BACKWARD N^*_{33} PRODUCTION IN πN COLLISIONS AND THE QUARK MODEL

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A quark model of the backward scattering in meson-baryon collisions at high energies is proposed. The predictions for the backward N^*_{33} production in pion-nucleon scattering are given.

1. Introduction

In this paper we would like to propose a quark model of the baryon-exchange processes at high energy[1]. We assume that the dominant mechanism of the interactions leading to backward scattering is analogous to the stripping processes in nuclear physics. In the language of particle physics it is a usual baryon-exchange model in which the vertices are calculated according to the rules of additivity[2].

The description of the meson-exchange processes by means of the additive quark model proved itself to work reasonably well. Therefore we think that it is rather interesting to develop the additive model of baryon-exchange processes and check its agreement with the experimental data. Since for the baryon-exchange reactions the additivity cannot be justified by the group-theory arguments[3], its experimental investigation may possibly provide information on the basic ideas of the quark model.

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[1] The basic ideas of the proposed model were presented by two of us (A. B. and K. Z.) in a series of lectures at the *VIIIth Cracow School of Theoretical Physics*, June 1968. See Ref. [1].

[2] There are also other possibilities of treating backward scattering in the general model. See Ref. [2].

[3] This was pointed to us by Dr M. Jacob.

The model described in this paper is rather simplified and should be considered only as a first approximation. We feel, however, that, since the measurements of inelastic reactions in backward directions are rather limited, it would not be justified to worry about fine details. They can be added if more data are available.

The mechanism we propose is summarized by the baryon-exchange diagram presented in the Figure 1. The initial baryon is considered as a three-quark system according to the usual rules of the quark model. In the first vertex, the baryon emits a meson. The four-

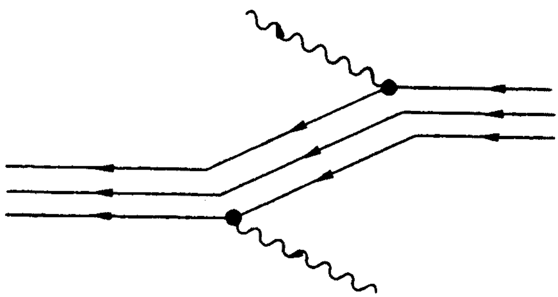

Fig. 1

-momentum transfer is small, therefore we assume that the meson is emitted by a single quark, as in quark theory of decay processes (see, *e.g.*, Ref. [3]). The virtual baryon left after the emission of the meson propagates and absorbs the incident meson (vertex 2). Then it becomes the real final baryon. Since the four-momentum transfer also in the second vertex is small, we use again additivity[4].

Following this procedure, one can express the amplitudes for all meson-baryon processes in terms of the unknown amplitudes describing the emission and absorption of the meson by a single quark and the propagators for virtual baryons. If the number of the unknown quantities is smaller than the number of processes to be considered, one obtains the relations which can be checked experimentally. In the present paper we apply this procedure to the reactions

$$\pi N \to N\pi \tag{1}$$

$$\pi N \to N^*(1236)\pi \tag{2}$$

It appears that, for the high-energy scattering close to 180°, the cross-sections for all possible processes (1) and (2) can be expressed in terms of three unknown parameters. It follows that, knowing the cross-sections for the processes

$$\pi^+ p \to p\pi^+ \tag{1.1}$$

$$\pi^- p \to p\pi^- \tag{1.2}$$

$$\pi^- p \to n\pi^0 \tag{1.3}$$

[4] The words "emission" and "absorption" are used here in the field-theoretical sense. That is to say, the emission of meson in the first vertex is equivalent to absorption of virtual antibaryon.

one is able to predict the absolute cross-sections for reactions (2) leading to resonance production.

There are no data on the high-energy N^* production close to backward direction. Therefore no significant test of the model can be made at the moment.

The plan of this paper is as follows. In Section 2, the necessary notation is introduced and an example of the calculation of scattering amplitude is given. In Section 3, the relations between the processes (1) and (2) at 180° are discussed. The numerical predictions and a discussion of the experimental data are presented in Section 4. Finally, we have collected in the Appendix the formulae expressing the amplitudes for all processes (1) and (2) in terms of the quark amplitudes.

2. Calculation of the amplitude for the process $\pi^- p \to \pi^+ N^{*-}$

The wave function of the initial proton in terms of quarks is (*cf*, *e.g.*, Ref. [3])

$$\frac{1}{\sqrt{18}}\,[2S(p_+p_+n_-) - S(p_+p_-n_+)]. \tag{2.1}$$

Here S denotes the symmetrizing operator, thus

$$S(p_+p_+n_-) = p_+p_+n_- + p_+n_-p_+ + n_-p_+p_+. \tag{2.2}$$

When the π^+ is emitted, one of the proton quarks goes over into the neutron quark. The amplitudes describing this transition will be denoted by

$$\begin{aligned} a &\equiv A(p_+ \to n_-\pi^+) \\ b &\equiv A(p_- \to n_+\pi^+) \end{aligned} \tag{2.3}$$

where the subscripts $+$ $-$ denote the sign of the transversities of the considered quarks.

Summing over all the possibilities, we obtain for the intermediate baryon

$$\frac{1}{\sqrt{18}}\,[4aS(p_+n_-n_-) - aS(n_-p_-n_+) - 2bS(p_+n_+n_+)]. \tag{2.4}$$

This wave function could be interpreted as a superposition of the wave functions of the neutron and the N^{*0}. More generally, however, we may consider (2.4) as a spin and unitary spin part of the wave function which can contain also orbital excitations, and which represents some very complicated baryonic state.

To find the amplitude for the considered baryon-exchange process

$$\pi^- p \to N^{*-}\pi^+ \tag{2.5}$$

this complicated intermediate state has to be decomposed into parts corresponding to the well-defined baryons and each part multiplied by the corresponding propagator. Thus we obtain for the contribution of each intermediate baryon to the scattering amplitude

$$\frac{1}{\sqrt{18}}\left\{\frac{5}{3}\,[2S(p_+n_-n_-) - S(p_-n_-n_+)]a\Phi_{1/2} + \right.$$
$$\left. + \frac{2}{3}\,[S(p_+n_-n_-) + S(p_-n_+n_-)]a\Phi_{3/2} - 2S(p_+n_+n_+)b\Phi'_{3/2}\right\} \tag{2.6}$$

where $\Phi_{1/2}$, $\Phi_{3/2}$ and $\Phi'_{3/2}$ are the propagators. The baryonic state (2.6) absorbs the initial meson and becomes the final baryon. Denoting the amplitudes for the absorption of the meson by a quark by

$$a \equiv A(\pi^- p_+ \to n_-); \quad \bar{b} = A(\pi^- p_- \to n_+) \tag{2.7}$$

we obtain from (2.6) the amplitude

$$\frac{1}{\sqrt{18}} \left\{ \frac{5}{3} [6(n_-n_-n_-)a\Phi_{1/2}\bar{a} - 2S(n_-n_+n_+)a\Phi_{1/2}\bar{b}] + \right.$$

$$\left. + \frac{2}{3} [3(n_-n_-n_-)a\Phi_{3/2}\bar{a} + 2S(n_+n_+n_-)a\Phi_{3/2}\bar{b}] - 2S(n_-n_+n_+)b\Phi'_{3/2}\bar{a}. \right\} \tag{2.8}$$

The amplitude (2.8) must now be summed over intermediate baryons. We introduce the notation

$$\sum a\Phi_I\bar{a} = a_{2I}; \quad \sum b\Phi_I\bar{b} = b_{2I}$$

$$\sum a\Phi_I\bar{b} = A_{2I}; \quad \sum b\Phi_I\bar{a} = B_{2I} \tag{2.9}$$

where the sum is extended over all intermediate baryons. Substituting the wave functions of the N^{*-}

$$N^{*-}_{1/2} = \frac{1}{\sqrt{3}} S(n_+n_+n_-) \tag{2.10a}$$

$$N^{*-}_{-3/2} = n_-n_-n_- \tag{2.10b}$$

we obtain

$$A(\pi^- p_{1/2} \to N^{*-}_{1/2}\pi^+) = \sqrt{\frac{2}{3}} \left\{ \frac{2}{3} A_3 - B_3 - \frac{5}{3} A_1 \right\} \tag{2.11a}$$

$$A(\pi^- p_{1/2} \to N^{*-}_{-3/2}\pi^+) = \frac{\sqrt{2}}{3} \{5a_1 + a_3\}. \tag{2.11b}$$

The amplitudes for other processes can be calculated in a similar way in terms of the amplitudes defined by formula (2.9). The results are given in the Appendix.

3. N^*_{33} *resonance production at 180°*

As seen from the formulae given in the Appendix, the amplitudes for the processes (1) and (2) can be expressed in terms of eight different quark amplitudes defined by (2.9). Such a large number of unknown parameters does not allow to make simple predictions for the experimentally measurable quantities, unless some further assumptions concerning the quark amplitudes are introduced.

The problem simplifies drastically, however, for the scattering close to 180°. In this reigon the angular momentum conservation allows only one independent amplitude in each of the processes (1) and (2). For the transversity amplitudes we use, the following relations have to be satisfied [4],

$$T_{++} = -T_{--} \tag{3.1}$$

for processes $\pi N \to N\pi$, and

$$T_{\frac{1}{2}\frac{1}{2}} = T_{-\frac{1}{2}-\frac{1}{2}} = \frac{1}{\sqrt{3}} T_{-\frac{3}{2}\frac{1}{2}} = \frac{1}{\sqrt{3}} T_{\frac{3}{2}-\frac{1}{2}} \tag{3.2}$$

for processes $\pi N \to N^*\pi$.

Applying the formulae (3.1) and (3.2) to all processes (1) and (2), we obtain the following restrictions for the quark amplitudes defined in (2.9):

$$-a_M(180°) = A_M(180°) = -b_M(180°) = B_M(180°) \tag{3.3}$$

$M = 1$ and 3.

It follows that, for scattering in the region close to 180°, where the amplitudes forbidden by helicity conservation can be neglected, all processes (1) and (2) can be described in terms of two independent quark amplitudes. Thus, at a given incident momentum, one can express the cross-sections for all processes (1) and (2) as linear combinations of three parameters, say $|a_1|^2$, $|a_3|^2$ and $\mathrm{Re}(a_1^* a_3)$. Consequently, the knowledge of the cross-sections for any three of the processes (1) and (2) allows one to predict the cross-sections for the remaining ones. We give below the formulae expressing the cross-sections for N^* production in terms of the cross-sections for elastic and charge-exchange scattering. They read

$$\sigma(\pi^- p \to N^{*+}\pi^-) = \frac{1}{8}\sigma_-$$

$$\sigma(\pi^+ p \to N^{*+}\pi^+) = \frac{2}{25}\sigma_+ - \frac{3}{50}\sigma_- + \frac{12}{25}\sigma_0$$

$$\sigma(\pi^- p \to N^{*0}\pi^0) = \frac{6}{25}\sigma_+ + \frac{3}{25}\sigma_- - \frac{4}{25}\sigma_0$$

$$\sigma(\pi^- p \to N^{*-}\pi^+) = \frac{21}{25}\sigma_+ - \frac{21}{200}\sigma_- + \frac{6}{25}\sigma_0$$

$$\sigma(\pi^+ p \to N^{*++}\pi^0) = \frac{3}{2}\sigma(\pi^+ p \to N^{*+}\pi^+) \tag{3.4}$$

where

$$\sigma_\pm = \sigma(\pi^\pm p \to p\pi^\pm)$$

$$\sigma_0 = \sigma(\pi^- p \to n\pi^0) \tag{3.5}$$

A discussion of these formulae in terms of the available experimental data is given in the next Section.

As remarked above, the formulae (3.4) are valid in the region close to 180°, where the helicity non-conserving amplitudes can be neglected. The mass difference between the nucleon and N^*_{33} resonance implies that one cannot reach in the elastic scattering the values of u which would correspond to N^* production at 180°. Thus it is rather difficult to test these predictions at low incident momenta, unless additional information on the spin-dependence is available. The relations (3.4) should be therefore considered only as valid in the asymptotic limit at very high energies. One would hope, however, that starting from $\sim$6 GeV/c, the corrections for this effect will be small enough to enable a reasonable comparison of our model with experiment.

Another obvious restriction for the validity of the formulae (3.4) is the possibility of strong form factor in the N^* vertex which can reduce the N^* production as compared to elastic scattering by a common factor. The discussion of the form factors goes, however, beyond the scope of the present model.

4. *The numerical predictions for the N^* production at 4 and 6 GeV/c*

The highest momentum at which all three cross-sections for backward πN elastic and charge-exchange scattering were measured is 6 GeV/c [5–7]. At this momentum the formulae (3.4) can be directly used for the calculation of the estimated cross-sections of N^* production. The results are given in the Table together with the input data. Since the measurements of backward πN charge-exchange cross-sections reported in Refs [6–9] differ in absolute normalization, we have calculated the predictions using two different input data.

The most striking feature in the obtained results is a very large cross-section expected for the process

$$\pi^- p \to N^{*-}\pi^+ \tag{4.1}$$

This cross-section should be comparable to the backward π^+p elastic cross-section. Other channels in the π^-p scattering are strongly reduced compared to (4.1). The N^* production in π^+p scattering is expected to be $\sim$ three times smaller than in the process (4.1)

Since there are no published data on the N^* production close to 180°, in the energy region above $\sim$5 GeV/c, we cannot test the validity of our model.

We would like to thank Dr M. Jacob, Dr J. Kwieciński and Dr R. J. N. Phillips for useful discussions.

APPENDIX

Here we would like to collect the formulae for the amplitudes of the processes (1) and (2) in terms of the quark amplitudes defined by (2.9). The transversity amplitudes are used throughout.

$$\pi^+ p \to p\pi^+$$

$$T_{++} = -\frac{2}{3}B_3 - \frac{2}{9}A_3 - \frac{25}{9}A_1$$

$$T_{--} = \frac{2}{3}A_3 + \frac{2}{9}B_3 + \frac{25}{9}B_1$$

$$\pi^- p \to p\pi^-$$

$$T_{++} = -2B_3 - \frac{2}{3} A_3$$

$$T_{--} = 2A_3 + \frac{2}{3} B_3$$

$$\pi^- p \to N^{*+}\pi^-$$

$$T_{{}^1/_2{}^1/_2} = \frac{2\sqrt{2}}{3} A_3 - \sqrt{2}\, B_3$$

$$T_{{}^1/_2 -{}^3/_2} = \sqrt{\frac{2}{3}}\, a_3$$

$$T_{-{}^1/_2 -{}^1/_2} = \frac{2\sqrt{2}}{3} B_3 - \sqrt{2}\, A_3$$

$$T_{-{}^1/_2{}^3/_2} = \sqrt{\frac{2}{3}}\, b_3$$

$$\pi^+ p \to N^{*+}\pi^+$$

$$T_{{}^1/_2{}^1/_2} = -\frac{4\sqrt{2}}{9} A_3 - \frac{5\sqrt{2}}{9} A_1 + \frac{2\sqrt{2}}{3} B_3$$

$$T_{{}^1/_2 -{}^3/_2} = -\frac{2}{3}\sqrt{\frac{2}{3}}\, a_3 + \frac{5}{3}\sqrt{\frac{2}{3}}\, a_1$$

$$T_{-{}^1/_2 -{}^1/_2} = -\frac{4\sqrt{2}}{9} B_3 - \frac{5\sqrt{2}}{9} B_1 + \frac{2\sqrt{2}}{3} A_3$$

$$T_{-{}^1/_2{}^3/_2} = -\frac{2}{3}\sqrt{\frac{2}{3}}\, b_3 + \frac{5}{3}\sqrt{\frac{2}{3}}\, b_1$$

The amplitudes for other processes can be calculated from isospin invariance:

$$T(\pi^- p \to n\pi^0) = \frac{1}{\sqrt{2}} [T(\pi^+ p \to p\pi^+) - T(\pi^- p \to p\pi^-)]$$

$$T(\pi^- p \to N^{*-}\pi^+) = \sqrt{3}\, [T(\pi^- p \to N^{*+}\pi^-) + T(\pi^+ p \to N^{*+}\pi^+)]$$

$$T(\pi^- p \to N^{*0}\pi^0) = -\frac{1}{\sqrt{2}} [2T(\pi^- p \to N^{*+}\pi^-) + T(\pi^+ p \to N^{*+}\pi^-)]$$

$$T(\pi^+ p \to N^{*++}\pi^0) = -\sqrt{\frac{3}{2}}\, T(\pi^+ p \to N^{*+}\pi^+).$$

TABLE

PREDICTIONS					INPUT		
$\pi^- p \to N^{*-}\pi^+$	$\pi^- p \to N^{*0}\pi^0$	$\pi^- p \to N^{*+}\pi^-$	$\pi^+ p \to N^{*++}\pi^0$	$\pi^+ p \to N^{*+}\pi^+$	$\pi^+ p \to p\pi^+$	$\pi^- p \to p\pi^-$	$\pi^- p \to n\pi^0$
35.5 ± 2	7.5 ± 0.5	0.7 ± 0.1	14.5 ± 2	10 ± 1.5	39 ± 2	5.3 ± 0.8	14.5 ± 3 [6], [8]
34.5 ± 2	8.5 ± 0.5	0.7 ± 0.1	11.5 ± 1.5	7.5 ± 1	39 ± 2	5.3 ± 0.8	10 ± 2 [7]

Predictions for the cross-section (in μb/GeV^2) in backward N^* production at 6 GeV/c.
The input data for $\pi^\pm p$ elastic scattering are taken from Ref. [5].
The data for $\pi^- \to \pi^0 n$ are taken from Refs [6, 7, 8] (as indicated in the Table).
The effect of $u_{\min} \neq 0$ is not included in the estimate of errors.

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