small contribution of the $^3S_1+{}^3D_1$ state and a triplet states contribution to $B_\Lambda(\infty)$

of about the same magnitude as the singlet states contribution. However, the resulting value of $B_\Lambda(\infty) = 45.0$ MeV is much larger than the empirical value, Eq. (1.1). The sizeable soft core produced by the ω makes the iterative method of solving the wave equation (2.33) divergent in all cases of the DP potentials except for the case (j). According to our estimates, the DP potentials with a strong coupling of the ω, *e.g.*, the potential (i), give a sizeable negative contribution of the $^3S_1+^3D_1$ state to $B_\Lambda(\infty)$, and one should expect to get for the resulting $B_\Lambda(\infty)$ a value close to the empirical estimate, Eq. (1.1). However, with the large size of the soft core of the DP potentials with a strong ω coupling, the three-body ΛNN diagrams, neglected in the present calculation, might be important. Furthermore it should be stressed that the values of a_s and a_t of the DP potentials do not agree with the more recent estimates.

Let us make the following final comment. The analysis of the tensor component in the ΛN interaction presented in this paper, as well as similar analyses in other papers, is a very incomplete one. What one should do is to adjust the whole ΛN interaction including its tensor component to the binding energies of the light hypernuclei, to $B_\Lambda(\infty)$, and to the Λp scattering data. Unfortunately, it is extremely difficult to solve the problem of the light hypernuclei with a realistic ΛN and NN interaction. The simplest case of the $^3H_\Lambda$ hypernucleus seems to be comparatively less important in the analysis of the ΛN tensor force since $B_\Lambda(^3H_\Lambda)$ is determined predominantly by the ΛN interaction in the singlet state. Consequently, the contribution of a ΛN tensor force to $B_\Lambda(^3H_\Lambda)$ is expected to be small [16]. More important in this respect seems to be the problem of $^4H_\Lambda$ and $^4He_\Lambda$ which, however, is a four-body problem. Obviously, the difficulties increase tremendously when we consider the heavier identified hypernuclei, in particular in the p shell. Thus, even without mentioning some other effects (ΛNN forces, isotopic spin suppression) we certainly must realize that at the moment we are in a very early stage of the phenomenological analysis of the ΛN interaction.

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