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RELATIONS BETWEEN THE DIFFERENTIAL CROSS-SECTIONS IN THE REGION OF THE SECOND DIFFRACTION MAXIMUM IN THE SU_6 QUARK MODEL

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The assumption of antiquark-quark scattering dominance in the formation of the second diffraction maximum is used to obtain relations between the differential cross-sections in the angular region of the secondary maximum in the framework of the additive quark model with spin. Some of the relations are compared with experimental data.

1. Recently Akhiezer and Rekalo [1] have proposed a description of two body hadron-hadron scattering in the region of the second diffraction maximum in the framework of the additive quark model. The assumption with which they supplement the usual quark model is that the formation of the second maximum in the angular distribution is dominated by antiquark-quark scattering. This explains in a natural way the absence of diffraction stucture in proton-proton elastic scattering and at the same time the appearance of a marked second maximum in antiproton-proton and pion-proton elastic and charge exchange differential cross-sections[1].

When the quark-quark contributions to the second maximum are neglected, many relations between second maximum cross-sections can be obtained immediately. Obviously, all the relations derived in the usual quark model still hold under this assumption. A number of sum rules relating second maximum cross-sections have been derived in reference [1] with the use of the SU_3 quark model.

In this paper we extend the model of Akhiezer and Rekalo by taking into account the spin dependence of quark-quark amplitudes. The technique we employ is the standard SU_6 quark model [2]. This extension enables one to relate cross-sections for scalar and vector meson production. Also predictions involving spin density matrix elements can be made in this version of the model.

2. By taking into account the spin dependence of quark-quark amplitudes additional parameters are introduced which have to be eliminated in order to obtain the parameter-free

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[1] See the discussion in Sec. *4*.

relations between the differential cross-sections. In general, this makes the resulting sum rules much more complicated. In our case however, the assumption of antiquark-quark scattering dominance reduces seriously the number of parameters entering the expressions for the differential cross-sections and the obtained relations are quite simple in form.

Below we present a list of relations between second maximum cross-sections obtained under the assumption of antiquark-quark scattering dominance. As mentioned earlier, they supplement the list of relations derived in the framework of the usual quark model with spin. For the complete set of such relations we refer the reader to the paper of Białas and Zalewski [3].

$$\sigma(\pi^- p \to \pi^- p) = \sigma(K^- p \to K^- p) \tag{1}$$

$$\sigma(\pi^- p \to \varrho^- p) = \sigma(K^- p \to K^{*-} p) \tag{2}$$

$$\sigma(\pi^- p \to \pi^0 n) = \frac{1}{2}\,\sigma(K^- p \to \bar{K}^0 n) \tag{3}$$

$$\sigma(\pi^- p \to \pi^\circ n) = \sigma(\pi^- p \to \eta n) + \sigma(\pi^- p \to \eta' n) \tag{4}$$

$$\sigma(\pi^- p \to \varrho^0 n) = \frac{1}{2}\,\sigma(K^- p \to \bar{K}^{*0} n) \tag{5}$$

$$\sigma(\pi^- p \to \varrho^\circ n) = \sigma(\pi^- p \to \omega n) + \sigma(\pi^- p \to \varphi n) \tag{6}$$

$$\sigma(\pi^+ p \to \pi^0 \Delta^{++}) = \frac{3}{2}\,\sigma(K^- p \to \bar{K}^0 \Delta^0) \tag{7}$$

$$\sigma(\pi^+ p \to \pi^\circ \Delta^{++}) = \sigma(\pi^+ p \to \eta \Delta^{++}) + \sigma(\pi^+ p \to \eta' \Delta^{++}) \tag{8}$$

$$\sigma(\pi^+ p \to \varrho^0 \Delta^{++}) = \frac{3}{2}\,\sigma(K^- p \to \bar{K}^{*0} \Delta^0) \tag{9}$$

$$\sigma(\pi^+ p \to \varrho^\circ \Delta^{++}) = \sigma(\pi^+ p \to \omega \Delta^{++}) + \sigma(\pi^+ p \to \varphi \Delta^{++}). \tag{10}$$

Other sum rules can be generated by combining relations (1)—(10) with those of reference [3].

We quote here as examples:

$$\sigma(\bar{p}p \to \bar{n}n) = 2\sigma(\pi^- p \to \pi^0 n) + \frac{50}{9}\,\sigma(\pi^- p \to \varrho^0 n) \tag{11}$$

$$\sigma(\bar{p}n \to \overline{\Delta^{++}} p) = \frac{16}{3}\,\sigma(\pi^- p \to \varrho^0 n) \tag{12}$$

$$\sigma(\bar{p}p \to \overline{\Delta^{++}} \Delta^{++}) = \frac{16}{3}\,\sigma(\pi^+ p \to \varrho^0 \Delta^{++}). \tag{13}$$

Relations (1)—(10) hold also when the differential cross-sections are replaced by the corresponding $\sigma_{ik} = \sigma\varrho_{ik}$ where ϱ_{ik} are the spin density matrix elements [4].

Out of the sum rules quoted above, the relation (1) has been derived already for the secondary maxima by Akhiezier and Rekalo in the framework of the SU_3 quark model [1]. The other part of their relation (1) connecting antiproton-proton to meson-proton scattering does not hold, however, without additional assumptions about antiquark-quark amplitudes, when spin of quarks is taken into account.

3. Experimental data in the region of $-t \approx 1\ (\text{GeV}/c)^2$ are still very inaccurate and do not allow to draw definite conclusions about the applicability of the model. In most cases it is still impossible to decide whether there is a diffraction structure in the angular distribution or not. Notwithstanding, we have used to test relations (1), (3), (4), (10) and (11) the data from the region: $0.5 \leqslant -t \leqslant 1.5\ (\text{GeV}/c)^2$.

First we consider equation (1). The data on elastic π^-p, K^-p and $\bar{p}p$ scattering show that the secondary maxima tend to disappear with increasing energy. This suggests that the the "non-peaked background" which is essentially energy independent and contains the contributions both from antiquark-quark and quark-quark amplitudes (high-energy limit or Pomeranchukon contribution) cannot be neglected in the discussion of secondary peaks in elastic scattering. In order to derive useful results from the model in this case, one has to make the assumption that there is no interference between this Pomeranchukon background and antiquark-quark amplitudes responsible for the secondary peak. We shall adopt tentatively this assumption.

Looking at the data at 5.9 GeV/c [5] one can see that at $-t \approx 1\ (\text{GeV}/c)^2$ there is an anomaly both in the K^-p and π^-p elastic angular distributions. In this region both cross-sections are of the same order of magnitude, although the π^-p one is appreciably greater than that for K^-p. The two plots approach each other with increasing $|t|$ and intersect at $-t \approx 2(\text{GeV}/c)^2$. However, if we subtract the background under the second maxima the heights of the two bumps become equal within the limits of experimental errors. This indicates that relation (1) agrees with the experimental data in this angular region. At higher energies the structure is no more visible and the differential cross-sections at $-t \approx 0.8$–$0.9\ (\text{GeV}/c)^2$ are equal within the limits of experimental errors in both cases [6].

In order to compare equation (3) with the data of references [7] and [8] we have calculated the ratio

$$k = \frac{\sigma_{\text{II}}(\pi^-p \to \pi^0 n)}{\sigma_{\text{II}}(K^-p \to \bar{K}^0 n)} \tag{14}$$

where [7]

$$\sigma_{\text{II}} = \int_{-1.5}^{-0.5} dt\sigma(t). \tag{15}$$

The data on $K^-p \to \bar{K}^0 n$ are available at 5, 7.1, 9.5 and 12.3 GeV/c [8]. To take into account the energy dependence of the σ_{II} we have extrapolated the data for $\pi^-p \to \pi^0 n$ from 4.83, 5.85 and 13.3 GeV/c using the formula [7]:

$$\sigma_{\text{II}}(\pi^-p \to \pi^0 n) \propto p^{02.65}. \tag{16}$$

The estimated values of k are:

$$k = 0.8 \pm 0.3 \text{ at } 5.0 \text{ GeV}/c$$

$$k = 0.5 \pm 0.2 \text{ at } 7.1 \text{ GeV}/c$$

$$k = 0.3 \pm 0.2 \text{ at } 9.5 \text{ GeV}/c$$

$$k = 0.5 \pm 0.2 \text{ at } 12.3 \text{ GeV}/c.$$

At the same time, the analogous ratio with σ_{II} replaced in equation (14) by (cf. reference [7]).

$$\sigma_{\text{I}} = \int_{-0.5}^{0} dt \sigma(t) \tag{17}$$

takes on values:

$$k' = 0.9 \pm 0.2 \text{ at } 5.0 \text{ GeV}/c$$

$$k' = 0.8 \pm 0.2 \text{ at } 7.1 \text{ GeV}/c$$

$$k' = 0.8 \pm 0.2 \text{ at } 9.5 \text{ GeV}/c$$

$$k' = 1.0 \pm 0.2 \text{ at } 12.3 \text{ GeV}/c.$$

The extrapolation has been performed according to the same formula (16) with the exponent -1.17 instead of -2.65 [7].

If we disregard the problem of the background from the forward peak we find the agreement with the prediction $k = 0.5$ rather satisfactory.

In the case of relation (10) the ratio

$$k = \frac{\sigma_{\text{II}}(\pi^+ p \to \varrho^0 \Delta^{++})}{\sigma_{\text{II}}(\pi^+ p \to \omega \Delta^{++})} \tag{18}$$

with σ_{II} defined by equation (15) assumes the values:

$$k = 1.0 \pm 0.4 \text{ at } 4 \text{ GeV}/c \text{ [9]}$$

$$k = 1.2 \pm 0.7 \text{ at } 8 \text{ GeV}/c \text{ [10]}.$$

At the same time, the analogous ratios of the rotal cross-sections are: 1.6 (no errors have been given in reference [9]) and 2.9 ± 0.6 respectively.

Since the cross-section for $\pi^+ p \to \varphi \Delta^{++}$ is known to be very small [11] we conclude that relation (10) is consistent with the experimental data.

There exist comparatively good data on the reaction $\pi^- p \to \eta n$ in the region of the second maximum [12]. However, there are no data concerning the η' production in $\pi^- p$ collisions. We have taken $\sigma_{\text{II}}(\pi^- p \to \eta n) = 4.7 \sigma_{\text{II}}(\pi^- p \to \eta' n)$ which is suggested by the data on $\pi^+ p$ at 3.7 GeV/c [11]. The values of the ratio

$$k = \frac{\sigma_{\text{II}}(\pi^- p \to \pi^0 n)}{\sigma_{\text{II}}(\pi^- p \to \eta n) + \sigma_{\text{II}}(\pi^- p \to \eta' n)} \tag{19}$$

obtained in this way are:

$$k = 1.5 \begin{matrix} +1.0 \\ -0.5 \end{matrix} \text{ at } 3.7 \text{ GeV}/c$$

$$k = 0.7 \begin{matrix} +0.5 \\ -0.2 \end{matrix} \text{ at } 5.9 \text{ GeV}/c$$

$$k = 0.6 \begin{matrix} +0.9 \\ -0.2 \end{matrix} \text{ at } 13.3 \text{ GeV}/c.$$

These values do not exclude the possibility of $k = 1$ as predicted by equation (4), but in view of large experimental uncertainties they do not constitute any serious argument for our prediction.

Relation (11) which links meson-baryon and antibaryon-baryon processes seems to disagree with experimental data due to the relatively large $\pi^- p \to \varrho^0 n$ contribution. This is suggested by the data on $\pi^- p \to \pi^0 n$ at 4.83 and 5.85 GeV/c [7] together with the data on $\pi^- p \to \pi^0 n$ at 4 GeV/c [13] and 8 GeV/c [14] and on $\bar{p}p \to \bar{n}n$ at 5–9 GeV/c [15]. Correcting the energies taken for the comparison for different Q-values and for the difference in projectile masses does not improve the agreement.

It is worth noting, that in reference [3] an analogous relation for near forward angles is also found to be violated. Therefore it is very likely that the discrepancy suggested by the data in the case of our relation (11) merely reflects the general tendency observed for the quark model relations linking meson-baryon and baryon-baryon cross-sections. Nevertheless, the data in the region of the secondary maximum are very inaccurate and the situation needs further experimental clarification.

4. The scarcity of experimental data in the region of the secondary maximum and large experimental errors make our considerations of the previous section rather speculative. Nevertheless, as seen from the arguments of the previous section, relations following from the assumption of antiquark-quark scattering dominance in the formation of the secondary maximum are compatible with the experimental data, as far as meson-baryon processes only are involved. On the other hand, the well known difficulties of the quark model in relating meson-baryon and baryon-baryon cross-sections [3] seem to be reproduced also in this modification of the model.

The attractive feature of the model proposed by Akhiezier and Rekalo is that it offers a very simple explanation of the presence or absence of secondary maxima in angular distributions. However, as pointed out by Morrison the K^+p elastic differential cross-section in the incident momentum region 3–4 GeV/c does not show any structure in the vicinity of $t = -0.5$ (GeV/c)^2 [16]. It implies that the secondary maximum has to be absent in $\bar{q}_\lambda q_p$ amplitudes entering the expression for K^+p elastic cross-section. This contradicts the tentative assumption made in reference [1] that all antiquark-quark amplitudes exhibit the secondary maxima[2]. The general idea of the model is not violated, however, by this observation.

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[2] It is predicted in reference [1] that the secondary structure in K^+p elastic scattering will be less pronounced than in pion-proton scattering due to weaker coupling of the strange quark.

The observation made above suggests that the secondary maxima appear only in the processes for which it is possible to draw an allowed Harrari-Rosner diagram[3]. It is therefore tempting to assume that the secondary structure will not appear also in other K^+N processes, like *e.g.* $K^+p \to K^{*+}p$. This does not follow directly from the model, because the antiquark-quark amplitudes with different spin structure may behave differently.

Finally, let us remark that the model does not apply to the description of the secondary structure in proton-proton elastic scattering which has a tendency to show up at very high energy [18]. The energy dependence of this structure suggests that it is rather due to different mechanism (*e.g.* the Chou-Yang model [19]).

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[3] The existence of the relation between duality and the Akhiezier-Rekalo model was pointed out by Prof. H. J. Lipkin in the discussion at the Lund Conference [17].