EVALUATION OF THE ABSORPTION CORRECTIONS TO EXCHANGE MODELS

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A method of evaluation of the absorption corrections by means of an expansion in a rapidly convergent power series is proposed. The resulting formulae are straightforward and easy for numerical calculations.

1. *Introduction*

In this paper we would like to propose a method of evaluating the absorption corrections[1]. The idea is to expand the exact expression for absorption corrections into a power series in a parameter which, at high energies, is equal to

$$\left[\frac{a+b}{2}s\right]^{-1}. \tag{1}$$

Here a and b are elastic and inelastic slopes, and s the total c. m. energy squared.

The method provides a simple analytic expression for the absorption corrections, which is quite accurate and very convenient for physical interpretation.

In the few BeV region the parameter (1) is already small and the convergence of the series is very good. The first two terms give quite reasonable accuracy.

We think that our approach may be useful for the following reasons:

a) If the required accuracy is of the order of, say, one percent, the use of our method of calculation would shorten considerably the time needed for the numerical evaluation of the

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[1] For a review of the different approaches to the absorption model calculations we refer the reader to the recent paper by Henyey *et al.* (Ref. [1]). We follow the notation and use the formulae of these authors.

absorption corrections. This may be of importance if (as usual) a fit with several parameters is performed.

b) The physical interpretation of the results is much clearer from the analytic formula provided by our method than, *e. g.*, from the numerical results obtained in the computer.

c) The investigation of general properties of the absorption corrections, like *e. g.* their dependence on the shape and strength of the interactions involved, is evidently greatly simplified if a simple analytic formula is available.

d) Our results are much more accurate than those obtained in the eikonal approximation, although the formulae are of comparable simplicity.

In the next Section we introduce our notation and the kinematics. Section 3 is devoted to the discussion of the scattering of spinless particles. The case of general spin is discussed in Section 4. In section 5 we apply our formalism to the discussion of the forward dynamical zeros of the scattering amplitudes. Our conclusions are listed in the last Section.

2. *Notation and kinematics*

An exact expression for the first order absorption corrections to an inelastic amplitude M reads [1]

$$\delta M_{\lambda'\mu';\lambda\mu}(z) = -\frac{i\Lambda}{32\pi^2\sqrt{s}}\int dx d\varphi_1 \,[k' M^{\mathrm{el}}(x)_{\lambda'\mu';\lambda''\mu''} M_{\lambda''\mu'';\lambda\mu}(y) + $$
$$+ k M_{\lambda'\mu';\lambda''\mu''}(x) M^{\mathrm{el}}_{\lambda''\mu'';\lambda\mu}(y)] \cos(h\varphi_3 + h''\varphi_2 + h'\varphi_1). \qquad (2.1)$$

Here the subscripts denote helicities. Thus *e. g.* $M_{\lambda'\mu';\lambda\mu}$ corresponds to a transition where the helicity of the first particle changed from λ to λ' and the helicity of the second particle from μ to μ'. z is the cosine of the centre of mass scattering angle. k and k'. are the c.m. initial and final momenta, Λ is a multiplicative factor which, in some models, is left as a free parameter. M^{el} is the amplitude for elastic scattering. M is the amplitude for the inelastic process, but without absorption corrections. In practical calculations both M^{el} and M are assumed known. $h = \lambda - \mu$ and analogous relations hold for h' and h''. The remaining variables y, φ_2 and φ_3 can be obtained by solving the spherical triangle on the unit sphere, as shown in Fig. 1. The integration extends over all of the unit sphere.

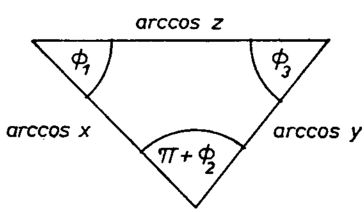

Fig. 1. Spherical triangle needed for calculating absorption correction

The formula (2.1) (with $\Lambda = 1$) is equivalent to the Jackson-Sopkovich prescription for absorption, provided the elastic phase-shifts in the initial and the final state are identical

An analogous formula was first derived for the case of πN scattering by Cohen-Tannoudji *et al.* [2]. The general case of arbitrary spin is discussed by Henyey *et al.* [1]. We follow the notation of these authors.

3. *Absorption corrections for spin 0 particles*

We will discuss first the absorption corrections for the scattering of spin zero particles. The calculation will be given in some detail, because the formulae obtained in this Section can be easily extended to the general case considered in the following Section. Let us note first that only one of the two terms in (1) has to be evaluated explicitly. Indeed, as seen from Fig. 1

$$dx d\varphi_1 = d\Omega = dy d\varphi_2 \tag{3.1}$$

where $d\Omega$ is the surface element on the unit sphere. Consequently, the first integral can be reduced to the second by a change of variables. The results for the first integral can be obtained from the results for the second one by replacing the initial lengths of the three-momenta by the final ones.

For spin zero particles all the subscripts denoting helicities may be omitted. Moreover the cosine drops out, because its argument equals zero. For the elastic amplitude we make the usual assumption

$$M^{el}(t) = Ce^{a/2\,t}. \tag{3.2}$$

Here t is the four momentum transfer squared and a is the slope of the diffraction peak. Thus

$$t = 2k^2(z-1). \tag{3.3}$$

Both C and a are assumed independent of t, but they may, in general, depend on s. In the normalization of Ref. [1], assuming that for forward scattering the amplitude is purely imaginary and using the optical theorem, we may rewrite (3.2) in the form

$$M^{el}(t) = -2\sqrt{s}\, ki\sigma_T e^{a/2\,t} \tag{3.2a}$$

where σ_T is the total cross-section.

For the inelastic amplitude we put

$$M^{inel}(t) = De^{b/2\,t'}. \tag{3.4}$$

Here t' denotes the difference between the square of the four-momentum transfer and its maximal value. We have

$$t' = 2kk'(z-1) \tag{3.5}$$

where k and k' are the initial and final three-momenta.

The form (3.4) of the inelastic amplitude is typical for Regge-pole models, and also it is very convenient for our analysis. As explained at the end of this Section, the generalization of our argument to the case when expression (3.4) is multiplied by a slowly varying function of t', *e. g.* by a polynomial, is very easy.

Substituting the formulae (3.2a) and (3.4) into formula (2.1), and using definitions (3.3) and (3.5) we obtain

$$\delta M^{\text{inel}} = -\frac{\lambda D}{16\pi^2}\,[k^2\sigma_T I + k'^2\sigma_T' I') \tag{3.6}$$

where σ_T' is the total cross-section for the final particles,

$$I(s, z) = \int_{-1}^{1} dx \int_{0}^{2\pi} d\varphi e^{\beta(x-1)+\alpha(y-1)} \tag{3.7}$$

and

$$I' = I(\alpha \to \alpha') \tag{3.7'}$$

$$\beta = bkk' \tag{3.8}$$

$$\alpha = ak^2,\ \alpha' = ak'^2. \tag{3.9}$$

To calculate the integral (3.7) we observe that, as seen from Fig. 1 (with substitution $\varphi_1 \to \varphi$)

$$y = xz + \sqrt{1-z^2}\,\sqrt{1-x^2}\cos\varphi \tag{3.10}$$

Expanding exp $(\alpha\sqrt{1-z^2}\,\sqrt{1-x^2}\cos\varphi)$ into a power series in $\cos\varphi$ and integrating term by term we obtain

$$I(s, z) = 2\pi \int_{-1}^{1} e^{cx-\alpha-\beta} \sum_{m=0}^{\infty} \frac{(\varepsilon c)^m}{(m!)^2} \left(\frac{1+x}{2}\right)^m (1-x)^m\, dx \tag{3.11}$$

where

$$c = \alpha z + \beta \tag{3.12}$$

$$\varepsilon = \frac{(1-z^2)\,\alpha^2}{2c}. \tag{3.13}$$

Each term in the series can be easily integrated using the identity

$$\int P_m(x) e^{cx} dx = c^{-1} e^{cx} \sum_{k=0}^{m} (-c)^{-k} P_m^{(k)}(x) \tag{3.14}$$

valid for an arbitrary polynomial $P_m(x)$. The subscript indicates the degree of the polynomial. $P_m^{(k)}(x)$ denotes the k-th derivative of $P_m(x)$. In our case c is a large parameter. Consequently the formula (3.14) evaluated at $x = -1$ yields a negligible contribution. Keeping only the contribution at $x = +1$ and ordering the terms according to powers of c^{-1} we obtain after simple rearrangements

$$I(s, z) = I_1(s, z) \sum_{n=0}^{\infty} \left(\frac{-\varepsilon}{2c}\right)^n L_n^n(-\varepsilon) \tag{3.15}$$

where

$$I_1 = 2\pi c^{-1} e^{\alpha(z-1)+\varepsilon} \tag{3.16}$$

and $L_n^n(x)$ are Laguerre polynomials defined by

$$L_n^\alpha(x) = \sum_{m=0}^{n} (-1)^m \binom{n+\alpha}{n-m} \frac{x^m}{m!} . \tag{3.17}$$

Note in particular that $L_0^0(x) \equiv 1$ and therefore I_1 is the first approximation to the absorption integral.

Since, as seen from (3.13), the parameter ε vanishes for forward scattering, the first term in the expansion (*i. e.* I_1) gives in this case the exact result. For scatterig angles different from zero higher terms contribute. The second and third corrections read

$$I_2 = I_1(2+\varepsilon)\left(-\frac{\varepsilon}{2c}\right) \tag{3.18}$$

$$I_3 = I_1\left(6+4\varepsilon+\frac{\varepsilon^2}{2}\right)\left(\frac{\varepsilon}{2c}\right)^2 . \tag{3.19}$$

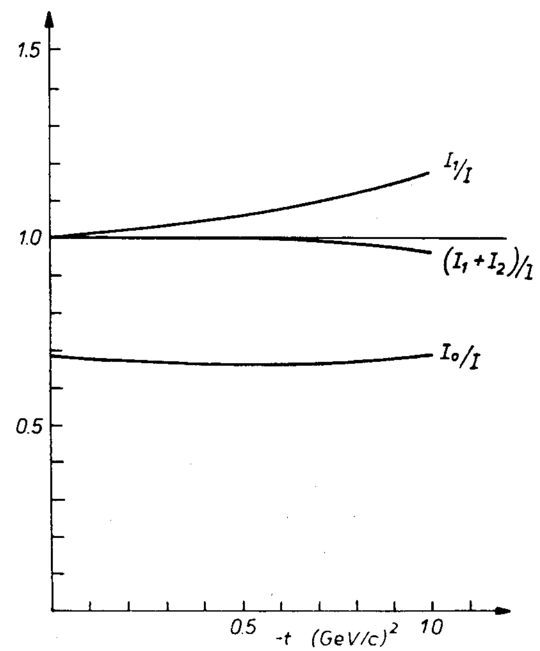

Fig. 2. Absorption corrections for spinless particles

In figure 2 the ratios $\frac{I}{I_1}$ and $\frac{I_1+I_2}{I}$ for NN scattering at 5 GeV/c are plotted *versus* t. It is seen that already I_3 can be practically neglected. At higher energies the convergence is still better.

In order to compare our result with the eikonal approximation we observe that I_1 can be rewritten in the form

$$I_1 = \frac{2\pi}{c} \exp\left\{\alpha c^{-1} \frac{bt'}{2} + \frac{1}{2} \gamma^2 c^{-1} \left(\frac{bt'}{2}\right)^2\right\} \tag{3.20}$$

where

$$\gamma = \frac{\alpha}{\beta}. \tag{3.21}$$

The eikonal approximation is obtained for high energies and small scattering angles. Then bt' is fixed, $\gamma = \frac{a}{b}$ and c tends to infinity. Thus the second term in the exponent can be neglected. Furthermore z is nearly one and $k^2 \approx k'^2 \approx \frac{s}{4}$, so that $c \approx \frac{(a+b)s}{4}$. In this approximation we obtain

$$I_1 \to I_0 = \frac{8\pi}{s(a+b)} \exp\left\{\frac{ab}{2\,(a+b)}\, t'\right\} \tag{3.22}$$

where I_0 is the usual eikonal approximation. In figure 2 the ratio I_0/I is plotted *versus* t. It is seen that in the $|t|$ range from $|t_{\text{min}}|$ up to 1 GeV/c^2 our first approximation is much better than the eikonal formula (3.22).

In order to find the convergence radius of the expansion (3.15) we use Cauchy's theorem and the inequalities

$$\binom{2n}{n} \leqslant L_n^n(-\varepsilon) < \binom{2n}{n} e^{\varepsilon}. \tag{3.23}$$

The first inequality is obtained by keeping only the first term in the formula (3.17). Since ε is positive, this underestimates the polynomial. The second ineguality is obtained by replacing each Newton symbol in (3.17) by the largest one and by extending the summation to infinity.

From (3.23) it is seen that the expansion (3.15) converges if

$$\left|\frac{2\varepsilon}{c}\right| \approx \frac{a^2}{\{(a+b)/2\}^2} \left|\frac{t}{s}\right| < 1. \tag{3.24}$$

Thus it is indeed a high energy small angle expansion.

It is easy to generalize our result to the case when the amplitude (3.2) and/or (3.4) is multiplied by a slowly varying function of the momentum transfer *e. g.* by a polynomial. The additional coefficients should be expanded in a power series around the point $x = +1$ and/or $y = 1$, and the resulting integrals can be obtained from the integral evaluated in this paper by differentiations with respect to α and/or β. This method gives reasonable results only if the series in question is convergent in the physical region $-1 \leqslant x \leqslant 1$. Therefore it is particularly suitable for polynomials in t. Another interesting case is the factor

$$\frac{1}{m^2 - t} \tag{3.25}$$

that is, the propagator of the exchanged particle. Here it is convenient to write

$$\frac{1}{m^2-t}=\frac{1}{2kk'}\frac{1}{z-1-\dfrac{m^2}{2kk'}}$$

and to use the identity

$$\int dxd\varphi\,\frac{e^{\beta(x-1)+\alpha(zx-1)+\alpha\sqrt{1-z^2}\sqrt{1-x^2}\cos\varphi}}{x-1-\delta}=-e^{\beta\delta}\int_{\beta}^{\infty}d\beta Ie^{-\beta\delta} \tag{3.26}$$

where $\delta=\dfrac{m^2}{2kk'}$.

Unfortunately, such integrals cannot be, in general, reduced to elementary functions. Nevertheless, they can be handled more easily than the double integral on the L. H. S. of Eq. (3.26).

In the forward direction the integral $\int_{\beta}^{\infty} d\beta Ie^{-\beta\delta}$ can be evaluated exactly. The result is

$$2\pi\, e^{-\beta\delta}e^{c\delta}Ei(-c\delta) \tag{3.27}$$

where $E_i(x)$ is exponential integral defined as

$$E_i(x)=\int_{-\infty}^{x}\frac{e^u}{u}\,du. \tag{3.28}$$

The case of the Regge propagator cosec $\pi\alpha(t)$ can be reduced to the one disussed above, by using the Mittag-Leffler expansion of coses $\pi\alpha(t)$ into rational functions.

For coefficients singular at $x=+1$ another approach is necessary. Fortunately, the square root branch point, which seems to be only plausible singularity for forward scattering, can be easily handled, as shown in the next section.

4. *Absorption corrections for particles with arbitrary spin*

When the particles involved in the scattering process have non zero spin, the absorptions corrections become more complicated. Firstly, the formula (3.4) for the inelastic amplitude becomes untenable, because it has no kinematical singularities. For the high--energy small angle scattering at least the singularities at $z=1$ (which follow from the angular momentum conservation) should be taken into account. We put

$$M^{\text{inel}}(s,t)=D\sqrt{-t'}^{\,|\nu|}e^{\frac{bt'}{2}}=D\left(\frac{2\beta}{b}\right)^{\frac{1}{2}|\nu|}\sqrt{1-z}^{\,|\nu|}e^{bt'} \tag{4.1}$$

where t' is given by the formula (3.5) and $\nu=h'-h$.

The elastic amplitude is usually assumed to be purely non spin flip in the helicity representation, therefore we keep formula (3.2).

Using (4.1) and (3.2), the formula (2.1) for the absorption corrections to the amplitude (4.1) reads

$$\delta M = -\frac{\lambda D}{16\pi^2}\left(\frac{2\beta}{b}\right)^{\frac{1}{2}|\nu|}(k^2\sigma_T A + k'^2\sigma'_T A') \tag{4.2}$$

where

$$A = \int dx d\varphi_1 e^{\beta(x-1)+\alpha(y-1)}(1-x)^{1/2|\nu|}\cos\{\nu\varphi_1 + h(\varphi_1+\varphi_2+\varphi_3)\}$$

$$A' = A(\alpha \to \alpha'). \tag{4.3}$$

Some complications arise because of the cosine factor

$$\cos\{\varphi_1\nu + h(\varphi_1+\varphi_2+\varphi_3)\}. \tag{4.4}$$

For a planar triangle the sum of the three angles equals zero, but for the spherical angle we have

$$\operatorname{tg}\frac{\varphi_1+\varphi_2+\varphi_3}{2} = \sin\varphi_1\left(\sqrt{\frac{(1+z)(1+x)}{(1-z)(1-x)}} + \cos\varphi_1\right)^{-1}. \tag{4.5}$$

Thus, our problem consists in evaluating the integral (4.3) where the sum of the angles $\varphi_1+\varphi_2+\varphi_3$ should be taken from (4.5). According to the discussion given in the preceding Section, the main contribution to the integral comes from the region where x is nearly equal to one. Since moreover we are interested in values of $z \approx 1$, the square root in the formula (4.5) is a large parameter. Consequently we expand the cosine from formula (4.4) into inverse powers of the square root. The first three terms of this expansion are

$$\cos\nu\varphi - 2h\sqrt{\frac{(1-z)(1-x)}{(1+z)(1+x)}}\sin\nu\varphi\sin\varphi +$$

$$+\frac{(1-z)(1-x)}{(1+z)(1+x)}(h\sin\nu\varphi\sin 2\varphi - 2h^2\cos\nu\varphi\sin^2\varphi). \tag{4.6}$$

Here φ stands for φ_1.

Formula (4.6), substituted into (4.3) provides integrals which can be reduced to those discussed in the preceding Section. The integrals arising from the second and third terms of (4.6) will be called "spherical corrections", since they vanish in the approximation in which the triangle of Fig. 1 can be considered planar.

The further discussion depends on the value of ν. For $\nu = 0$ the kinematical factor in the formula (4.2) drops out. The first term in the expansion (4.6) equals one and in this approximation the absorption correction coincides with that for spin zero particles. The second term in expansion (4.6) vanishes and the third contributes

$$A^{(2)} = -2h^2\frac{1-z}{1+z}\int dx d\varphi\, e^{\beta(x-1)+\alpha(y-1)}\sin^2\varphi\,\frac{1-x}{1+x}. \tag{4.7}$$

Here and in the following the upper index of A denotes the order of the term in the expansion (4.6). Integral (4.7) would diverge if the integration over x were extended over the whole range $[-1, 1]$. Since, however, it is an approximate expression valid for x close to 1, and since this region gives the main contribution to the integral (4.3), it is legitimate to expand $\frac{1}{1+x}$ into a power series in $1-x$ and to integrate this series term by term. Keeping only the first term in the expansion and substituting y from (3.10) we have

$$A^{(2)} = -h^2 \frac{1-z}{1+z} \int dx d\varphi (1-x)(1-\cos^2\varphi) e^{\beta(x-1)+\alpha(xz-1)+\alpha\sqrt{1-x^2}\sqrt{1-z^2}\cos\varphi} + O(c^{-4}) \quad (4.8)$$

where use has been made of the fact that at ε fixed, $1-z$ is of order c^{-1}. The estimate of the error will be explained later. This integral could be evaluated directly, but it is simpler to differentiate the basic integral (3.7) with respect to parameters. The result is

$$A^{(2)} = h^2 \frac{1-z}{1+z} \left\{ \frac{\partial I}{\partial \beta} + \frac{1}{2} e^{-\alpha} \left(\frac{\partial^2 (I e^{\alpha})}{\partial \mu^2} \right)_{\alpha z} \right\} \quad (4.9)$$

$$\mu = \alpha \sqrt{1-z^2} .$$

Here the subscript αz means that the differentiation should be performed keeping αz constant. In order to obtain the first term of the expansion of $A^{(2)}$ into inverse powers of c, it is enough to substitute for I the approximation (3.16). The result is

$$A_3^{(2)} = -h^2 \frac{\varepsilon}{(1+z)^2 \alpha^2} I_1. \quad (4.10)$$

The subscript 3 reminds that this contribution is of the order c^{-3}. The error in (4.8) and (4.9) is estimated as follows. Each correction term to the integral written explicitly contains higher powers of $1-x$. Consequently, in order to calculate it, it is necessary to perform more differentiations with respect to β. Since each differentiation brings in the factor c^{-1} and the main term is of order c^{-3}, the corrections are at most of the order c^{-4}.

A similar calculation for $|\nu| = 1$ gives

$$A_1^{(0)} = \sqrt{\frac{\varepsilon}{c}} I_1 \quad (4.11)$$

$$A_2^{(0)} = -\frac{2\varepsilon^2+7\varepsilon+2}{4c} A_1^{(0)} \quad (4.12)$$

$$A_3^{(0)} = \frac{2\varepsilon^4+22\varepsilon^3+57\varepsilon^2+30\varepsilon+6}{16c^2} A_1^{(0)} \quad (4.13)$$

$$A_2^{(1)} = -\frac{h\nu}{\alpha(1+z)} A_1^{(0)} \quad (4.14)$$

$$A_3^{(1)} = \frac{h\nu}{2\alpha c(1+z)} A_1^{(0)} \left(\varepsilon^2 + \frac{5}{2}\varepsilon - 1 \right) \quad (4.15)$$

$$A_3^{(2)} = -(h^2 - h\nu) \frac{\varepsilon}{\alpha^2(1+z)^2} A_1^{(0)}. \quad (4.16)$$

For $|\nu| = 2$

$$A_1^{(0)} = \frac{\varepsilon}{c} I_1 \tag{4.17}$$

$$A_2^{(0)} = -\frac{\varepsilon^2+5\varepsilon+3}{2c}\frac{\varepsilon}{c} I_1 \tag{4.18}$$

$$A_3^{(0)} = \frac{\varepsilon}{c} I_1 \frac{\varepsilon}{4c^2}\left(\frac{1}{2}\varepsilon^3+7\varepsilon^2+25\varepsilon+20\right) \tag{4.19}$$

$$A_2^{(1)} = -\frac{h\nu}{\alpha(1+z)}\frac{\varepsilon}{c} I_1 \tag{4.20}$$

$$A_3^{(1)} = \frac{h\nu}{2\alpha(1+z)}\frac{(\varepsilon+4)\varepsilon}{c}\frac{\varepsilon}{c} I_1 \tag{4.21}$$

$$A_3^{(2)} = \left(\frac{1}{\alpha(1+z)}\right)^2\left\{h^2(1-\varepsilon)+h\nu\left(\varepsilon+\frac{1}{2}\right)\right\}\frac{\varepsilon}{c} I_1. \tag{4.22}$$

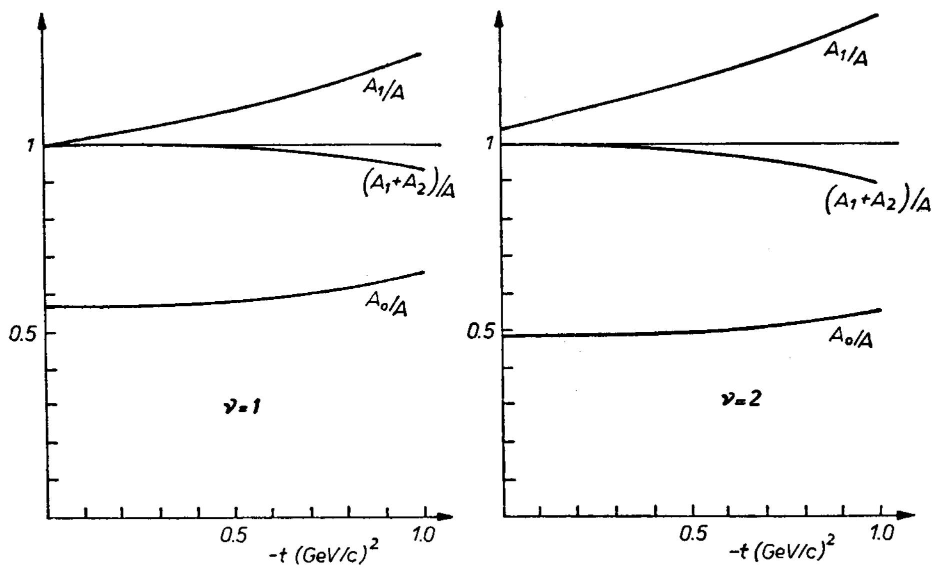

Fig. 3. Absorption corrections for $\nu = 1$ and $\nu = 2$.

The main conclusion one can draw from the formulae (4.11)–(4.22) is that, already at comparatively low energies the spherical corrections are very small. *E. g.* for the *NN* scattering at 5 GeV/c and the $|t|$ region $0-1$ GeV/c^2 the spherical corrections for $\nu = 1$ and 2 do not exceed 3% and 6%, respectively of the total absorption term. The corrections $A_1^{(0)}$, $A_2^{(0)}$ and the eikonal approximation for $\nu = 1$ and 2 are presented in Fig. 3. The kinematical conditions are the same as in Fig. 2.

The formulae for higher helicity flips are not given, because they seem of little practical importance. In each case the high energy, small angle approximation for $A^{(0)}$ yields the result of the eikonal approximation, and the expansion for $A^{(0)}$ converges if the condition (3.24) is satisfied.

5. *Dynamical zeros in the forward scattering amplitude*

Up to now we have considered only the simplified cases when the amplitudes are simple exponentials in momentum transfer multiplied by factors required from angular momentum conservation. However, as already mentioned at the end of Section 3, the method is quite easy to generalize for more general forms of amplitudes. In particular, it applies easily if the amplitudes are multiplied by polynomials in momentum transfer.

A particularly interesting case, which we would like to discuss now, is the problem of dynamical zeros in the forward amplitude. As is well known, the absorption has a dramatic effect: in general, the forward dynamical zeros are completely removed.

We will consider the amplitude with the dynamical zero of order n in the form

$$M^{\mathrm{inel}} = D(-t+t_{\max})^n(-t+t_{\max})^{\frac{1}{2}|\nu|}e^{\frac{bt'}{2}}. \tag{5.1}$$

It is just the amplitude (4.1) multiplied by an additional factor

$$(-t+t_{\max})^n \tag{5.2}$$

which is not required by the angular momentum conservation, but may be imposed by a specific dynamics of the interaction. Well-known examples of such a behaviour of the amplitudes are one pion exchange models of np charge-exchange scattering and charged π photoproduction.

According to the general formula (2.1) and using the abbreviations introduced in Section 3, the formula for the absorption correction to the amplitude (5.1) becomes

$$\delta M = -\frac{AD}{16\pi^2}\left(\frac{2\beta}{b}\right)^{n+\frac{1}{2}|\nu|}\{k^2\sigma_T A(n)+k'^2\sigma'_T A'(n) \tag{5.3}$$

where

$$A(n) = \int dx d\varphi_1\,(1-x)^{n+\frac{1}{2}|\nu|}e^{\beta(x-1)+\alpha(xz-1)+\alpha\sqrt{1-z^2}\sqrt{1-x^2}\cos\varphi_1}\cos(\nu\varphi_1+h(\varphi_1+\varphi_2+\varphi_3)) \tag{5.4}$$

and, according to the general rule, $A'(n)$ can be obtained from $A(n)$ by the replacement $\alpha \to \alpha'$.

The integrals (5.4) ban be easily evaluated, once the basic integrals $A = A(0)$, which were discussed in the previous Section, are known. We have

$$A(n) = (-1)^n\frac{\partial^n A(0)}{\partial\beta^n}. \tag{5.5}$$

It is clear that, since the integral $A(0)$ does not have any dynamical zeros in the forward direction, also $A(n)$ will not have such zeros. Consequently the total amplitude after absorption does not have a dynamical zero.

In the practically most important case $\nu = 0$, $n = 1$ (a dynamical zero of the first order in the no-flip amplitude) the formula (5.5) gives in the first approximation

$$\delta M = \frac{AD}{4\pi} \frac{\beta}{bc^2} (1+\varepsilon)\,(k^2\sigma_r e^{\alpha(z-1)+\varepsilon} + k'^2\sigma_r' e^{\alpha'(z-1)+\varepsilon'}). \tag{5.6}$$

According to the discussion given in Section 3, this formula is exact in the forward direction. Thus we have

$$\delta M(z=1) = \frac{AD}{4\pi} \frac{\sigma_T + \sigma_T'}{\left(a\dfrac{k}{k'} + b\right)\left(a\dfrac{k'}{k} + b\right)} \tag{5.7}$$

It is instructive to compare the value (5.7) with the maximum value of the unabsorbed amplitude (5.1), which reads

$$M_{\max} = -\frac{2}{b} e^{-1}. \tag{5.8}$$

For high energies we obtain

$$\frac{\delta M(z=1)}{M_{\max}} = -\frac{Ae}{\pi} \frac{b}{b+a} \frac{\sigma_T}{a+b}. \tag{5.9}$$

For reasonable values of the parameters a, b, and σ_T the formula (5.9) gives values of the order 1 or larger, thus showing that the absorption correction fills in completely the dip of the unabsorbed amplitude.

Finally, starting from the unabsorbed inelastic amplitude of the form

$$M_{\text{inel}} = \frac{Dt'}{m^2 - t'} e^{bt'/2} \tag{5.10}$$

we would like to give the formula for the absorption correction in the forward direction It reads

$$\delta M(z=1) = -\frac{AD}{4\pi} k^2\sigma_T \left[\delta e^{c\delta} E_i(-c\delta) + \frac{1}{c}\right] + (\alpha \leftrightarrow \alpha') \tag{5.11}$$

where

$$\delta = \frac{m^2}{2kk'}.$$

6. Conclusions

We presented a method for evaluating the first absorption term to the scattering amplitude by expanding it into a power series. This method is applicable to models, like the Regge pole model, where the uncorrected amplitude is, in the momentum transfer, an exponential multiplied by a slowly varying function. The energy dependence is irrelevant.

The first term of our expansion gives for forward scattering the exact value of the absorption correction.

Corrections to the first approximation can be divided into two groups:

a) Ordinary corrections are higher order terms obtained when evaluating integral (4.3) with the cosine replaced by cos $\nu\varphi$.

b) Spherical corrections result from a more careful evaluation of the cosine. The term "spherical" is proposed, because these corrections vanish in the approximation, where the spherical triangle shown in Fig. 1 is considered planar.

Ordinary corrections build up an alternating series, which converges when

$$\frac{2\varepsilon}{c} \approx \left\{\frac{a}{(a+b)/2}\right\}^2 \frac{|t|}{s} < 1.$$

The convergence radius does not depend on the amount of helicity flip, but the convergence rate improves with the decreasing helicity flip. Even for double spin flip, however, the convergence is rather rapid. We have calculated numerically the corrections for NN scattering at 5 GeV/c primary momentum in the momentum transfer region 0–1 GeV/c^2. The first three terms give a result valid within 1–2%.

Spherical corrections are small compared to ordinary ones. This is an important observation, because only the spherical corrections depend on individual helicities and not just on the amount of helicity flip.

As an example of application of our technique, we have discussed the problem of forward dynamical zeros in the inelastic amplitude.

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