AMBIGUITIES IN THE EXPERIMENTAL DETERMINATION OF RESONANCE PRODUCTION AMPLITUDES

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Experiments with unpolarized particles determine only partially the scattering amplitudes. For reactions $0^-\frac{1}{2}^+ \rightarrow 0^-\frac{3}{2}^+$ and $0^-\frac{1}{2}^+ \rightarrow 1^-\frac{3}{2}^+$ we give the most general transformation of the amplitudes, which leaves the experimental results unchanged. Implications for the quark model are also discussed.

1. Introduction

In this paper we discuss the transversity amplitudes for the reactions

$$PB \rightarrow PB^* \tag{1}$$

and

$$PB \rightarrow VB^*. \tag{2}$$

Here P denotes a pseudoscalar meson, B denotes a $1/2^+$ baryon, B^* is a $3/2^+$ isobar and V stands for a vector meson. The problem is stated as follows. What can be said about the transversity amplitudes in the present experimental situation *i. e.* without the measurements on polarized targets and without polarization measurements for the final particles.

Our discussion for each reaction consists of two parts. First, without making any dynamical assumption, we propose a simple formalism. Then we assume that double transversity flip amplitudes vanish. This assumption seems to be consistent with the experiment.

Further, we use our formalism to settle which of the formulae given by the quark model for the transversity amplitudes are supported by experiment and discuss an extension of the quark model proposed recently by Lipkin [1].

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In Section 2 we consider the processes (1). The formalism in this case is very simple. We present it in some detail, however, in order to prepare for the more complicated case of the reactions (2), which are investigated in Section 3. Finally, Section 4 contains our conclusions.

2. *Reactions $PB \to PB^*$*

For a reaction $PB \to PB^*$ the most general set of parity conserving amplitudes is

$$\mathrm{T} = \begin{pmatrix} 0 & \bar{F} \\ f_0 & 0 \\ 0 & \bar{f}_0 \\ \mathrm{F} & 0 \end{pmatrix}. \tag{3}$$

Here the columns are labelled by the transversities of the initial baryon and the rows by the transversities of the final isobar. Thus *e. g.* F is the amplitude for the transition of a baryon with transversity $+1/2$ into an isobar with transversity $-3/2$. This set of amplitudes corresponds to eight real independent parameters, each of them a function of the Mandelstam variables s and t.

In experiments with unpolarized targets four real quantities are measured [2] *e. g.* in the transversity frame: the differential cross-section, the real statistical tensor T_0^2 and the complex statistical tensor T_2^2.

For further reference it is convenient to introduce the two-dimensional complex vectors

$$\hat{e} = \begin{pmatrix} f_0 \\ \bar{f}_0^* \end{pmatrix} \quad \text{and} \quad \hat{g} = \begin{pmatrix} F \\ \bar{F}^* \end{pmatrix} \tag{4}$$

where the asterisk means complex conjugation. All the measurable quantities can be expressed in terms of these vectors as follows:

$$\frac{d\sigma}{dt} = \hat{e}^+\hat{e} + \hat{g}^+\hat{g} \equiv N,$$

$$T_0^2 = \frac{1}{2N}(-\hat{e}^+\hat{e} + \hat{g}^+\hat{g}),$$

$$T_2^2 = \frac{1}{N}\,\hat{e}^+\hat{g}. \tag{5}$$

Here the superscript $+$ means hermitian conjugation.

It is easily seen that the measurable quantities are invariant under the simultaneous transformations

$$\hat{e} \to U\hat{e} \quad \text{and} \quad \hat{g} \to U\hat{g} \tag{6}$$

where U is an arbitrary 2×2 unitary matrix. Such a matrix is parametrized by four independent real parameters. These parameters can be functions of s and t. One possible parametrization is

$$U = e^{i\alpha}\Phi_1 B \Phi_2,$$

$$\Phi_k = \begin{pmatrix} e^{i\varphi_k} & 0 \\ 0 & e^{-i\varphi_k} \end{pmatrix}, \quad B = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}. \tag{7}$$

The transformation Φ_1 introduces an overall phase, which, even in polarization experiments, cannot be measured.

The transformation

$$\hat{e} \to e^{i\alpha}\hat{e} \qquad \hat{g} \to e^{i\alpha}\hat{g} \tag{8}$$

corresponds to the following transformation of the amplitudes

$$f_0 \to e^{i\alpha} f_0 \qquad F \to e^{i\alpha} F$$

$$\bar{f}_0 \to e^{-i\alpha} \bar{f}_0 \qquad \bar{F} \to e^{-i\alpha} \bar{F}. \tag{8a}$$

This has a simple physical interpretation. As long as the spin density matrix for the initial baryon is diagonal, the scattering amplitudes for spin-up and spin-down baryons add incoherently. Consequently their relative phase is not measurable. In the framework of the quark model transformation (8) corresponds to a change of the additivity frame[1]. 2α is the angle between the additivity frame and the transversity frame for the initial baryon, assuming that the transversity frame is used for the final isobar. The parameters α, β and φ_2 can in general be obtained from experiments with polarized particles. In particular, ignoring the overall phase, it is possible to fix β and φ_2 from experiment with particles polarized perpendicularly to the reaction plane.

The freedom of the transformation (7) enables one to choose the parametrization of the amplitudes in such a way that all the parameters can be in principle determined from experiment with an unpolarized target. The choice of parametrization is of course not unique.

Let us now assume that the transitions with double transversity flip are forbidden:

$$F = \bar{F} = 0. \tag{9}$$

In terms of the statistical tensors this assumption is equivalent to

$$T_0^2 = -1/2 \tag{10}$$

and implies also the equality

$$T_2^2 = 0. \tag{11}$$

This gives the Stodolsky-Sakurai distribution which is consistent with experiment (for reference, see *e. g.* [4]). Now the only remaining measurable quantity is the differential cross-

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[1] For a discussion of this problem and references to earlier work see Ref. [3].

-section, which reduces to

$$\frac{d\sigma}{dt} = \hat{e}^{+}\hat{e}. \tag{12}$$

Choosing properly the parameters of transformation (7) we can always get

$$f_0 = \bar{f}_0 \, . \tag{13}$$

The relations (9) and (13) are the predictions of the additive quark model in which the additivity and transversity frames coincide [4]. We conclude that experiment is consistent with (13), however, it is also consistent with any pair of amplitudes obtained from (13) applying the transformation (7). The freedom of phases is usually incorporated in the model but there remains the untested prediction $\beta = 0$.

Recently Lipkin [1] suggested that the quark amplitudes consistent with (13) should be multiplied by formfactors which are arbitrary functions of the initial spin projections. For reaction (1) this is equivalent with our transformation, which gives the most general amplitudes consistent with (9).

3. *Reactions* $PB \to VB^*$

For the reaction $PB \to VB^*$ the most general set of transversity amplitudes consistent with parity conservation reads:

$$T = \left\| \begin{array}{cc} -f_7 & 0 \\ 0 & -\bar{f}_7/\sqrt{3} \\ f_6/\sqrt{3} & 0 \\ 0 & \bar{f}_6 \\ \hline 0 & \bar{F} \\ -f_0/\sqrt{6} & 0 \\ 0 & -\bar{f}_0/\sqrt{6} \\ F & 0 \\ \hline f_5 & 0 \\ 0 & \bar{f}_5/\sqrt{3} \\ -f_8/\sqrt{3} & 0 \\ 0 & -\bar{f}_8 \end{array} \right\| . \tag{14}$$

Here the columns are labelled by the transversities of the initial baryon. The groups of elements separated by the dashed lines correspond to the meson transversities +1, 0 and —1 respectively. Within each group the rows are labelled by the transversities of the isobar. Thus *e. g.* f_5 is the amplitude for the transition of a baryon with transversity +1/2 and a pseudoscalar meson into an isobar with transversity +3/2 and a vector meson with transversity —1. The amplitudes for the production of mesons with zero transversity are denoted by the same symbols which were used in formula (3). This does not mean that the corresponding amplitudes are equal.

Like in the preceding section it is convenient to define a set of two-dimensional vectors

$$\hat{a} = \begin{pmatrix} f_5 \\ \bar{f}_6^* \end{pmatrix} \quad \hat{b} = \begin{pmatrix} f_6 \\ \bar{f}_5^* \end{pmatrix} \quad \hat{c} = \begin{pmatrix} f_7 \\ \bar{f}_8^* \end{pmatrix}$$

$$\hat{d} = \begin{pmatrix} f_8 \\ \bar{f}_7^* \end{pmatrix} \quad \hat{e} = \begin{pmatrix} f_0 \\ \bar{f}_0^* \end{pmatrix} \quad \hat{g} = \begin{pmatrix} F \\ \bar{F}^* \end{pmatrix}. \tag{15}$$

All the quantities to be measured in an experiment with an unpolarized target can be expressed in terms of the scalar products of these vectors. The list of the measurable quantities and the detailed formulae can be found in the Appendix.

It can be seen that only the following combinations of the vectors (15) are measurable:

$$\begin{gathered} \hat{a}^+\hat{a} + \hat{c}^+\hat{c}, \quad \hat{b}^+\hat{b} + \hat{d}^+\hat{d}, \quad \hat{e}^+\hat{e}, \quad \hat{g}^+\hat{g}, \\ \hat{c}^+\hat{a}, \quad \hat{b}^+\hat{d}, \quad \hat{c}^+\hat{d}, \quad \hat{b}^+\hat{a}, \quad \hat{e}^+\hat{g}, \\ \hat{c}^+\hat{b} + \hat{a}^+\hat{d}, \quad \hat{c}^+\hat{e} + \sqrt{2}\,\hat{b}^+\hat{g}, \quad -\hat{e}^+\hat{a} + \sqrt{2}\,\hat{g}^+\hat{d}. \end{gathered} \tag{16}$$

This gives twenty real numbers for every s and t.

The combinations (16) obviously do not change when each vector is transformed by the matrix defined in Eq. (7). Thus the transversity amplitudes for a reaction (2) can be fully determined from experiment if all the parameters entering the transformation (7) *i. e.* if $\alpha(s, t)$, $\beta(s, t)$, $\varphi_1(s, t)$, and $\varphi_2(s, t)$ are fixed. The interpretation of the transformation remains unchanged.

Let us assume that also for the reactions $PB \to VB^*$ the double transversity flip amplitudes vanish *i. e.*

$$F = \bar{F} = 0. \tag{17}$$

This assumption can be directly checked by measuring the combination of statistical tensors

$$T_{00}^{22} - \frac{1}{\sqrt{2}} T_{00}^{02} + T_{00}^{20} - \frac{1}{2\sqrt{6}} \sim \hat{g}^+\hat{g}. \tag{18}$$

The conjecture $\hat{g}^+\hat{g} = 0$ equivalent to (17) is well supported by experiment (see Ref. [4]). The equality (17) implies another relation

$$T_{02}^{02} = \frac{1}{\sqrt{2}} T_{02}^{22} \tag{19}$$

which is also consistent with experiment [4]. Thus there remain apparently seventeen real numbers to measure. However, these numbers have to obey one real relation because there exists the freedom of the four-parameter transformation and the nonvanishing amplitudes need only twenty real numbers to be entirely known.

The additive quark model contains the assumption (17) but it predicts also the following equalities among the transversity amplitudes

$$f_i = \bar{f}_i, \quad (i = 0, 5, 6, 7, 8). \tag{20}$$

The resulting class (a) relations among the statistical tensors seem to be consistent with experiment [5]. The experiment is of course consistent also with any set of amplitudes obtained from (17) and (20) by applying the transformation (7).

We conclude that if the class (a) relations are valid for the reactions $PB \to VB^*$, the general amplitudes can be obtained from the quark model amplitudes introducing two additional functions $\beta(s, t)$ and $\varphi_2(s, t)$. The remaining degrees of freedom of the transformation (7) *i.e.* the overall phase and orientation of the additivity frame are usually incorporated in the quark model.

Lipkin's suggestion [1] implies

$$f_i \to \lambda f_i, \quad \bar{f}_i \to \mu \bar{f}_i. \tag{21}$$

If

$$|\lambda| = |\mu| = 1 \tag{22}$$

this is a particular case of our transformation with $\beta = 0$. If (22) is not fulfilled, (21) does not reduce to transformation (7), and experimentally deviations from the class (a) relations predicted by the quark model should be observed. We assumed that the differential cross-sections are given correctly both before and after transformation satisfying (22). This excludes $|\lambda| = |\mu| \neq 1$.

4. *Conclusions*

1. In experiments with unpolarized particles the amplitudes for the reactions $PB \to PB^*$ and $PB \to VB^*$ cannot be determined unambiguously. We have found the most general transformation of the amplitudes, which leaves all the measurable quantities unchanged. It is the four-parameter transformation given by formula (7). The freedom of this transformation makes it possible to choose in the framework of any dynamical model the most suitable parametrization of the transversity amplitudes.

2. It is possible to test directly the quark model prediction about the vanishing of the double transversity flip amplitudes. This can be done in a model independent way by measuring certain linear combinations of statistical tensors (Eqs (10) and (18)). Here experiment supports the model.

3. For reactions (1) and (2) the amplitudes given by the quark model seem to be one possible set consistent with present day experimental data [5]. According to our discussion any other set of amplitudes obtained from this set by applying transformation (7) is also consistent with experiment. Transformation (7) depends on four functions of energy and momentum transfer. The overall phase $\varphi_1(s, t)$ is irrelevant. The function $\alpha(s, t)$ is incorporated into the quark model. It fixes the orientation of the additivity frame. According to the quark model the function $\beta(s, t)$ should vanish. This is the only prediction of the model, which can be tested in experiments with polarized particles only. If $\beta = 0$, the function $\varphi_2(s, t)$ can be absorbed into the function $\varphi_1(s, t)$.

4. The formfactors proposed by Lipkin [1] are sufficient to obtain the general amplitudes consistent with the vanishing of double spin flip amplitudes for reactions $PB \to PB^*$. For reactions $PB \to VB^*$ if the class (a) relations are valid, Lipkin's formfactors have to reduce to those already inherent in the usual version of the quark model.

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APPENDIX

We list here the formulae relating the statistical tensors of the VB^* system to the vectors defined by Eq. (15). Only the tensors measurable in experiments with unpolarized particles are included.

$$\frac{d\sigma}{dt} = \frac{1}{12}[6(\hat{a}^+\hat{a}+\hat{c}^+\hat{c})+2(\hat{b}^+\hat{b}+\hat{d}^+\hat{d})+\hat{e}^+\hat{e}+6\hat{g}^+\hat{g}] \equiv N, \tag{A1}$$

$$T^{02}_{00} = \frac{1}{24\sqrt{3}N}[6(\hat{a}^+\hat{a}+\hat{c}^+\hat{c})-2(\hat{b}^+\hat{b}+\hat{d}^+\hat{d})-\hat{e}^+\hat{e}+6\hat{g}^+\hat{g}], \tag{A2}$$

$$T^{20}_{00} = \frac{1}{12\sqrt{6}N}[3(\hat{a}^+\hat{a}+\hat{c}^+\hat{c})+(\hat{b}^+\hat{b}+\hat{d}^+\hat{d})-\hat{e}^+\hat{e}-6\hat{g}^+\hat{g}], \tag{A3}$$

$$T^{22}_{00} = \frac{1}{12\sqrt{6}N}[3(\hat{a}^+\hat{a}+\hat{c}^+\hat{c})-(\hat{b}^+\hat{b}+\hat{d}^+\hat{d})+\hat{e}^+\hat{e}-6\hat{g}^+\hat{g}], \tag{A4}$$

$$T^{02}_{02} = -\frac{1}{12N}[\sqrt{2}(\hat{a}^+\hat{d}+\hat{c}^+\hat{b})+\hat{e}^+\hat{g}], \tag{A5}$$

$$T^{22}_{02} = -\frac{1}{12N}[(\hat{a}^+\hat{d}+\hat{c}^+\hat{b})-\sqrt{2}\hat{e}^+\hat{g}], \tag{A6}$$

$$T^{20}_{20} = -\frac{1}{12N}[3\hat{c}^+\hat{a}+\hat{b}^+\hat{d}], \tag{A7}$$

$$T^{22}_{20} = -\frac{1}{12N}[3\hat{c}^+\hat{a}-\hat{b}^+\hat{d}], \tag{A8}$$

$$T^{22}_{22} = \frac{1}{2\sqrt{6}N}\hat{c}^+\hat{d}, \tag{A9}$$

$$T^{2\,2}_{2-2} = \frac{1}{2\sqrt{6}N}\hat{b}^+\hat{a}, \tag{A10}$$

$$T^{22}_{11} = \frac{1}{4\sqrt{6}N}[\hat{c}^+\hat{e}+\sqrt{2}\hat{b}^+\hat{g}], \tag{A11}$$

$$T^{2\,2}_{1-1} = \frac{1}{4\sqrt{6}N}[-\hat{e}^+\hat{a}+\sqrt{2}\hat{g}^+\hat{d}]; \tag{A12}$$

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