REGGE POLE MODEL WITH VENEZIANO RESIDUES FOR REACTIONS $0^{-}\frac{1}{2}^{+}$ $0^{-}\frac{3}{2}^{+}$

By P. Gizbert-Studnicki, A. Golemo

Institute of Physics, Jagellonian University, Cracow*

And K. Zalewski

Institute of Nuclear Physics, Cracow

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A Regge pole model for the reactions $\pi N \to \pi\Delta$, $\pi N \to \eta\Delta$ and $KN \to K\Delta$ is proposed. The description of the decay distributions and differential cross-sections is good. The model contains six free parameters: the intercepts of the ϱ and A_2 trajectories, a universal slope, the η, η' mixing angle, and two coupling constants. The number of free parameters is kept low by the use of the quark model and of the Veneziano formula for the residues. The predictions for the differential cross-sections of the π^+p processes at 16 GeV/c are also given.

1. Introduction

In this paper we present a Regge-fit for the reactions

$$PB \to P\Delta \tag{1}$$

where P denotes a pseudoscalar meson, B stands for a $\frac{1}{2}^{+}$ baryon and Δ is the $(1236, \frac{3}{2}^{+})$ isobar. For a discussion of earlier fits see Refs [1] and [2].

The additive quark model gives for the decay distribution of the Δ isobar produced in the reactions (1) predictions consistent with experiment. (For a discussion and for references see *e.g.* Ref. [3].) In order to preserve this nice feature we parametrize a set of amplitudes obeying the quark relations.

Assuming a Regge pole parametrization of amplitudes we fit simultaneously the following reactions

$$\pi^+p \to \pi^0\Delta^{++} \tag{2}$$

$$\pi^-p \to \pi^-\Delta^{+} \tag{3}$$

$$\pi^+p \to \eta\Delta^{++} \tag{4}$$

$$K^+p \to K^0\Delta^{++} \tag{5}$$

in terms of exchanged ϱ and A_2 trajectories.

* Address: Instytut Fizyki UJ, Kraków 16, Reymonta 4, Polska.

The momentum-transfer dependence of Regge-pole residues suggested by the Veneziano model [4] was found to work reasonably well in elastic reactions (see *e.g.* Ref. [5]). We adopt such a parametrization for the residues of both trajectories. However, the Veneziano model is not applied here as a definite prescription for constructing the scattering amplitudes. Therefore we do not have to keep the trajectories degenerated. We only take equal slopes.

Further, we conjecture that Regge-couplings are $SU(3)$ symmetric in the framework of the nonet structure for the pseudoscalar mesons. This last assumption enables us to obtain the value of the $\eta-\eta'$ mixing angle from the fit.

The results of the fit indicate that the model proposed here gives in general a satisfactory description of the experimental data. The fit, however, does not work in the case of the reaction

$$K^- n \to \bar{K}^0 \Lambda^- \tag{6}$$

related to (5) by line reversal.

In Section 2 we give the detailed formulation of the model. Section 3 contains the list of experimental data used in the fit. The results of the fit are discussed in Section 4 and finally, Section 5 presents our conclusions.

2. The model

We assume that the additivity of the quark interactions holds in a fixed spin reference frame, in which the spins of both particles are projected on the direction of the momentum of the final isobar[1]. Using formulae from Ref. [7] we can write the helicity amplitudes for the reactions (1) in the following form:

$$H = \sin\vartheta\, a(s,t) \begin{Vmatrix} \sqrt{3}\cos\frac{\vartheta}{2} & \sqrt{3}\sin\frac{\vartheta}{2} \\ -\sin\frac{\vartheta}{2} & \cos\frac{\vartheta}{2} \\ \cos\frac{\vartheta}{2} & \sin\frac{\vartheta}{2} \\ -\sqrt{3}\sin\frac{\vartheta}{2} & \sqrt{3}\cos\frac{\vartheta}{2} \end{Vmatrix}. \tag{7}$$

Here the columns are labelled by the helicities of the initial baryon and the rows by the helicities of the isobar, *e.g.* $\sqrt{3}\sin\vartheta\sin\frac{\vartheta}{2}\, a(s,t)$ is the amplitude for the transition of the initial baryon with helicity $-1/2$ into the final isobar with helicity $+3/2$. ϑ is the

[1] Other choices of the additivity frame give the same results. More generally, from amplitude (7) it is possible to obtain a two-parameter family of amplitudes giving the same predictions for the measured quantities [6].

centre-of-mass scattering angle, *i.e.* the angle between the momenta of the incoming baryon and outgoing isobar, s denotes the total CMS energy squared and t is the square of the four-momentum transfer between baryon and isobar.

The amplitude (7) has a correct kinematical behaviour for forward and backward scattering. It also includes explicitly an additional dynamical zero for forward scattering which results from the additivity assumption. We assume $a(s, t)$ to be free of any singularities on the boundaries of the physical region. We also believe that possible singularities of $a(s, t)$ for $s = 0$ and on the thresholds and pseudothresholds in the s-channel are not important in the analysis of high-energy data.

In order to obtain a definite parametrization of $a(s, t)$ we conjecture that the processes (2)–(5) are dominated by exchange of the t-channel Regge poles. Thus

$$a(s, t) = \sum_r a_r(s, t) \tag{8}$$

where $a_r(s, t)$ is the contribution of a single Regge pole and can be written in the form

$$a_r(s, t) = \beta_r(t) \frac{1+\tau_r e^{-i\pi\alpha_r(t)}}{\sin \pi\alpha_r(t)} \left(\frac{s}{s_0}\right)^{\alpha_r(t)}. \tag{9}$$

Here $\alpha_r(t)$, $\beta_r(t)$ and τ_r are the trajectory, residue and signature of the r-th Regge pole, respectively; s_0 is an energy-scale factor. We assume all the trajectories exchanged to be linear with a universal slope α' *i.e.*

$$\alpha_r(t) = \alpha_r(0) + \alpha' t. \tag{10}$$

However, we do not expect the ϱ and A_2 trajectories to be degenerated, *i.e.* their intercepts do not have to be equal.

The parametrization of the residues is assumed according the Veneziano model[2]:

$$\beta_r(t) = g_r \frac{\pi}{\Gamma(\alpha_r(t))} \tag{11}$$

where g_r is a constant.

Another result of the Veneziano theory is also used here by choosing

$$s_0 = \frac{1}{\alpha'}. \tag{12}$$

Inserting (11) and (12) into (9) we obtain the parametrization of the r-th Regge pole contribution to the scalar amplitude

$$a_r(s, t) = g_r \Gamma(1-\alpha_r)(1+\tau_r e^{-i\pi\alpha_r})(\alpha' s)^{\alpha_r}. \tag{13}$$

Possible contributions from trajectories other than ϱ and A_2 and absorptive corrections are neglected.

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[2] For a discussion and references see *e. g.* [5].

We define the couplings g_ϱ and g_{A_2} as those appropriate to the reaction (5). Assuming $SU(3)$ symmetry we get

$$a(K^+p \to K^0\Delta^{++}) = a_\varrho + a_{A_2} \tag{14a}$$

$$a(\pi^+p \to \pi^0\Delta^{++}) = -\sqrt{2}a_\varrho \tag{14b}$$

$$a(\pi^-p \to \pi^-\Delta^+) = \frac{2}{\sqrt{3}}a_\varrho, \tag{14c}$$

$$a(\pi^+p \to \eta\Delta^{++}) = \sqrt{2}\sin(\theta_0-\theta)a_{A_2}. \tag{14d}$$

Here θ denotes the $\eta-\eta'$ mixing angle and θ_0 is the ideal mixing angle (for conventions used for $SU(3)$ Clebsch-Gordan coefficients see Appendix). The line reversal yields

$$a(K^-n \to \bar{K}^0\Delta^-) = -a_\varrho + a_{A_2}. \tag{14e}$$

The free parameters of the model stated above are $\alpha_\varrho(0)$, $\alpha_{A_2}(0)$, α', g_ϱ, g_{A_2} and $\sin(\theta_0-\theta)$. The normalization of amplitudes is chosen so that the differential cross-section is

$$\frac{d\sigma}{dt} = \frac{1}{8\pi q_i^2}\sin^2\vartheta|a|^2 \tag{15}$$

where q_i is the momentum of the initial baryon in CMS. We consistently use the following units: s in GeV^2, t in (GeV/c)^2, g_ϱ and g_{A_2} in 1/GeV ($\hbar = c = 1$).

3. *Experimental data*

In the fits we use the following set of experimental differential cross-sections.
For $\pi^+p \to \pi^0\Delta^{++}$
$P_{\text{lab}} = 4$ GeV/c: 11 data points from Ref. [8] as quoted in [15].
$P_{\text{lab}} = 8$ GeV/c: 8 data points from Ref. [9].
For $\pi^-p \to \pi^-\Delta^+$
$P_{\text{lab}} = 8$ GeV/c: 17 data points from Ref. [10] as quoted in [1].
For $\pi^+p \to \eta\Delta^{++}$
$P_{\text{lab}} = 3.5$ GeV/c: 7 data points from Ref. [11].
$P_{\text{lab}} = 8$ GeV/c: 7 data points from Ref. [9] as quoted in [16].
For $K^+p \to K^0\Delta^{++}$
$P_{\text{lab}} = 3$ GeV/c: 11 data points from Ref. [12].
$P_{\text{lab}} = 3.5$ GeV/c: 5 data points from Ref. [13].
$P_{\text{lab}} = 5$ GeV/c: 10 data points from Ref. [13] as quoted in [17].
$P_{\text{lab}} = 12.7$ GeV/c: 4 data points from Ref. [14].
Altogether 80 data points.

Only the data on the differential cross-sections are taken, because the quark model adopted here gives automatically the angular decay distribution of the Δ isobar.

4. *Results of the fit*

We determine the free parameters of the model described in Section 2 by a simultaneous, six parameter fit to the data on the reactions (2)–(5). It is an experimental fact (see *e.g.* [18]) that the differential cross-section for the reaction (5) is, near the forward direction, much larger than that for reaction (6). This indicates that $\varrho - A_2$ interference in (14a) should be constructive. The positivity of the interference term is incorporated here as an additional constraint for the parameters of the fit[3].

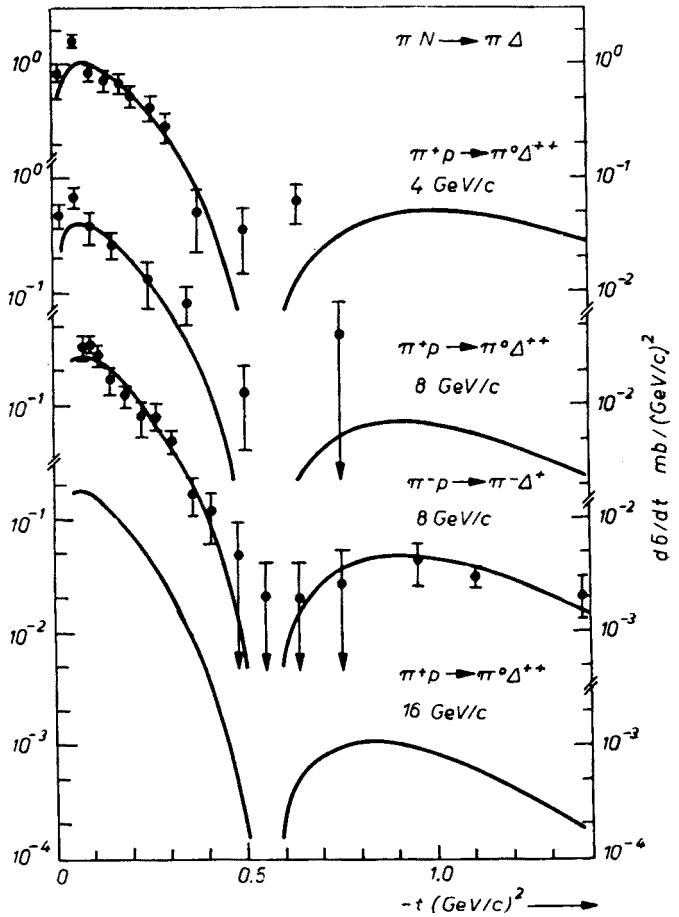

Fig. 1. Differential cross-sections for the processes $\pi N \to \pi \Delta$. The curves for $P_{\text{lab}} = 4$ GeV/c and for $P_{\text{lab}} = 8$ GeV/c are fitted as described in the text. The curve for $P_{\text{lab}} = 16$ GeV/c is the prediction

The fitted cross-sections are compared with experimental data in Figs 1–3. The curves for the $\pi^+ p$ processes at 16 GeV/c are predictions. The agreement is, in general, good. The model satisfactorily describes qualitatively the dips in the forward direction. These dips are an immediate result of the quark additivity assumption. The quantitative deviations,

[3] We tested that it is possible to get a good fit also with negative interference.

visible especially for reaction (2), for $|t| < 0.1$ indicate either that our residue functions are too simple, or that amplitudes with a different spin dependence are admixed for small angle scattering. Anyway, we think that with the present data these deviations would not justify the introduction of a further parameter. From Figs 1–3 it is also visible that the ghost-killing mechanism chosen by Veneziano residues works well producing a dip for $\alpha_\varrho = 0$ in the reactions (2) and (3) and giving a smooth behaviour for reactions (4) and (5).

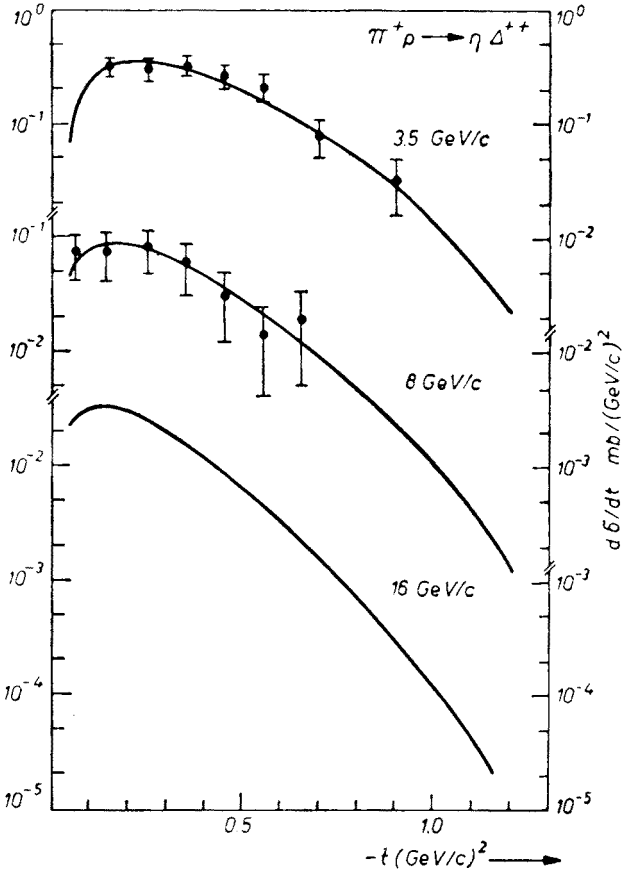

Fig. 2. Differential cross-sections for the process $\pi^+ p \to \eta \Delta^{++}$. The curves for $P_{\mathrm{lab}} = 3.5$ GeV/c and for $P_{\mathrm{lab}} = 8$ GeV/c are fitted as described in the text. The curve for $P_{\mathrm{lab}} = 16$ GeV/c is the prediction

The fit gives the best χ^2($\chi^2 = 91.5$ for the 80 data points, which corresponds to a confidence level of 8 per cent) with the following values of the parameters[4]:

$$\alpha_\varrho(0) = 0.555 \pm 0.010, \quad \alpha_{A_2}(0) = 0.457 \pm 0.029,$$

$$g_\varrho = 5.18 \pm 0.44 \frac{1}{\mathrm{GeV}}, \quad g_{A_2} = 5.34 \pm 0.39 \frac{1}{\mathrm{GeV}},$$

$$\alpha' = 1.027 \pm 0.014 \frac{1}{\mathrm{GeV}^2}, \quad |\sin(\theta_0 - \theta)| = 0.550 \pm 0.035. \tag{16}$$

[4] Three-figure accuracy is given only to allow the reconstruction of the fit.

The numbers in (16) written as errors give correctly only the order of magnitude of possible deviations from the best values of the parameters.

A fit to processes without resonance production [19] gives

$$\alpha_\varrho = 0.566+1.057t,\ \alpha_{A_2} = 0.340+0.350t. \tag{17}$$

Thus our ϱ trajectory coincides with the elastic one, while our A_2 trajectory is somewhat higher and has a larger slope. Both this and the fact that we could get no good fit with

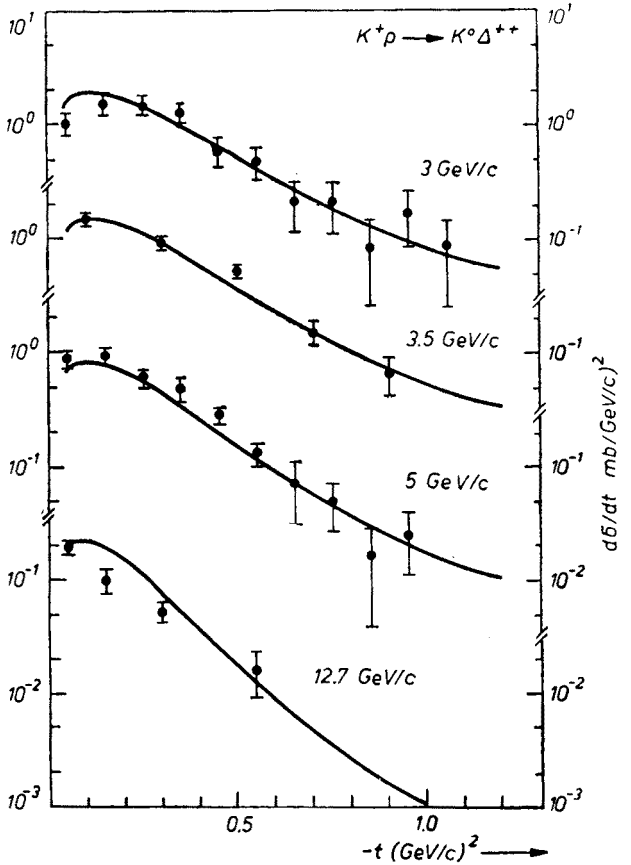

Fig. 3. Differential cross-sections for the process $K^+p \to K^0\Delta^{++}$. The curves are fitted as described in the text

exchange degenerate trajectories, suggest that we deal with some effective trajectories including partly the effect of cuts. It is pleasing that the conjecture of the universal slope, being simultaneously the energy-scale factor, is well supported.

From (16) we get two solutions for the $\eta-\eta'$ mixing angle

$$\theta = 1.9^\circ \pm 2.4^\circ \quad \text{and} \quad \theta = 68.7^\circ \pm 2.4^\circ. \tag{18}$$

On the other hand, the linear Gell-Mann/Okubo mass formula gives $|\theta| = 23.7^\circ \pm 0.3^\circ$, the quadratic GMO formula gives $|\theta| = 10.4^\circ \pm 0.2^\circ$ and the measured ratio $\sigma(\pi^+p \to$

$\to \eta\Delta^{++})/\sigma(\pi^+p \to \eta'\Delta^{++})$ indicates that $\theta = -29.0° \pm 3.3°$. (For recent results and references see [20].) Thus we reject the second solution (18) and conclude that the best fit corresponds to an η being almost purely octet.

The predictions of the model with parameters (16) were also compared with the experimental differential cross-sections for reaction (6) at 3 GeV/c and 5 GeV/c as quoted in [18]. The predicted cross-sections are systematically larger than those from experiment. This difference is especially striking for 3 GeV/c[5]. Also the energy dependence of cross-sections seems to be quite different from the dependence for reaction (5).

The difficulty in correlating line reversed reactions like (5) and (6) in one fit is well known. It has been discussed recently by Auvil *et al.* [22], who found it necessary to include not only broken exchange degeneracy, which we do, but also a secondary, lower, trajectory. Thus the quantitative description of both reaction (5) and (6) at comparatively low energies cannot be done in the framework of a high-energy fit, and is beyond the scope of our model.

5. *Conclusions*

1. A simple model is proposed for isospin exchange $0^{-\frac{1}{2}+} \to 0^{-\frac{3}{2}+}$ reactions. For the isobar the Stodolsky-Sakurai decay distribution is predicted in agreement with experiment. The fits to the differential cross-sections are shown in Figs 1–3. χ^2 for this fit equals 91.5, which for the 80 data points, corresponds to a confidence level of 8 per cent. For reactions (2) and (4) we included in the Figs the predictions for the 16 GeV/c experiment.

2. The model is a Regge pole model with ϱ and A_2 exchange. The trajectories are linear with a universal slope equal to the inverse of the scale parameter s_0. Thus the two trajectories bring in three parameters $\alpha_\varrho(0)$, $\alpha_{A_2}(0)$ and α'. The residue functions are taken from the Veneziano model, thus once the trajectories are fixed, each pole contribution contains only one free parameter — the overall coupling constant.

3. The model is a particular realization of the quark model. Besides the Stodolsky-Sakurai decay distribution this implies $SU(3)$ symmetry for the coupling constants. Thus all the residue functions depend on three further parameters: the coupling constants g_ϱ and g_{A_2} and the $\eta - \eta'$ mixing angle θ. Our best fit corresponds to η almost purely octet ($\theta = 1.9° \pm 2.4°$). Since other estimates of this angle range from about $-10°$ to about $-30°$, our result suggests some $SU(3)$ breaking. This is not conclusive, however, because small corrections *e.g.* from cuts could restore θ to its usual value.

4. The reaction $K^-n \to \overline{K}^0\Delta^-$ is particularly interesting. We predict correctly that its cross-section should be smaller than that for its line reversal reaction $K^+p \to K^0\Delta^{++}$. Quantitatively, however, the measured cross-section is unexpectedly small. Unfortunately there are no high energy data, which could be used to decide whether with increasing energy the effect persists, which would probably correspond to cuts, or disappears, which would correspond to secondary poles.

5. The fit could be made even more impressive by assuming *a priori* that the η is pure

[5] This could be connected with the $\Sigma(2595)$ resonance seen in total KN cross-section near this energy [21].

octet, which is often done, and taking the trajectories from other fits. Thus, formally one could reduce the number of parameters to two: g_ϱ and g_{A_2}.

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APPENDIX

Conventions used for the SU(3) Clebsch-Gordan coefficients

As is well known (*cf. e.g.* [23]) the $SU(3)$ results for vertices can be derived from the simple quark calculations. Putting as usual (*cf. e.g.* [24])

$$\pi^+ = p\bar{n},\ \pi^0 = \frac{1}{\sqrt{2}}(p\bar{p} - n\bar{n}),\ K^+ = p\bar{\lambda},\ K^0 = n\bar{\lambda},$$

$$\eta = \frac{1}{\sqrt{2}}(p\bar{p} + n\bar{n})\sin\psi - \lambda\bar{\lambda}\cos\psi, \tag{A1}$$

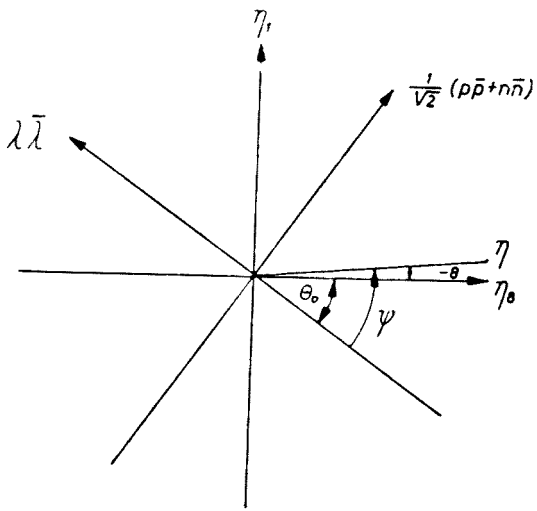

Fig. 4. The $\eta - \eta'$ mixing angles used in Appendix

we obtain

$$A(\pi^+ \to \pi^0) = \frac{1}{\sqrt{2}}[(\bar{n}|\bar{p}) - (p|n)] = -\sqrt{2}A_\varrho \tag{A2}$$

$$A(\pi^+ \to \eta) = \frac{1}{\sqrt{2}}[(\bar{n}|\bar{p}) + (p|n)]\sin\psi = \sqrt{2}A_{A_2}\sin\psi \tag{A3}$$

$$A(K^+ \to K^0) = (p|n) = A_\varrho + A_{A_2}. \tag{A4}$$

This gives the relations (14).

The angle ψ can be related to the more usual mixing angle θ defined by (*cf. e.g.* [25])

$$\eta = \eta_8 \cos\theta - \eta_1 \sin\theta \tag{A5}$$

where

$$\eta_1 = \frac{1}{\sqrt{3}}(p\bar{p}+n\bar{n}+\lambda\bar{\lambda}) \quad \text{and} \quad \eta_8 = \frac{1}{\sqrt{6}}(p\bar{p}+n\bar{n}-2\lambda\bar{\lambda}). \tag{A6}$$

Comparing (A1) and (A5)

$$\psi = \text{arc sin}\frac{1}{\sqrt{3}} - \theta = \theta_0 - \theta = 35.3° - \theta. \tag{A7}$$

The meaning of the angles θ, θ_0 and ψ can be seen from Fig. 4. The combination $\lambda\bar{\lambda}$ is not produced from incident pions. Thus θ negative means that η contains less $\lambda\bar{\lambda}$ and is more copiously produced than if it were pure octet.

REFERENCES

[1] G. H. Renninger, K. V. L. Sarma, *Phys. Rev.*, **178**, 2201 (1968).
[2] M. Krammer, U. Maor, *Nuclear Phys.*, **B13**, 659 (1969).
[3] A. Białas, K. Zalewski, *Nuclear Phys.*, **B6**, 465 (1968).
[4] G. Veneziano, *Nuovo Cimento*, **57A**, 190 (1968).
[5] C. Lovelace, *Nuclear Phys.*, **B12**, 263 (1969).
[6] P. Gizbert-Studnicki, A. Golemo, K. Zalewski, *Acta Phys. Polon.* **B1**, 227 (1970).
[7] A. Kotański, K. Zalewski, *Nuclear Phys.*, **B12**, 72 (1969).
[8] Aachen–Berlin–Birmingham–Bonn–Hamburg–London (I. C.)–München Collaboration, *Phys. Letters*, **10**, 229 (1964) and *Nuovo Cimento*, **34**, 495 (1964).
[9] M. Deutschman, D. Kropp, R. Schulte, H. Weber, W. Woisching, C. Grote, J. Klugow, H. W. Meier, A. Pose, S. Brandt, V. T. Cocconi, O. Czyżewski, P. F. Dalpiaz, E. Flaminio, H. Hromadnik, G. Kellner, D. R. O. Morrison, S. Nowak, *Phys. Letters*, **19**, 608 (1965).
[10] Preliminary data at 8 GeV/c from *BNL-Carnegie Mellon Univ. collaboration*, N. C. Hien, *Bull. Amer. Phys. Soc.*, **13**, 713 (1968).
[11] D. Brown, G. Gidal, R. W. Birge, R. Bacastow, Sun Yiu Fung, W. Jackson, R. Pu, *Phys. Rev. Letters*, **19**, 664 (1967).
[12] M. Ferro-Luzzi, R. George, Y. Goldschmidt-Clermont, V. P. Henri, B. Jongejans, D. W. G. Leith, G. R. Lynch, F. Muller, J. M. Perreau, *Nuovo Cimento*, **36**, 1101 (1965).
[13] Y. Goldschmidt-Clermont, V. P. Henri, B. Jongejans, A. Moisseev, F. Muller, J. M. Perreau, A. Prokes, V. Yarba, W. DeBaere, J. Debaisieux, P. Dufour, F. Grard, J. Henghebaert, L. Pape, P. Peeters, F. Varbeure, R. Windmolders, *Nuovo Cimento*, **46A**, 539 (1966).
[14] J. C. Berlinghieri, M. S. Farber, T. Ferbel, B. E. Forman, A. C. Melissinos, P. F. Slattery, H. Yuta, *Nuclear Phys.*, **B8**, 333 (1968).
[15] M. Krammer, U. Maor, *Nuovo Cimento*, **56A**, 963 (1967).
[16] M. Krammer, *Nuovo Cimento*, **52A**, 931 (1967).
[17] M. Krammer, U. Maor, *Nuovo Cimento*, **52A**, 308 (1967).
[18] Kwan Wu Lai, J. Louie, *Irvine Conference on Regge Poles*, December 1969.
[19] G. V. Dass, C. Michael, R. J. N. Phillips, *Nuclear Phys.*, **B9**, 549 (1969).

[20] W. R. Butler, D. G. Coyne, G. Goldhaber, B. H. Hall, J. N. Mac-Naughton, G. H. Trilling, *paper presented at the Lund International Conf. on Elementary Particles*, June 1969.
[21] N. Barash-Schmidt, A. Barbaro-Galtieri, G. Conforto, L. R. Price, M. Ross, A. H. Rosenfeld, P. Soding, C. G. Wohl, *Rev. Mod. Phys.*, **41**, 109 (1969).
[22] P. R. Auvil, F. Halzen, C. Michael, J. Weyers, *CERN preprint* TH. 1121, January 1970.
[23] H. J. Lipkin, *Nuclear Phys.*, **B1**, 597 (1967).
[24] R. Van Royen, V. F. Weisskopf, *Nuovo Cimento* **50A**, 617 (1967).
[25] S. L. Glashow, *Phys. Rev. Letters*, **11**, 48 (1969).