SHOWER COUNTERS OF GAMMA-QUANTA AND ELECTRONS

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A discussion of the methods of energy measurement of gamma-quanta and electrons is given basing on results of a systematic ivestigation of the electron-photon propagation process in matter. Typical of gamma and electron shower counters are presented.

In this article we report on the systematic search for the methods of energy measurement of gamma-quanta and electrons in a wide range of values — from a few MeV to the highest met in physical investigations. Only those methods and counters whose operation is based on the energy dependence of characteristics of electron-photon shower propagation process in matter shall be considered.

As the basis for the analysis performed the results of experimental investigations of the electron-photon showers [1–4] and the results of the theory [5–6] are used.

The progress in the development of gamma-quanta and electrons energy determination methods allows for progress in a wide range of investigations in elementary particle physics, cosmic ray physics, and gamma-ray astronomy. Hence the continued interest in methodical problems is observed.

1. Method

From the wide range of results of experimental investigation on electron-photon showers, the sample of characteristics of the cascade process being in clear dependence on the energy E_0 of the shower initiating particles was selected. Those characteristics are the base for the analysis of the methods of gamma-quanta and electrons energy measurement, and provide the fundamental principle of operation of the corresponding shower counters.

The investigations of the shower propagation process were made using the 26 litre JINR xenon bubble chamber. The use of this chamber to measure the longitudinal and radial shower propagation has proved to be quite effective. The ratio of the chamber length, $l = 55$ cm, to the radiation length, $t_0 = 4.05$ cm, is equal to 13.7, therefore the possibility

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exists to study different shower development processes at shower initiating particle energies less than 5 GeV. The minimum length, in projection on the film plane, of the electron tracks which may be detected has been found to be 0.5 ± 0.2 cm, being independent on the shower energy in the total energy interval. This minimum track length corresponds to the minimum electron energy $E = 3 \pm 1.2$ MeV. The accuracy of E_0 determination is about 10%. The localization of the depth t of the cascade development, and the localization of some point on shower electron tracks may be performed with the accuracy, in average, 1–2 mm.

A typical photograph of the events investigated is shown in Fig. 1. The shower propagation can be observed starting from the primary gamma-quantum generation point or from the point of its conversion. For any set of electron-photon showers at some primary

Fig. 1. A typical photograph of the shower investigated

energy the following experimental information have been obtained: about the longitudinal and lateral development of the showers, on the fluctuations of the average characteristics of the shower propagation process, and about the rate of the primary energy loss in material and on the fluctuations of this energy loss as well [1–4].

A typical set of fundamental experimental data for the sample of showers at energies E_0 is shown in Fig. 2. The energy range of shower analysed was 20–4000 MeV. However, the experimentally determined features of the shower propagation process may be extrapolated to the higher energies, because of the regularity of the cascade process.

Everywhere the comparisons of experimental results with the cascade theory results were performed, this was necessary and possibile.

2. The basic Characteristics of the electron-photon shower propagation process

2.1. The shower development length

The development length d of the electron-photon shower is defined as the distance in the medium in which the cascade propagate, starting from the point of conversion of primary gamma-quantum to the point at which the shower axis intersects the first perpendicular plane which is not reached by secondary shower electrons. For electron-induced showers we accept as the starting point the point at which the primary electron goes into the medium. The average development length is a function of the shower energy E_0.

For a definite value of E_0 this length fluctuates within the interval of 20% of average value of $\bar{d}(E_0)$ [1].

In Table I the energy dependence of $\bar{d}$ is presented. The theoretically [5–6] evaluated values concern the E_0 interval from 1 to 10^4 GeV. The experimental data include the energy region from 20 to 4000 MeV.

TABLE I

Energy dependence of the shower propagation length $\bar{d}$[1]

E_0 eV	$\bar{d}_e$ t_0	$\bar{d}_t$ t_0
$2\ 10^7$	1	—
$4\ 10^7$	1.5	—
$8\ 10^7$	4.5	—
$1.5\ 10^8$	5	—
$3\ 10^8$	6	—
10^9	10	11
$2\ 10^9$	11	—
$4\ 10^9$	12	13
10^{10}	—	19
$5\ 10^{10}$	—	25
10^{11}	—	28
$3\ 10^{11}$	—	32
10^{12}	—	36
$3\ 10^{12}$	—	40
10^{13}	—	45

[1] $\bar{d}_e$ — experimental data, $E \geqslant \frac{\beta}{3}$, β — critical energy. The accuracy of $\bar{d}_e$ is equal to 20%. $\bar{d}_t$ — theoretical data, $E > 0$ [5–6].

2.2. The radial propagation of showers

The collisions of shower electrons with nuclei of the matter leads to the lateral spread of shower particles. The approximate and simple expression for the radius R of the shower, according the cascade theory, is the following [5]:

$$\sqrt{\overline{R_t^2}} = \frac{0.91\ E_k}{\beta}. \tag{1}$$

R in t_0, $E_k = 21$ MeV, β-critical energy in MeV.

The investigations of the fluctuations of the radial shower development [2–3] show the significant spread of the shower diameter reaching 90%. The experimentally evaluated [2–3] and determinated according to the formula (1) as well average values of the shower radius $\bar{R}_e$ and $\sqrt{\overline{R_t^2}}$ at different E_0 are shown in Table II.

2.3. The cascade curves

The dependence on t of the average number $\bar{N}(E_0, E, t)$ of shower electrons with energies no less than E in showers induced by electrons or gamma-quanta with energies E_0

TABLE II

The shower radius

E_0 eV	$\overline{R}_e$ t_0	$\sqrt{\overline{R_t^2}}$ t_0
2 10^7	0.5	1.9
4 10^7	0.8	1.9
8 10^7	1	1.9
1.5 10^8	1.5	1.9
3 10^8	1.3	1.9
10^9	1.5	1.9
1.5 10^9	1.5	1.9
2 10^9	1.5	1.9
2 10^{13}	—	1.9

fluctuate. At shower maximum, *i. e.* at $\overline{N}(E_0, E, t) = \overline{N}(E_0, E, t_{max})$ the fluctuations are approximately Poissonian.

The spread on the conversion length of the primary gamma-quantum has an influence on fluctuations at $t < t_{max}$ in shower with E_0 greater than 1 GeV [2]. In showers with E_0 less than 1 GeV the fluctuations depend on the distribution of the primary gamma-quantum conversion length along the whole shower development length [3].

2.4. The location of shower maximum

The location of the cascade curve maximum can be evaluated from the cascade theory [5–6]. The experimental estimate of the location of this maximum shows a very strong fluctuation of the average depth $\bar{t}_{max}$.

In Table III the calculated values of $\bar{t}_{max}$ for showers at E_0 from 1 to 10 000 GeV are shown. The experimental values of $\bar{t}_{max}$ at E_0 from 1 to 4 GeV as well are also given.

2.5. Number of particles at shower maximum

The average number of shower particles at its maximum, N_{max}, depends on E_0 and E. In showers with energies greater than a few hundred MeV N_{max} fluctuates within $\pm \sqrt{N_{max}}$.

One can evaluate the approximate number $\overline{N}_{max}$ $(E_0, E > 0, t)$ of the shower particles whose energies are greater than 0 using the relation [6]:

$$\overline{N}_{max} = \frac{0.3\, E_0}{\sqrt{\ln \frac{E_0}{\beta} \cdot \beta}}\,. \tag{2}$$

Usually it is necessary to determine the number of shower electrons of energies greater than some definite value E. To determine quickly $N_{max}(E_0, 'E, t_{max})$ we can use the relation (2) and the energy spectrum of shower electrons at t_{max} (Fig. 3 [6]).

TABLE III

Shower maximum location energy dependence

E_0 eV	$\frac{\bar{t}_{max}}{t_0}$ theory[1]	$\frac{\bar{t}_{max}}{t_0}$ experiment[2]
10^9	3	$3.5{+4.5 \atop -1.5}$
$2\ 10^9$	—	$4{+3 \atop -2}$
$4\ 10^9$	—	$5{+3 \atop -2}$
$5\ 10^9$	4	—
10^{10}	5	—
$5\ 10^{10}$	6.5	—
10^{11}	7	—
$3\ 10^{11}$	8.4	—
10^{12}	9.5	—
$3\ 10^{12}$	11	—
10^{13}	12	

[1] at $E > 0$ MeV.
[2] at $E \geqslant \frac{1}{3}\beta$.

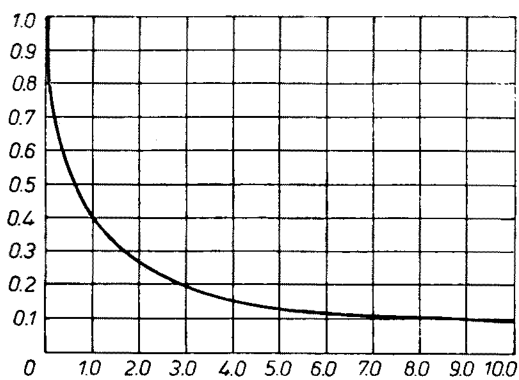

Fig. 3. Shower electrons energy spectrum at shower maximum [6]

2.6. Fluctuations of the shower energy loss in matter

The fraction A of the shower initiating particle energy lost in the matter was estimated by xenon bubble chamber photographs for shower elctrons of energies greater than $E = 3 \pm 1.2$ MeV [7].

The fluctuations $\frac{\sigma_A}{\bar{A}}$ of the shower energy loss are a function of the depth of shower propagation [7] being no higher than 5%, if this depth is equal to the shower length [1].

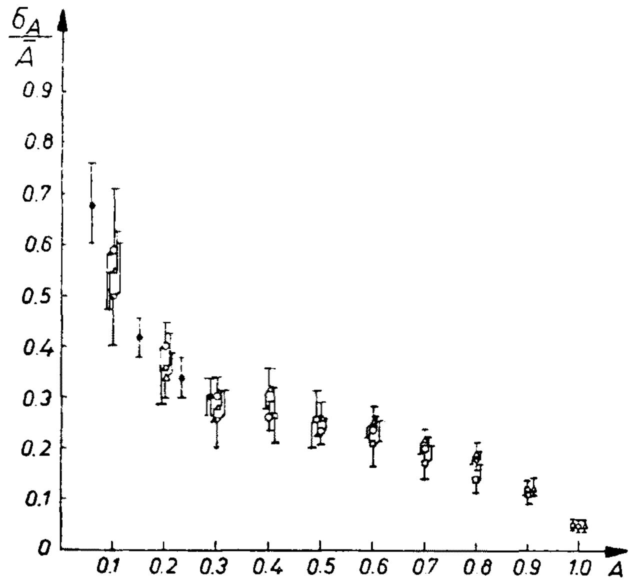

Fig. 4. The dependence of $\frac{\sigma_A}{\bar{A}}$ on the shower cut-off

The dependence of $\frac{\sigma_A}{\bar{A}}$ on the shower cut-off is shown in Fig. 4. The shower initiating particle energies were no greater than 2 GeV. One should expect the fluctuations to be no greater at higher energies.

2.7. Integral cascade curves

The integral cascade curve is defined here as the sum of the following terms:

$$\sum \bar{N} = \bar{N}(E_0, E, \Delta t) + \bar{N}(E_0, E, 2\Delta t) + \dots \tag{3}$$
$$\dots + \bar{N}(E_0, E, \bar{d} - \Delta t) + \bar{N}(E_0, E, \bar{d}).$$

$N(E_0, E, i\Delta t)$ denotes the average number of shower electrons of energies higher than E crossing the axis plane perpendicular to the shower at the distance $i\Delta t$ from the primary gamma-quantum conversion point ($i = 1, 2, \dots$).

Making Δt sufficiently small, the estimation of $\Sigma\bar{N}$ gives the same information as the measurement of the shower electron total track length ΣR. The $\Sigma\bar{N}$ and ΣR are simple functions of the shower energy. Putting $\Delta t \to 0$ and $i \to \infty$, we get:

$$\sum_{\substack{i=1 \\ \Delta t \to 0}}^{\infty} \bar{N}(E_0, E, i\Delta t) \to \sum R(E_0, E, d). \tag{4}$$

Practically, within the frame of the attainable measurement accuracy, being about 10%, this relation is fulfilled if $\Delta t = \frac{1}{2} t_0$ for E_0 higher than 300 MeV, and if $\Delta t = \frac{1}{4} t_0$ for

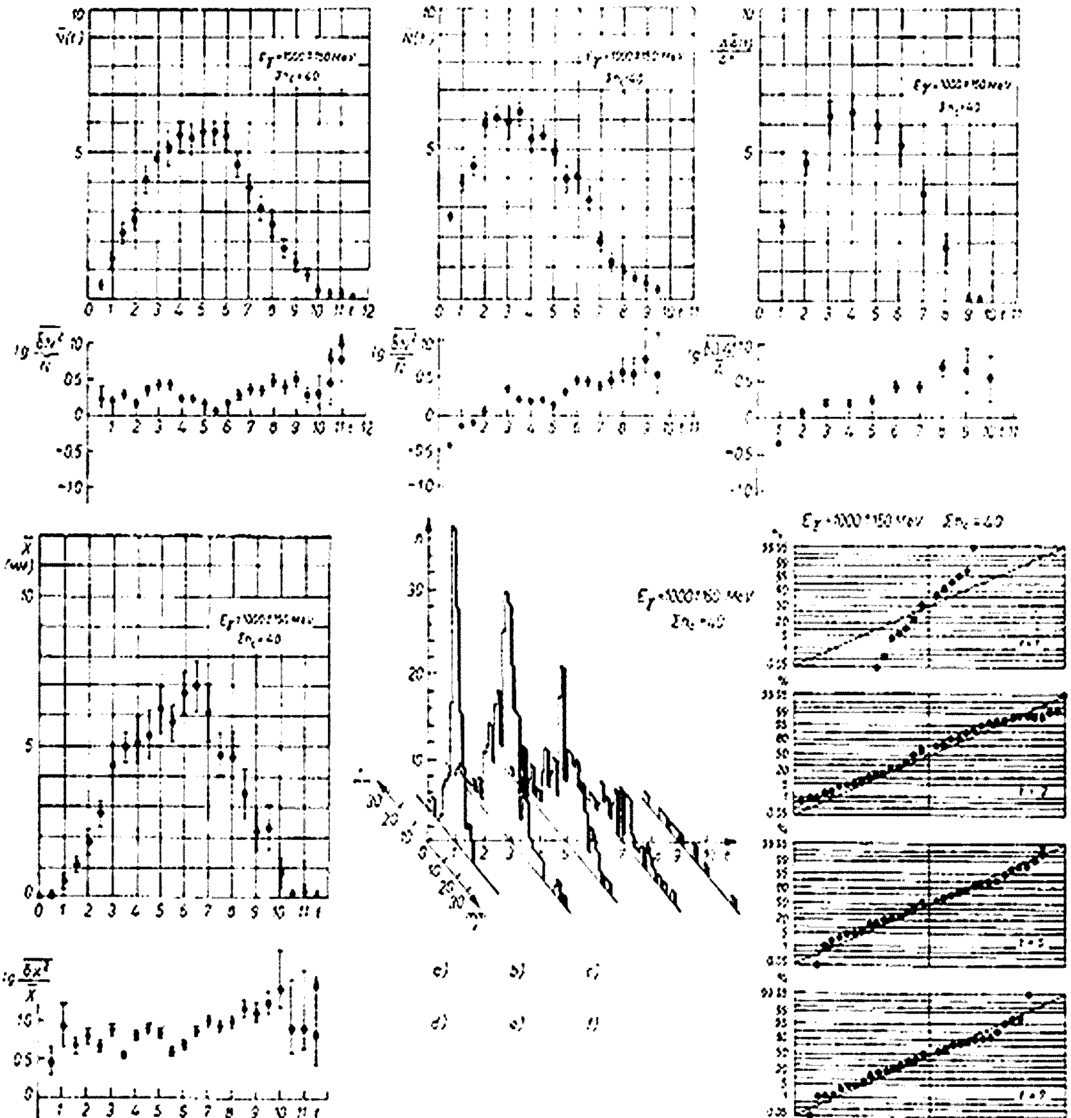

Fig. 2. A typical set of experimental data for the sample of gamma initiated showers at $E_\gamma = 1000$ MeV [2, 3]: *a*) The cascade curve and fluctuations of the shower particle number. The starting point — the point of conversion of the primary gamma-quantum. *b*) The cascade curve and fluctuations of the shower particle number. The shower starting point — the point of generation of the primary gamma-quantum. *c*) Longitudinal energy deposition in matter. The shower starting point — the point of gamma-quantum conversion. *d*) The average shower width longitudinal dependence $x = f(t)$. The shower starting point — the point of gamma-quantum conversion. *e*) Longitudinal dependence of the shower particle distances from the shower axis. *f*) Characteristics of the lateral distributions of shower particles

E_0 values from 20 MeV to 300 MeV [8]. The dependence of N on t at definite E_0 is shown in Fig. 5.

The fluctuations of $\overline{N}$ depend on E_0 and on Δt. For showers of energies above 1 GeV the ratio $\frac{\sigma_{\Sigma N}}{\Sigma \overline{N}}$ amounts to 10% for $\Delta t = \frac{1}{2} t_0$. This value can be reduced, in fact, to be equal to the fluctuations of total energy loss in matter, making $\Delta t \to 0$.

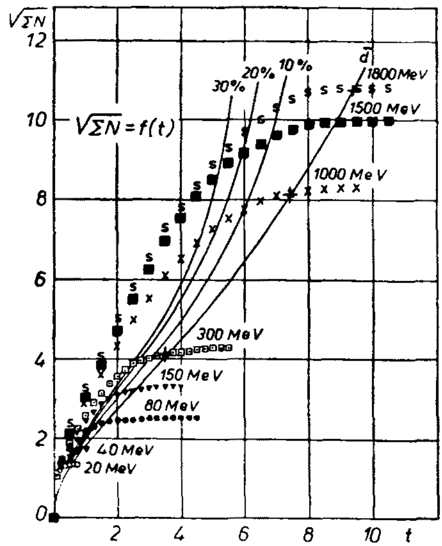

Fig. 5. The dependence of $\Sigma \overline{N}$ on t at different E_0 [8]

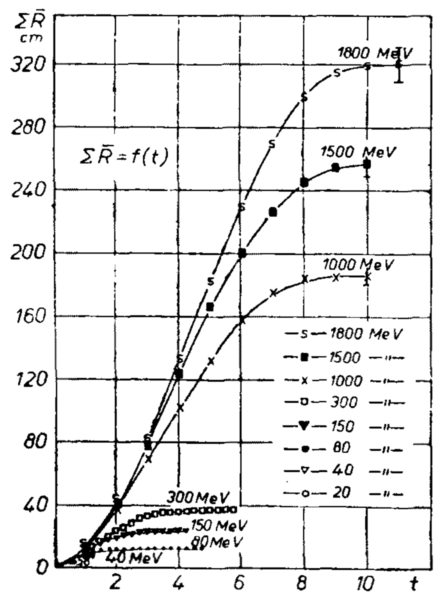

Fig. 6. Energy dependence of ΣR [3]

2.8. Energy dependence of the shower electron total track length

The average total track length ΣR of shower electrons is a simple function of the shower energy [7]. As an example of such a dependence the relation between ΣR and E_0 in liquid xenon is presented in Fig. 6 [3].

The fluctuations of ΣR are practically the same as the fluctuations of the shower energy deposited in matter. This follows from the simple relation between ΣR and E_0.

3. Energy measurements of gamma-quanta and electrons

The analysis of the electron-photon showers leads to the following methods of shower initiating gamma-quantum and electron energy determination:

1. Using the information on the total energy loss of shower electrons in matter.
2. Using the information about some rate of total shower energy loss starting from a definite shower depth.
3. Using the integral cascade curves.
4. From the energy dependence of the shower electron number at shower maximum.
5. From the shower length energy dependence.

Each of the five possible methods are valid at some energy range of shower initiating particle and can give a definite accuracy of the energy measurement. Each of these methods leads to a specific construction of the shower detectors.

4. The general characteristics of shower counters of gamma-quanta and electrons

We now make some analysis of the gamma-quanta and electrons energy deteimination methods mentioned above. The consequences of the analysis will be noted concerning the technical solutions of the shower counters.

4.1. Total energy absorption shower counters

By means of the total absorption counters we can estimate the energy of gamma-quanta or electrons using the information on total shower energy loss in matter. From the § 2.6 it follows that the energies E_0 can be measured by means of such a counter with an accuracy better than 5%.

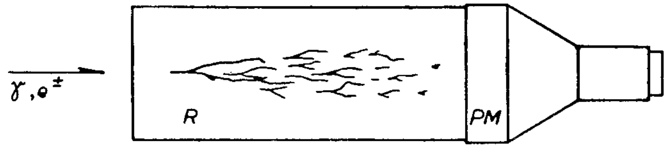

Fig. 7. A typical scheme of total absorption shower counter. *R* — absorber, *PM* — photomultiplier

A typical construction of this type of counter is shown in Fig. 7. The piece of transparent material-absorber is the medium in which the shower propagates. Scintillation effects or Čerenkov radiation in absorber are observed by means of a photomultiplier. Total absorption counters were proposed some years ago [9]. Good results, especially for energy resolution were presented by Hofstadter concerning the counters with monocrystalline absorbers.

The energy range for total absorption counters is limited by the convertor dimensions.

4.2. Multiplate shower counters

In these counters the numbers N of shower electrons at shower propagation depths Δt, $2\Delta t$, $3\Delta t$, ... are estimated. For E_0 estimation the relation between E_0 and $\Sigma\overline{N}$ is used (Fig. 5).

Typical construction of the counter is shown in Fig. 8. The counter is composed of some number of plates of scintillating plastic Δd thick, separated by plates of lead Δt thick.

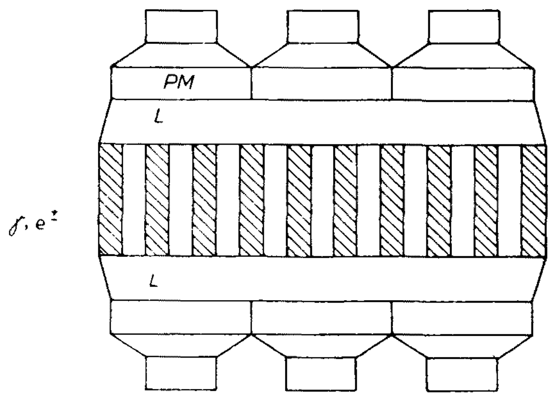

Fig. 8. A typical construction scheme of shower multiplate counter. The shaded areas represent lead plates, non-shaded areas represent scintillating plastic plates. L — light conductors PM — photomultipliers

If the thickness of scintillating plastic plates is small in comparison with the thickness of lead plates, measuring in t_0 units, the showers will develope mainly in lead. By means of the thin scintillators it is possible to measure accurately the numbers of particles at shower propagation depths Δt, $2\Delta t$, $3\Delta t$, ...,. A shower developing in lead plates will, therefore, be sampled by the scintillating sheets sufficiently often to give a good estimate of the total energy of the shower initiating particle.

An estimation was made of the error in a shower energy measurement based on sampling electron numbers at regularly spaced distance intervals [8]. The energy resolution of this counter can be better than 10%.

The multiplate shower detector composed of alternate sheet of scintillating plastic and lead was built and tested by placing it in a beam of monoenergetic electrons [10].

4.3. Non-total absorption shower counters

In non-total energy absorption shower counters the energy of shower initiating particle is measured using information about the rate of total shower energy deposited in absorber.

It follows from the total shower electron track length dependence on the shower propagation length t, at some E_0 range, that it is possible to estimate the shower energy taking into account only a part of the total energy deposition in material. For example, from Fig. 6 one can see that for shower energy estimation in the energy range 1500–1800 MeV it is sufficient to measure the energy loss in the absorber at shower development depth $t > 4t_0$, *i. e.* at $t > t_{max}$.

A schematic representation of the typical non-total absorption shower counter is shown in Fig. 9. The scintillator counter S_1 restricts the gamma-quanta conversion area to the small thickness lead absorber. It is working in coincidence with the scintillator counter S_2 which is connected with the transparent part of the absorber. In this part of absorber the rate of

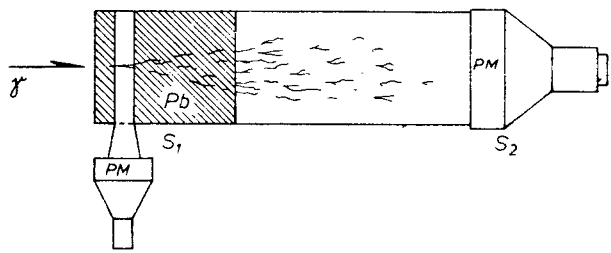

Fig. 9. Schematic representation of the typical non-total absorption shower counter

shower energy corresponding to the shower development length above $t > 4t_0$ is deposited. The approximate length *l* of the transparerent part of the absorber can be estimated from the relation $i \simeq \bar{d} - t_{max}$, where $\bar{d}$ denotes the average shower development depth. It is expected that the percentage probable error of shower energy determination will be higher, compared to the total absorption counters, being 10–20%.

The optimum dimensions of lead absorber and that of transparent part absorber can be determined using the information of § 2.1 and of § 2.4. The very large error expected of the shower energy estimation follows from strong fluctuations of the t_{max} location.

4.4. Shower particle maximum counters

The construction of the counter should give the possibility to determine the number of shower electrons at the shower maximum. It should be possible to estimate the numbers of shower electrons at some region Δt of the shower development depth t around the t_{max}, because of large fluctuations of the location of the shower maximum. It is easy to estimate roughly a sufficient accurate location of t_{max} from the cascade theory.

A typical shower particle maximum counter is schematically shown in Fig. 10. The electron or photon initiated shower propagates in the lead absorber *1* and in the sheet of lead plates separated by plastic plate counters $S_1, S_2, S_3, \ldots, S_i$. Changing the lengths of the blocks of absorber *1* we can prepare the counter for measurements at some E_0 energy region. The thickness of the lead plates between scientillator counters $S_1 - S_i$ and the number of counters should be determined by the t_{max} fluctuations. The lead plates between the counters S_i should be of small thickness and the number of counters should be as large as possible. It is necessary to obtain a maximum accuracy of particle number determination. The error of the maximum number of shower particle estimation is expected to be no higher than $\pm\sqrt{N_{max}}$.

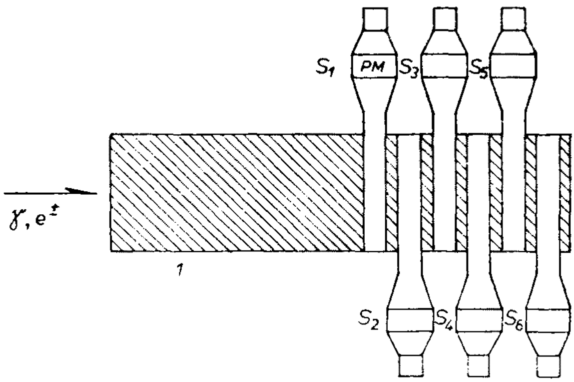

Fig. 10. Schematic representation of the typical shower particle maximum counter

Having the number N_{max} it is easy to determine the shower energy E_0. Expected accuracy equals $\frac{\Delta E_0}{E_0} = \frac{\sqrt{N_{max}}}{N_{max}}$.

4.5. Shower length counters

The shower length conter should give the possibility to estimate the shower development length in absorber. A typical technical solution of the counter is shown in Fig. 11. The propagation of the electron-photon cascade occur in the lead absorber *2*. The scintillator counter S_1 shows that the gamma-quantum conversion occurs in the lead plate *1*. The scin-

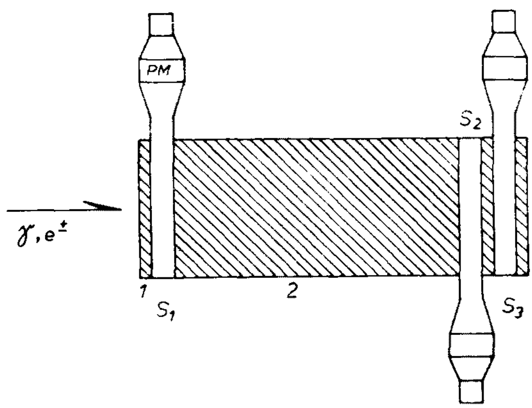

Fig. 11. Schematic representation of the typical shower length counter

tillator counter S_3 shows that the electrons do not escape from the lead plate separating the counters S_2 and S_3. The coincidence of signals from the counters S_1 and S_2 and simultaneos of signal from the counter S_3 is necessary.

For each E_0 value a definite length of absorber *2* is necessary. The shower length counter gives an approximate value of E_0 only, within 20%. These counters can be used for the estimation of electron or photon beam energy spectrum.

4.6. General remarks

The informations concerning the technical solutions of the shower counters follow from the electron-photon cascade characteristics (§ 2.1 – § 2.8). For accurate measurements of gamma-quanta and electron energies the counters should be, however, tested in a beam of monoenergetic electrons.

5. Limits of the applicability of shower counters

It is expected that the efficiency of gamma-quantum or electron registration by means of shower counters will be closed to 100%.

The most accurate measurements can be performed by means of the total absorption showers counters. The E_0 estimation accuracy is about a few percent, in average, and can approach the value of less than 1% as well. The accuracy is determined by the fluctuations of the shower energy deposited in matter. According to the approximate estimation, on xenon bubble chamber photographs, the fluctuations should be no higher than 5%. The measurements performed in recent works show these fluctuations to be smaller than 5% being closer rather to 2% [11].

Less accurate measurements can be performed by means of the multiplate shower counters. The information about the shower characteristics indicate the that E_0 measurement accuracies are expected to be better than 10%. The E_0 estimation region for such counters is wide. The lower limit is about a few tens MeV and the upper limit is determined by the absorber dimensions only. In practice no upper limit for E_0 measured exists.

The non-total absorption counters can be used for very small energy ranges at E_0 values above a few GeV. The shower energy determination accuracy is worse than that for the multiplate counters, being about 15%.

The shower particle maximum counters are designed for measurements at a very wide energy range above some tens GeV. The percentage error expected will be no higher than

$$\frac{\Delta E_0}{E_0} = \frac{\sqrt{N_{max}}}{N_{max}}.$$

The measurements of E_0 by means of shower length counters can be performed with the accuracy of no better than 20–30%, at the energy range above a few GeV.

For very high energies it is expected to be convenient to use multiplate counters or shower particle maximum counters.

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