QUARK MODEL PREDICTIONS FOR PROCESSES $0^- \frac{1}{2}^+ \to J^P \frac{3}{2}^+$

By K. Zalewski

Institute of Nuclear Research, Cracow*

(Received May 9, 1970)

A simple method of deriving the quark model predictions for processes $0^- \frac{1}{2}^+ \to J^P \frac{3}{2}^+$ is presented. The cases $J^P = 0^\pm$, $1^\pm$, and 2^+ are discussed explicitly. The method is extended to cover the description of interfering resonances. As an example the 2^+—0^+ interference is described.

In this note we present a simple formalism, which gives the quark model predictions for an arbitrary process of the type

$$0^- \frac{1}{2}^+ \to J^P \frac{3}{2}^+ \tag{1}$$

where a pseudoscalar meson and a $\frac{1}{2}^+$ baryon form a meson of spin J and parity P, and a $\frac{3}{2}^+$ isobar. In the version of the quark model, which is used here, the baryonic vertex is described consistently with the additivity assumption. No special assumption is made, however, about the mesonic vertex. Thus J^P may be any resonance, or system of particles, produced peripherally. In particular the formalism may be used in the framework of a general partial wave expansion, with no reference to resonances.

Applying the additivity assumption of the quark model to the baryonic vertex, we find according to the method described in Ref. [1] the following formula for the transition amplitude:

$$A^{\mu'0}_{\nu'\nu} = \sqrt{2}\, a^{\mu'} \Sigma^+_{\nu'\nu} + \sqrt{2}\, b^{\mu'} \Sigma^-_{\nu'\nu} + c^{\mu'} \Sigma^0_{\nu'\nu}. \tag{2}$$

Here the upper (lower) indices refer to the spin projections of the meson (baryon). The matrices Σ were described in Ref. [1]:

$$\sqrt{2}\,\Sigma^+ = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \sqrt{2}\,\Sigma^- = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & \sqrt{3} \end{pmatrix}, \quad \Sigma^0 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{3}$$

and $a^{\mu'}$, $b^{\mu'}$, $c^{\mu'}$ are arbitrary functions.

* Address: Instytut Fizyki Jądrowej, Kraków, Al. Mickiewicza 30, Polska.

Formula (2) will be used in order to simplify the general expressions for the double statistical tensors describing process (1).

Double statistical tensors are defined by the formula (*cf.* Ref. [2])

$$T^{J_1J_2}_{M_1M_2} = \sum_{\substack{m'm \\ n'n}} (-1)^{s_1+s_2+m+n-J_1-J_2}\, C(s_1, -m; s_1, m'|J_1M_1) \times$$

$$\times C(s_2, -m; s_2, m'|J_2M_2)\, \varrho^{mm'}_{nn'}. \tag{4}$$

Here $\varrho^{mm'}_{nn'}$ is the spin density matrix for the final state, which can be expressed in terms of the amplitudes (2) using standard formulae. The symbols labelled "1" refer to the meson, and the symbols labelled "2" refer to the isobar.

Performing the summation over the baryonic spin projections n and n′, one gets

$$T^{J_1J_2}_{M_1M_2} = \sum_{m,\, m'} (-1)^{s_1+m-J_1}\, C(s_1, -m; s_1, m'|J_1M_1)\, R^{mm'}_{J_2M_2}. \tag{5}$$

This is very similar to the formula for the single statistical tensor $T^{J_1}_{M_1}$. The only difference is that the meson spin density matrix $\varrho^{mm'}$ is replaced by the more complicated matrices $R^{mm'}_{J_2M_2}$. Formula (2) is now used to evaluate the matrices R. The relevant matrices are

$$R^{mm'}_{00} = a^m a^{m'*} + b^m b^{m'*} + \tfrac{1}{2}\, c^m c^{m'*} \tag{6}$$

$$R^{mm'}_{20} = \tfrac{1}{2}\, (a^m a^{m'*} + b^m b^{m'*} - c^m c^{m'*}) \tag{7}$$

$$R^{mm'}_{21} = \sqrt{\frac{3}{8}}\, (b^m c^{m'*} - c^m a^{m'*}) \tag{8}$$

$$R^{mm'}_{22} = \sqrt{\frac{3}{2}}\, b^m a^{m'*}. \tag{9}$$

The matrices for M_2 negative are obtained from the corresponding matrices for M_2 positive by substituting b for a and a for b. All the simplification brought in by the quark model is contained in the formulae (6)–(9) independently of the spin of the meson. Without the quark model formula (5) would hold, but the expressions for the matrices R would be much more complicated.

The quark model, in the version used here, gives no predictions about the amplitudes a^m, b^m, and c^m. Therefore we simply substitute the most general amplitudes consistent with parity conservation. In order to illustrate the use of our method, we present a few examples. All the calculations are performed in a spin reference frame with the z axis normal to the reaction plane.

a) Reaction $0^- \tfrac{1}{2}^+ \to 0^- \tfrac{3}{2}^+$

In this case there is only one nonvanishing amplitude: c^0. Using the identity [2]

$$T^{0J_2}_{0M_2} = T^{J_2}_{M_2} \tag{10}$$

valid whenever the spin of the first particle equals zero, we find

$$T^0_0 = \tfrac{1}{2}\, |c^0|^2,\; T^2_0 = -\tfrac{1}{2} |c^0|^2,\; T^2_2 = 0. \tag{11}$$

Using the normalization condition [2]

$$T_0^0 = (2s+1)^{-\frac{1}{2}} \tag{12}$$

we find that (11) corresponds to the Stodolsky-Sakurai decay distribution. This is a well-known result of the quark model.

b) Reaction $0^- \frac{1}{2}^+ \to 0^+ \frac{3}{2}^+$

The nonzero amplitudes are now a^0 and b^0. Thus when (10) and (12) are used,

$$T_0^2 = \frac{1}{4}, \quad T_2^2 = \sqrt{\frac{3}{8}}\, ba^*. \tag{13}$$

c) Reaction $0^- \frac{1}{2}^+ \to 1^- \frac{3}{2}^+$

The nonvanishing amplitudes are

$$a^1 = -f_7/\sqrt{3},\ a^{-1} = f_5/\sqrt{3},\ b^1 = f_6/\sqrt{3}$$
$$b^{-1} = -f_8/\sqrt{3},\ c^0 = -f_0/\sqrt{6}. \tag{14}$$

The notation is chosen in such a way that substituting into the formulae (5)–(9) one obtains the results given in Ref. [3].

d) Reaction $0^- \frac{1}{2}^+ \to 1^+ \frac{3}{2}^+$

The nonvanishing amplitudes are a^0, b^0, and $c^{\pm 1}$. Substituting into the formulae (6)–(9) one reproduces the formulae derived by Muryn [4].

e) Reaction $0^- \frac{1}{2}^+ \to 2^+ \frac{3}{2}^+$

The nonvanishing amplitudes are a^0, $a^{\pm 2}$, b^0, $b^{\pm 2}$ and $c^{\pm 1}$. Again Muryn's results [4] are reproduced.

The present formalism can be easily extended to the study of interfering mesonic resonances. Using the formulae from Ref. [5] one finds

$$T^{J_1 J_2}_{M_1 M_2}(\tfrac{3}{2}, \tfrac{3}{2}; k, l) = \sum_{m_k, m_l} (-1)^{s_k + m_k - J_1} C(s_s, -m_k; s_l, m_l | J_2 M_2)\, R^{m_k m_l}_{J_1 M_1}(k, l) \tag{15}$$

where the labels k and l refer to the two interfering bosonic resonances and the matrices $R^{m_k m_l}_{J_1 M_1}(k, l)$ are obtained from the formulae (6)–(9) by attaching the index k to the first and the index l to the second amplitude in each of the monomials on the right hand sides.

As an example let us consider the interference of a 0^+ meson with a 2^+ meson. From the examples (b) and (e) discussed previously, we find the nonvanishing amplitudes: a_k^0, b_k^0, a_l^0, $a_l^{\pm 2}$, b_l^0, $b_l^{\pm 2}$ and $c_l^{\pm 1}$. Substituting these amplitudes into the formulae (6)–(9) supplemented by the indices k, l, and substituting the resulting expressions into (15), we obtain the formulae from Ref. [4].

The author thanks Dr A. Białas, Dr A. Kotański and Mr B. Muryn for discussions.

REFERENCES

[1] A. Kotański, K. Zalewski, *Nuclear Phys.*, **B12**, 72 (1969).
[2] A. Kotański, K. Zalewski, *Nuclear Phys.*, **B4**, 559 (1968), **B20** 236 (1970) (E).
[3] A. Białas, K. Zalewski, *Nuclear Phys.*, **B6**, 465 (1968).
[4] B. Muryn, private communications and to be published.
[5] A. Kotański, K. Zalewski, *Nuclear Phys.*, **B22**, 317 (1970).