MULTIMESONIC DECAYS OF CHARMONIUM STATES IN A CONSTANT MATRIX ELEMENT QUARK MODEL

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The presently known experimental data on multimesonic decays of ψ and χ states are fitted in a constant matrix element quark model taking into account also resonances and both strong and second order electromagnetic processes. The known data are well reproduced and the branching ratios for the rest of multimesonic channels are predicted. The fit leaves about 40% for baryonic and radiative channels in the case of J/ψ. The parameters of the J/ψ decays are used to predict the mesonic decays of the pseudoscalar η_c. Some multiparticle production aspects of these decays are also emphasized.

1. *Introduction*

During the last three years a considerable amount of experimental information was collected concerning the decay modes of the charmonium states (for recent reviews of data see Refs. [1–4]). A large number of branching ratios for the $J/\psi(3095)$ state is already known and there are lots of data also for excited charmonium states like $\psi'(3684)$, $\chi(3415)$, $\chi(3505)$, $\chi(3550)$ etc. A general characteristics of the decays is that strong and electromagnetic processes are competing with each other with comparable strength due to the Okubo-Zweig-Iizuka (OZI) [5] rule violation of the strong decays. The majority of decay modes is in multiparticle channels. This is the reason why the identified exclusive decay channels make up only a relatively small fraction of the total width. In the best known case of $J/\psi(3095)$ decays only something like 25% of the decay modes are unambiguously identified (the same number e. g. for $\chi(3415)$ is 15%).

In this situation model calculations incorporating present knowledge and predicting the unknown channels may be useful. A number of authors tried recently to estimate multiparticle branching ratios of the different particles containing charmed quarks using some simple models. The first simple statistical model with constant matrix element for pions was proposed already in the prophetic paper of Gaillard, Lee and Rosner [6]. Other statistical models, usually with constant pion matrix elements or Poissonian multiplicity

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distributions were used in Refs. [7–11]. (In Ref. [7] the embrionic form of our present model was used to predict D and F meson decays.)

In the present paper we use a constant matrix element quark model [12–14] to fit the known mesonic branching ratios of the J/ψ(3095) and χ(3415, 3505, 3550) states and to predict the unobserved ones. We shall consider final states containing pseudoscalar, vector and tensor mesons only resulting from the OZI-rule violating hadronic and second order electromagnetic processes. (The interference between the two processes will be neglected.) The essential features of the model are that we shall

(*i*) take constant *S*-matrix element for (pseudoscalar, vector and tensor meson) resonances;

(*ii*) take SU(3)-symmetry factors from the quark model (consistent with the OZI-rule [5]);

(*iii*) neglect spins and resonance widths as well as interferences among resonances.

The assumption of constant matrix elements for resonances is better than to assume constant matrix elements for pions [6, 8–11], as an essential part of the strong interaction is contained in resonance formation. Besides, the importance of resonances seems to be a general trend in multiparticle production (for a recent experiment see [15]). The constant matrix element implies that the hadronic final state is spherical, there are no hadron jets present. Hadron jets manifest themselves in e^+e^- annihilation only at higher energies (about 7 GeV) therefore at 3–4 GeV the spherical final state may be still a good approximation. Concerning the SU(3)-symmetry factors an explicit example of a detailed quark model with this SU(3) structure (in first approximation) is given in Ref. [16]. Due to the neglection of spins from the point of view of the constant matrix element quark model the difference between the charmonium states is only in mass and in the internal quantum numbers (*G*-parity and octet-singlet mixing angle). For the same reason the nonresonant background in e^+e^- annihilation into hadrons also differs only in these respects from the charmonium resonances [14]. Hence, for comparison, we calculate also the mesonic channels in e^+e^- annihilation at 3095 MeV (a little bit off the resonance) using the parameters of the J/ψ-decay. The mesonic decays of the η_c state (assumed to be the X(2830) are calculated in the same way.

In Section 2 some details of the method of calculation are given (more details are contained in the Appendix). The comparison with experiment is in Section 3 whereas in Section 4 some concluding remarks are collected.

2. *The calculation of the branching ratios*

The constant matrix element quark model was given in terms of the density operator corresponding to the final state in Ref. [12] and in the equivalent form of the Hilbert space element in the space of outgoing states in Ref. [13]. The more conventional way of treating quantum transitions is in terms of transition amplitudes. This form was given in Ref. [14] where the applications to multiparticle decays of new heavy particles were also outlined.

In order to simplify things let us first consider the case of a single SU(3)-nonet of final state particles, say pseudoscalar mesons: $P = \pi^+, \pi^0, \pi^-, \eta, \eta', K^+, K^-, K^0, \bar{K}^0$.

The creation operator of the outgoing state with four-momentum p (three-momentum $\boldsymbol{p}$) and SU(3) index P is $a^{\dagger}(p, \mathrm{P})$ satisfying the commutation relation

$$[a(p, \mathrm{P}), a^{\dagger}(p', \mathrm{P}')] = 2p_0 N \delta^3(\boldsymbol{p}-\boldsymbol{p}')\delta_{\mathrm{PP}'}. \tag{2.1}$$

Here N is an arbitrary normalization factor (say $N = (2\pi)^3$) and $p_0 = \sqrt{m_{\mathrm{P}}^2+\boldsymbol{p}^2}$ where m_{P} is the mass of the P meson. The resolution of the identity in the Fock-space of outgoing particles is

$$I = \sum_{=0}^{\infty} \frac{1}{n!} \sum_{\mathrm{P}_1 \dots \mathrm{P}_n} \int \frac{d^3 p_1}{2p_{10}N} \cdots \frac{d^3 p_n}{2p_{n0}N} a^{\dagger}(p_n\mathrm{P}_n) \cdots a^{\dagger}(p_1\mathrm{P}_1) \,|0\rangle \langle 0| a(p_1\mathrm{P}_1) \cdots a(p_n\mathrm{P}_n). \tag{2.2}$$

The transition amplitude T_I from the decaying state $|I\rangle$ (with four-momentum p) to the final states given by the S-matrix is defined as

$$\langle 0| a(p_1\mathrm{P}_1) \cdots a(p_n\mathrm{P}_n) S |I\rangle = -i(2\pi)^4 \delta^4(p-p_1- \cdots -p_n) T_I(p_1\mathrm{P}_1, \cdots, p_n\mathrm{P}_n). \tag{2.3}$$

The SU(3) quantum numbers of a meson in the final state are expressed by the matrix $\overline{M}(\mathrm{P})$ generated by

$$\overline{M} \equiv \sum_{\mathrm{P}} x(\mathrm{P})\overline{M}(\mathrm{P}) = \begin{pmatrix} \frac{1}{\sqrt{2}}[x(\pi^0)+\cos\varphi x(\eta)+\sin\varphi x(\eta')], & x(\pi^+), & x(\mathrm{K}^+) \\ x(\pi^-) & \frac{1}{\sqrt{2}}[-x(\pi^0)+\cos\varphi x(\eta)+\sin\varphi x(\eta')], & x(\mathrm{K}^0) \\ x(\mathrm{K}^-) & x(\overline{\mathrm{K}}^0) & \cos\varphi x(\eta')-\sin\varphi x(\eta) \end{pmatrix}. \tag{2.4}$$

Here φ is the octet-singlet mixing angle measured from the "ideal" mixing. If the SU(3)--properties of the initial state $|I\rangle$ are specified by the matrix $M_I = \overline{M}_I^{\dagger}$ then the transition amplitude T_I in the constant matrix element quark model is [14]:

$$T_I(p_1\mathrm{P}_1, \cdots, p_n\mathrm{P}_n) = c_n \sqrt{\frac{B^{n-3}}{n!}} \sum_{\pi(!)n} \mathrm{Tr}\,\{M_I \overline{M}(\mathrm{P}_{\pi(1)})\overline{M}(\mathrm{P}_{\pi(2)}) \cdots \overline{M}(\mathrm{P}_{\pi(n)})\}. \tag{2.5}$$

Here $\sum_{\pi(!)n}$ denotes a summation over the permutations $\pi(1), \pi(2), \dots, \pi(n)$ of the numbers $1, 2, \dots, n$; B is a parameter of dimension mass^{-2} and c_n is the constant, dimensionless amplitude of the n-meson final state. In a quark model like in Ref. [16] c_n has to be understood as an average (in modulus) of the amplitude over the momentum space. As it was pointed out in Ref. [14] the important point is that the SU(3) coupling scheme in Eq. (2.5) is common to a large class of diagrams in the quark model and by virtue of the OZI-rule this class is dominating.

As is shown in the Appendix, the branching ratio of a channel with a number $n(\mathrm{P})$ of P-type hadrons (i. e. occupation number of SU(3)-states $n(\cdot) \equiv \{n(\mathrm{P})\}$ and total particle number $n = \sum_{\mathrm{P}} n(\mathrm{P})$) is

$$\frac{\prod_{\mathrm{P}} n(\mathrm{P})!}{n!} |G_I[n(\cdot)]|^2 \varrho[n(\cdot); p; B] \tag{2.6}$$

multiplied by an overall normalization factor. Here the factor $G_I[n(\cdot)]$ is determined by the generating function

$$G_I \equiv \sum_{n=2}^{\infty} c_n \operatorname{Tr}\{M_I \overline{M}^n\} = \sum_{n(\cdot)} G_I[n(\cdot)] \{\prod_{\mathrm{P}} x(\mathrm{P})^{n(\mathrm{P})}\}, \tag{2.7}$$

and ϱ is the relativistic phase space integral corresponding to the occupation number $n(\cdot)$:

$$\varrho[n(\cdot); p; B] = \int \prod_{\mathrm{P}} \prod_{i(\mathrm{P})=1}^{n(\mathrm{P})} \frac{B d^3 p\{\mathrm{P}, i(\mathrm{P})\}}{2p\{\mathrm{P}, i(\mathrm{P})\}_0 N} \delta^4 \left(p - \sum_{\mathrm{P}, i(\mathrm{P})} p\{\mathrm{P}, i(\mathrm{P})\}\right). \tag{2.8}$$

The numerical calculation of the trace factor $\operatorname{Tr}\{M_I \overline{M}^n\}$ can be done directly on a computer but we found it more convenient to use an explicit form following from

$$\operatorname{Tr}\{M_I(1-\overline{M})^{-1}\} = \sum_{n=0}^{\infty} \operatorname{Tr}\{M_I \overline{M}^n\}. \tag{2.9}$$

Abbreviating now $x(\mathrm{P})$ by P we have

$$1-\overline{M} = \begin{pmatrix} 1-\eta_u & -\pi^+ & -K^+ \\ -\pi^- & 1-\eta_d & -K^0 \\ -K^- & -\overline{K}^0 & 1-\eta_s \end{pmatrix}. \tag{2.10}$$

The states $\eta_{u,d,s}$ are the diagonal elements in the matrix $\overline{M}$, that is:

$$\eta_u = \frac{\pi^0}{\sqrt{2}} + \eta \frac{\cos\varphi}{\sqrt{2}} + \eta' \frac{\sin\varphi}{\sqrt{2}},$$

$$\eta_d = -\frac{\pi^0}{\sqrt{2}} + \eta \frac{\cos\varphi}{\sqrt{2}} + \eta' \frac{\sin\varphi}{\sqrt{2}},$$

$$\eta_s = -\eta \sin\varphi + \eta' \cos\varphi. \tag{2.11}$$

From Eq. (2.10) it follows that

$$(1-\overline{M})^{-1} = [\det(1-\overline{M})]^{-1}$$

$$\times\begin{pmatrix} 1-\eta_d-\eta_s+\eta_d\eta_s-K^0\overline{K}^0 & \pi^+-\pi^+\eta_s+K^+\overline{K}^0 & \pi^+K^0+K^+-K^+\eta_d \\ \pi^--\pi^-\eta_s+K^-K^0 & 1-\eta_u-\eta_s+\eta_u\eta_s-K^+K^- & \pi^-K^++K^0-K^0\eta_u \\ \pi^-\overline{K}^0+K^--K^-\eta_d & \pi^+K^-+\overline{K}^0-\overline{K}^0\eta_u & 1-\eta_u-\eta_d+\eta_u\eta_d-\pi^+\pi^- \end{pmatrix}. \quad (2.12)$$

The determinant is given by

$$\det(1-\overline{M}) = (1-\eta_u)(1-\eta_d)(1-\eta_s)-\pi^+K^0K^--\pi^-\overline{K}^0K^+$$
$$-(1-\eta_s)\pi^+\pi^--(1-\eta_d)K^+K^--(1-\eta_u)K^0\overline{K}^0. \quad (2.13)$$

By the repeated use of the identity

$$(1-x)^{-1-L} = \sum_{l=0}^{\infty} x\,\frac{l(L+l)!}{L!\,l!} \quad (2.14)$$

one can easily show that

$$[\det(1-\overline{M})]^{-1} = \sum_{n_1,\ldots,n_8=0}^{\infty} \eta_u^{n_6}\eta_d^{n_7}\eta_s^{n_8}(\pi^+K^0K^-)^{n_1}(\pi^-\overline{K}^0K^+)^{n_2}(K^+K^-)^{n_3}(K^0\overline{K}^0)^{n_4}$$
$$\times(\pi^+\pi^-)^{n_5}\frac{(n_1+n_2+n_3+n_4+n_5)!(n_1+n_2+n_3+n_5+n_6)!}{n_1!n_2!n_3!n_4!n_5!n_6!(n_1+n_2+n_3+n_5)!}$$
$$\times\frac{(n_1+n_2+n_4+n_5+n_7)!(n_1+n_2+n_3+n_4+n_8)!}{n_7\,(n_1+n_2+n_4+n_5)!n_8!(n_1+n_2+n_3+n_4)!}. \quad (2.15)$$

The powers of the mixed states $\eta_{u,d,s}$ can be calculated from the multinomial series like e. g.

$$\eta_u^k = \sum_{k_1+k_2+k_3=k} \pi_0^{k_1}\eta^{k_2}\eta'^{k_3}\frac{k!}{k_1!k_2!k_3!}\left(\frac{1}{\sqrt{2}}\right)^k \cos^{k_2}\varphi\,\sin^{k_3}\varphi. \quad (2.16)$$

The number multiplying c_n at a given occupation number $n(\cdot)$ of the SU(3) states can be read off from Eqs. (2.15–2.16) as the coefficient of $\prod_P P^{n(P)}$. It is easy to write a computer program for this and to use the result in Eq. (2.6). The multibody phase space integrals in Eq. (2.8) can be calculated either by a Monte-Carlo method or (as we did) by the statistical method [17].

Before proceeding to the case of several SU(3)-multiplets let us mention the distribution (2.6) is not of a simple multinomial type for the different charge branching ratios.

In fact, the closer inspection of the factor G_I reveals that, for a single SU(3)-multiplet, (2.6) is rather far from being multinomial as e. g. the distribution is essentially independent from the number of neutral pions. This, however, refers only to the case of a single (say pseudoscalar) multiplet, whereas in multiparticle final states resonance multiplets also play an important role. This alters the distribution over charge configurations in a non--trivial way (presumably making it more similar to the "statistical" multinomial distribution).

The case of several (a number k) different spinless SU(3) nonets can be treated similarly. Let us denote the SU(3)-matrix of the i^{th} nonet by $\overline{M}(i, \text{P})$ (P stands for the different states within a nonet, the dependence of $\overline{M}(i, \text{P})$ on i is due to the mixing angle φ_i which may be different for the different nonets). The number of P-type particles in the i^{th} nonet will be denoted by $n(i, \text{P})$, hence there are altogether $\sum_{\text{P}} n(i, \text{P}) \equiv n(i)$ particles in the i^{th} nonet. The simplest generalization of Eq. (2.5) for the case of more nonets is to assume that c_n is replaced by a constant depending on the occupation numbers $n(\cdot)$ only: $c[n(\cdot)]$. The transition amplitude is in this case

$$T_I[i_1\text{P}_1, i_2\text{P}_2, \cdots, i_n\text{P}_n] = c[n(\cdot)] \sqrt{\frac{B^{n-3}}{n!}}$$

$$\times \sum_{\pi(!)n} \text{Tr}\, \{M_I \overline{M}(i_{\pi(1)}, \text{P}_{\pi(1)}) \overline{M}(i_{\pi(2)}, \text{P}_{\pi(2)}) \cdots \overline{M}(i_{\pi(n)}, \text{P}_{\pi(n)})\}. \tag{2.17}$$

In order to reduce the number of parameters we shall assume that there is a common n-particle amplitude c_n for all the nonets ($n = \sum_i n(i) = \sum_{i,\text{P}} n(i, \text{P})$) and the dependence on the numbers $n(i)$ is simply given by a weight factor $\xi(i)$ to the power $n(i)$, that is

$$c[n(\cdot)] = c_n \prod_{i=1}^{k} \xi(i)^{n(i)}. \tag{2.18}$$

In this case the branching ratio of a channel with occupation numbers $n(\cdot\cdot) \equiv \{n(i, \text{P})\}$ is (apart from an overall normalization factor) proportional to

$$\frac{\prod_{i,\text{P}} n(i, \text{P})!}{n!} |G_I[n(\cdot\cdot)]|^2 \varrho[n(\cdot\cdot); p; B]. \tag{2.19}$$

The generating function of the factors $G_I[n(\cdot\cdot)]$ is now

$$G_I = \sum_{n=0}^{\infty} c_n \text{Tr}\, \{M_I [\sum_{i,\text{P}} \xi(i) x(i, \text{P}) \overline{M}(i, \text{P})]^n\} = \sum_{n(\cdot\cdot)} G_I[n(\cdot\cdot)] \prod_{i,\text{P}} x(i, \text{P})^{n(i,\text{P})}. \tag{2.20}$$

The phase space factor ϱ is, of course, defined in full analogy with Eq. (2.8). The calculation of $G_I[n(\cdot\cdot)]$ is facilitated by an expression like in Eq. (2.15) where, in our case of $\{i\} = \{\text{pseudoscalar, vector, tensor}\}$ nonets, for instance, π^+ is replaced by $\pi^+ + x\varrho^+ + yA_2^+$, and in general P is replaced by $\text{P} + x\text{V} + y\text{T}$ if $\text{V} = \{\varrho^+, \varrho^0, \ldots, \overline{\text{K}}^{*0}\}$ stands for vector mesons and $\text{T} = \{A_2^+, A_2^0, \ldots, \overline{\text{K}}^{**0}\}$ for the tensor ones.

3. *Comparison with experiment*

The mesonic decays of some charmonium states below the charm threshold, namely of the J/ψ, χ(3.415), χ(3.505), χ(3.550), X(2.830) states and the mesonic final states in the e^+e^- annihilation at $\sqrt{s} = 3.095$ GeV c. m. energy (off the resonance) are studied in this section. The electromagnetic and the OZI-rule violating strong decays into mesons take a prominent part in the hadronic decays of the J/ψ. The M_I matrix characterizing the SU(3) structure of the decaying state was chosen in the OZI-rule violating strong (M_{ST}) and in the electromagnetic (M_{EM}) decay as follows:

$$M_{ST} = \begin{bmatrix} \frac{\sin\varphi}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{\sin\varphi}{\sqrt{2}} & 0 \\ 0 & 0 & \cos\varphi \end{bmatrix}, \tag{3.1}$$

$$M_{EM} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}. \tag{3.2}$$

The mixing angle φ introduced in Eq. (3.1) is specifying whether the J/ψ is really an SU(3) singlet state. The special choice of M_{EM} in Eq. (3.2) reflects that the second order electromagnetic decay amplitude of a $c\bar{c}$ bound state into SU(3) quarks is proportional to the charge of the corresponding SU(3) quark. If we assume that there is no interference between the two processes, then $|G_{J/\psi}[n(\cdot\cdot)]|^2$ included in Eq. (2.19) is the following:

$$|G_{J/\psi}[n(\cdot\cdot)]|^2 = |G_{ST}^-[n(\cdot\cdot)]|^2 + \alpha|G_{EM}[n(\cdot\cdot)]|^2, \tag{3.3}$$

here the parameter α takes into account the relative weight of the electromagnetic in comparison with the strong interaction. The generating functional of the factors $G_{EM}[n(\cdot\cdot)]$ can be obtained from Eq. (2.20) by substituting EM instead of I. The generating functional of the factors $G_{ST}^-[n(\cdot\cdot)]$ takes into account the G-parity conservation in the strong interaction

$$G_{ST}^{\pm} = \sum_{n=0}^{\infty} c_n \operatorname{Tr}\{\sum_{i,P}[\xi(i)x(i,P)\overline{M}(i,P)]^n\} \pm \sum_{n=0}^{\infty} c_n \operatorname{Tr}\{\sum_{i,P}[\xi(i)x(i,G_iP)\overline{M}(i,P)]^n\}, \tag{3.4}$$

here C_i is the C-parity of the i-th multiplet and $C_i = C_i \exp(i\pi I_2)$ (I_2 is the second component of the isospin operator).

Let us denote the expression in Eq. (2.19) by $v_I[n(\cdot\cdot)]$ (where I = EM or ST). In this case the following expression can be obtained for the branching ratio of a resonance state $|n(\cdot\cdot)\rangle$ arising in the first step of the decay

$$w[n(\cdot\cdot)] = \lambda\{v_{ST}[n(\cdot\cdot)] + \alpha v_{EM}[n(\cdot\cdot)]\}, \tag{3.5}$$

λ is the normalization factor. The sum of the branching ratios of electromagnetic processes giving meson resonances in the final state is

$$\lambda_{\mathrm{EM}} = \lambda\alpha \sum_{n(\cdot\cdot)} v_{\mathrm{EM}}[n(\cdot\cdot)], \tag{3.6}$$

and the same quantity for the OZI-rule violating strong processes is:

$$\lambda_{\mathrm{ST}} = \lambda \sum_{n(\cdot\cdot)} v_{\mathrm{ST}}[n(\cdot\cdot)]. \tag{3.7}$$

The equations analogous with Eqs. (3.5)–(3.7) characterizing the decay of χ(and X(2.830)) particles and the final states containing only meson resonances in the e^+e^- annihilation in the off resonance case can be obtained similarly. In the former case we have to take into account that the *G*-parity of the χ and X particles is even and there is no second order electromagnetic decay, and in the latter case, that the mesonic final states arise only via electromagnetic interaction.

The experimentally detected final states originate from the decay of resonance states. In our calculations the decay rates of the individual resonances were taken from the Rosenfeld Table (the interference among the resonances was neglected). Our model can be characterized by the parameters B, x, y, φ, α, λ, $g_n \equiv |c_n|^2$ $(n \geqslant 2)$, which in general may have different values in the different processes. When sufficient experimentally known data for the branching ratios is available we can determine these parameters with high precision. The parameter values (or some of them) obtained in this way can be used to predict branching ratios in other experimentally still unknown (or less well known) decays.

Studying the decay of J/ψ $g_n = 1$ was assumed for $n \geqslant 7$ as the states with $n \geqslant 7$ do not contribute appreciably to the observed channels. The branching ratios of the final states coming from the decay of $\chi(3.415)$, $\chi(3.505)$ and $\chi(3.550)$ were fitted together assuming that (1) B, x, y are the same for the three processes; (2) x and y are equal to those values obtained from the fit to the J/ψ decay; (3) $g_n = 1$ for $n \geqslant 2$; (4) $\lambda_{\chi(3.415)} \leqslant 3 \cdot 10^{-4}$ (this assumption was needed to satisfy the relation: $\lambda \sum_{n(\cdot\cdot)} v_{\mathrm{ST}}[n(\cdot\cdot)] < 1$ for $\chi(3.415)$); (5) we do not make too large a mistake in the numerical calculation by neglecting states with the total rest mass of the resonances larger than the mass of the J/ψ. We consider assumption (5) well motivated by the fact that in the case of J/λ decay (assuming the values tabulated in the Table I for the parameters) the sum of the branching ratios of all the resonance channels having total rest mass larger than 2.6 GeV is equal only to 1.3%. In order to study the e^+e^- annihilation the parameters B, x, y, φ, g_n were chosen to be equal to the ones obtained by the fit to the final states of J/ψ decay. In this case, of course, only the electromagnetic interaction is present. The best value for $\alpha\lambda$ in Eq. (3.6) can be determined from the following experimental data taken from Ref. [1]:

$$a[n(\cdot\cdot)] = \frac{\sigma[\mathrm{J}/\psi \to n(\cdot\cdot)]/\sigma[\mathrm{J}/\psi \to \mu^+\mu^-]}{\sigma[e^+e^- \to n(\cdot\cdot)]/\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$= \begin{cases} 0.8 \pm 0.3 \text{ for } n(\cdot\cdot) = 2(\pi^+\pi^-), \\ 1.17 \pm 0.57 \text{ for } n(\cdot\cdot) = 3(\pi^+\pi^-); \text{ at } \sqrt{s} = 3 \text{ GeV}. \end{cases} \tag{3.8}$$

Here we have used that the branching ratio of the process $J/\psi \to \mu^+\mu^-$ is equal to 0.069 and the branching ratio of the process $e^+e^- \to \mu^+\mu^-$ is equal to 0.226 at $\sqrt{s} = 3$ GeV c. m. energy [1]. In the fitting procedure we did not take into account the possibility that the $(\pi^+\pi^-)$ pairs may come from the decay of K_{0S}. The subsequent control showed that this does not influence the agreement with experimental data in an essential way. Upper limits were not taken into account in the fit.

The results of the fit under the above assumptions for the different processes are collected in Table I. Assuming the values of Table I for the parameters we compared in Table II the predictions of the model with experimental data on the branching ratios of all purely mesonic channels. The predicted and the experimentally determined values for the quantities in Eq. (3.8) are also tabulated here.

TABLE I

Parameter values in different processes

	B (GeV^{-2})	x	y	φ	α	λ
J/ψ	3.21	0.36	1.33	71.0	0.99	$0.750 \cdot 10^{-4}$
e^+e^-	3.21	0.36	1.33	71.0	$\to$	$0.27 \cdot 10^{-3}$ $(= \lambda\alpha)$
$\chi(3.415)$	1.43	0.36	1.33	38.3	0	0.0003
$\chi(3.505)$	1.43	0.36	1.33	56.8	0	$0.91 \cdot 10^{-4}$
$\chi(3.550)$	1.43	0.36	1.33	30.3	0	$0.22 \cdot 10^{-3}$
	g_2	g_3	g_4	g_5	g_6	g_n $n \geqslant 7$
J/ψ, e^+e^-	5.14	0.51	0.58	0.089	0.49	1
χ	1	1	1	1	1	1

From Tables I and II we can infere the following:

1. In the decay of J/ψ the "characteristic hadron size" is $\sqrt{B} = 0.35$ fermi. This value is equal to about 1.5 times the corresponding value in the decay of the χ particles: $\sqrt{B} = 0.24$ fermi.

2. In the J/ψ decay the tensor mesons are produced with relatively large amplitude,

3. The $\chi(3.505)$ particle can be considered as mostly SU(3) singlet state. In J/ψ the $(u\bar{u})$ and the $(d\bar{d})$ states, in the decay of $\chi(3.415)$ and $\chi(3.550)$ the $(s\bar{s})$ state are dominant.

4. The agreement with experimental data is generally rather good. An exception is the decay rate of the $J/\psi \to \phi\pi^+\pi^-$ channel: the ratio of the experimental to the model predicted value is about 20. In the model this channel is suppressed by the OZI-rule. We refer in this respect to the experimental paper of Vanucci et al. [18]. According to it the very different behaviour of the invariant mass spectrum of the $\pi^+\pi^-$ pairs below and above the $K\bar{K}$ threshold makes likely that the mechanism producing this final state is incompatible with the basic assumptions of our model. In Table II the measured upper limits are also included. These limits are generally compatible with the branching ratios computed from the model. The exception is $K_0\bar{K}_0$.

TABLE IIA

Branching ratios for χ decays and the value of $a[n(\cdot\cdot)]$ defined in Eq. (3.8). The numbers in the second and third columns show the contribution of the strong and of the second order electromagnetic processes, respectively, to the theoretical value w_{th}. The numbers in brackets refer to the case of taking into account also pions from K_{S0} decay

Final state	$\lambda \cdot v_{ST}/w_{th}$ (%)	$\lambda\alpha v_{EM}/w_{th}$ (%)	w_{th} (%)	w_{exp} (%)	Ref.
			Decay of $\chi(3.415)$		
$\pi^+\pi^-$	100	0	0.15	1 ±0.3	[2]
K^+K^-	100	0	0.28	1.4 ±0.3	[2]
$2(\pi^+\pi^-)$	100	0	3.53 (3.56)	4.6 ±0.9	[2]
$\pi^+\pi^-K^+K^-$	100	0	2.75	3.7 ±0.9	[2]
$3(\pi^+\pi^-)$	100	0	3.03 (3.35)	1.9 ±0.7	[2]
			Decay of $\chi(3.505)$		
$2(\pi^+\pi^-)$	100	0	2.17	2 ±0.6	[2]
$\pi^+\pi^-K^+K^-$	100	0	1.1	1.1±0.4	[2]
$3(\pi^+\pi^-$	100	0	2.13 (2.26)	2.7±1.1	[2]
			Decay of $\chi(3.550)$		
$\pi^+\pi^-+K^+K^-$	100	0	0.27	0.27±0.11	[2]
$2(\pi^+\pi^-)$	100	0	1.95 (1.98)	2.4±0.6	[2]
$\pi^+\pi^-K^+K^-$	100	0	2.11	2.1±0.6	[2]
$3(\pi^+\pi^-)$	100	0	2.06 (2.30)	1.3±0.8	[2]
			e^+e^- $\sqrt{s} = 3.095$ GeV		
$2(\pi^\pm\pi^-)$	0	100	0.98 (1.08)	1.03±0.4	[1]
$3(\pi^+\pi^-)$	0	100	1.12 (1.22)	1.28±0.5	[1]

$a[n(\cdot\cdot)]$

	model		experiment	Ref.
$\pi^0\pi^+\pi^-$	26.7	(25.4)	>3.5	[1]
$2(\pi^+\pi^-)$	0.95	(0.91)	0.8±0.3	[1]
$\pi^0 2(\pi^+\pi^-)$	35.5	(30.5)	>5.4	[1]
$3(\pi^+\pi^-)$	0.95	(1.02)	1.17±0.57	[1]
$\pi^0 3(\pi^+\pi^-)$	23.5	(21.1)	>4.6	[1]

TABLE IIB

The same as Table IIA for J/ψ decays

Final state	$\lambda v_{ST}/w_{th}$ (%)	$\lambda\alpha v_{EM}/w_{th}$ (%)	w_{th} (%)	w_{exp} (%)	Ref.
$\pi^+\pi^-$	0	100	0.034	0.016 ± 0.016	[18]
$\pi^0\pi^+\pi^-$	96.6	3.4	0.85	1.3 ± 0.3	[18]
$2(\pi^+\pi^-)$	3.6	96.4	0.29	0.4 ± 0.1	[18]
	(3.5)	(96.5)	(0.30)		
$\pi^0 2(\pi^+\pi^-)$	97.4	2.6	5.20	4 ± 1	[18]
	(97.2)	(2.9)	(5.23)		
$3(\pi^+\pi^-)$	3.6	96.4	0.33	0.4 ± 0.2	[18]
	(14.0)	(86)	(0.38)		
$\pi^0 3(\pi^+\pi^-)$	96.1	3.9	3.10	2.9 ± 0.7	[18]
	(95.8)	(4.2)	(3.20)		
$\pi^0 4(\pi^+\pi^-)$	93.7	6.3	0.90	0.9 ± 0.3	[18]
	(93.5)	(6.5)	(0.91)		
$\pi\rho$	98.5	1.5	0.83	1 ± 0.2	[4]
ρA_2	98.5	1.5	1.15	0.84 ± 0.45	[18]
$A_2^+\pi^-$	0	100	0.05	<0.43	[1]
$\omega\pi^+\pi^-$	98.5	1.5	0.81	0.78 ± 0.16	[4]
$\omega 2(\pi^+\pi^-)$	98.3	1.7	0.92	0.85 ± 0.34	[18]
K^+K^-	0	100	0.033	0.02 ± 0.016	[18]
$K^0\overline{K}^0$	0	100	0.13	<0.018	[18]
$K^0\overline{K}^{0**}$	0	100	0.37	<0.2	[18]
$K^0K^-\pi^+$+c. c.	65.3	34.7	0.59	0.52 ± 0.14	[18]
$\pi^+\pi^-K^+K^-$	69.6	30.4	0.50	0.72 ± 0.23	[18]
$\pi^0\pi^+\pi^-K^+K^-$	84.8	15.2	0.91	1.2 ± 0.3	[18]
$2(K^+K^-)$	53.1	46.9	0.081	0.07 ± 0.03	[18]
$2(\pi^+\pi^-)K^+K^-$	63.5	36.5	1.10	0.31 ± 0.13	[18]
	(62.8)	(37.2)	(0.12)		
K^+K^{*-}+ c. c.	97.3	2.7	0.30	0.32 ± 0.06	[18]
$K^0\overline{K}^{*0}$+ c. c.	90	10	0.32	0.27 ± 0.06	[18]
K^+K^{**-}+ c. c.	0	100	0.092	<0.15	[18]
$K^{*0}\overline{K}^{*0}$	0	100	0.0019	<0.5	[1]
$K^{*0}\overline{K}^{**0}$	90	10	0.42	0.67 ± 0.26	[18]
$K^{**0}K^{**0}$	0	100	0.17	<0.29	[18]
$\phi\pi^+\pi^-$	86.9	13.1	0.0073	0.14 ± 0.06	[18]
ωK^+K^-	85.1	14.9	0.063	0.08 ± 0.05	[18]
ϕK^+K^-	90.5	9.5	0.10	0.09 ± 0.04	[18]
$\phi\eta$	70.2	29.8	0.027	0.1 ± 0.06	[18]
$\phi\eta'$	85.4	14.6	0.044	0.05 ± 0.04	[1]
ϕf	99.7	0.3	0.0051	<0.037	[18]
$\phi f'$	79.0	21	0.086	0.08 ± 0.05	[18]
ωf	98.4	1.6	0.38	0.40 ± 0.14	[4]
$\omega f'$	98.7	1.3	0.0043	<0.016	[18]

TABLE III

Branching ratios after the resonance decay which are above 1% in X(2830) mesonic decays. The numbers are normalized to the sum of purely mesonic channels. The parameter values are given in Table I, the ones for X(2830) are taken to be equal to those of the J/ψ

	e^+e^- %	J/ψ %	X(2.830) %	$\chi(3.415)$ %	$\chi(3.505)$ %	$\chi(3.550)$ %
$4\pi^0$	1.3	0.25	2.9	1.2	1.4	0.85
$6\pi^0$	1.8	0.34	1.7	0.54	0.71	0.48
$\pi^+\pi^-$	0.42	0.08	1.7	0.15	0.16	0.093
$(\pi^-K^+ + \pi^+K^-)K_{2\,0L}$	1.2	0.69	1.9	1.9	1.2	1.7
$\pi^+\pi^-\pi^0\pi^0$	3.1	0.60	8.3	2.7	3.1	1.9
$\pi^+\pi^-\gamma\gamma$	0.40	0.079	1.2	0.50	0.56	0.33
$K^+K^-\pi^0\pi^0$	1.3	0.42	1.1	1.2	0.93	1.2
$\pi^+\pi^-\pi^0\pi^0\pi^0$	1.9	6.4	1.4	0.63	0.66	0.44
$\pi^+\pi^-4\pi^0$	3.8	1.5	3.8	2.4	3.0	2.1
$\pi^+\pi^-2(\pi^0\gamma)$	1.05	0.29	1.4	1.6	1.9	1.2
$\pi^+\pi^-5\pi^0$	1.9	2.7	1.3	1.7	2.1	1.4
$2(\pi^+\pi^-)$	3.5	0.71	9.8	3.7	4.3	2.6
$\pi^+\pi^-(\pi^+K^- + \pi^-K^+)$	0.85	0.47	1.3	1.3	0.83	1.2
$\pi^+\pi^-K^+K^-$	1.9	1.2	2.5	2.9	2.2	2.8
$2(\pi^+\pi^-\pi^0)$	4.5	1.6	7.1	4.4	5.6	3.8
$2(\pi^+\pi^-\gamma)$	0.81	0.18	1.5	2.0	2.5	1.6
$2(\pi^+\pi^-)3\pi^0$	2.5	5.9	2.1	2.7	3.3	2.2
$2(\pi^+\pi^-)4\pi^0$	2.0	2.7	1.2	0.81	1.1	0.78
$3(\pi^+\pi^-)$	3.97	0.89	6.8	3.5	4.5	3.1
$3(\pi^+\pi^-)2\pi^0$	1.5	1.6	1.3	1.0	1.4	1.0
$3(\pi^+\pi^-)3\pi^0$	1.2	1.7	1.0	0.88	1.2	0.87

TABLE IV

Average multiplicities of the first generation ("direct") resonances in the mesonic decays of the J/ψ and in the mesonic final states of the e^+e^- annihilation at $\sqrt{s} = 3.095$ GeV off the resonance

	π^0	π^+	K^+	K^0	η	η'
J/ψ	0.91	0.78	0.08	0.08	0.26	0.068
e^+e^-	1.25	0.70	0.092	0.11	0.41	0.10
	ρ^0	ρ^+	K^{*+}	K^{*0}	ω	ϕ
J/ψ	0.15	0.14	0.032	0.032	0.26	0.014
e^+e^-	0.025	0.013	0.0041	0.0071	0.031	0.011
	A_2^0	A_2^+	K^{**+}	K^{**0}	f	f′
J/ψ	0.041	0.032	0.011	0.017	0.065	0.015
e^+e^-	0.084	0.039	0.017	0.049	0.085	0.069

5. The known branching ratios in the decay of $\chi(3.505)$ and $\chi(3.550)$ are well reproduced by the model. Some discrepancy can be found in the case of $\chi(3.415)$ but the differences between the model predicted and the experimentally measured values are also

TABLE V

Average multiplicity as the function of the number of charged particles n_{ch} and the total average multiplicity in the mesonic final states after the decay of resonances. w_{ch} is the sum of the probabilities of all channels containing charged particles of number n_{ch}

n_{ch}	$w_{ch}(\%)$		$\langle\gamma\rangle_{n_{ch}}$	$\langle\pi^0\rangle_{n_{ch}}$	$\langle\pi^+\rangle_{n_{ch}}$	$\langle K^+\rangle_{n_{ch}}$	$\langle K_{0L}\rangle_{n_{ch}}$
0	J/ψ	3.2	0.80	4.40	0	0	0.45
	e^+e^-	12.8	0.76	4.55	0	0	0.35
2	J/ψ	29.3	0.42	2.98	0.87	0.13	0.24
	e^+e^-	35.2	0.55	2.75	0.79	0.21	0.27
4	J/ψ	43.5	0.29	1.82	1.81	0.19	0.11
	e^+e^-	33.3	0.39	1.65	1.75	0.25	0.11
6	J/ψ	18.8	0.24	1.42	2.90	0.10	0.03
	e^+e^-	13.3	0.34	1.06	2.90	0.10	0.03
8	J/ψ	3.6	0.09	1.14	3.96	0.04	0.004
	e^+e^-	2.4	0.19	0.69	3.97	0.03	0.001
10	J/ψ	0.05	0.12	1.23	5.00	0	0
	e^+e^-	0.06	0.16	0.83	5.00	0	0
			$\langle\gamma\rangle$	$\langle\pi^0\rangle$	$\langle\pi^+\rangle$	$\langle K^+\rangle$	$\langle K_{0L}\rangle$
Total av. multipl.		J/ψ	0.32	2.12	1.73	0.14	0.14
		e^+e^-	0.47	2.26	1.35	0.17	0.18

smaller than twice the experimental error with the exception of the $\chi(3.415) \to \pi^+\pi^-$ and K^+K^- channels.

In Table III the channels arising from the decay of X(2.830), $\chi(3.415)$, $\chi(3.505)$, $\chi(3.550)$ and J/ψ (e^+e^-) having the largest branching ratios in X(2.830) are collected. Tables IV and V contain the average multiplicities of the direct pseudoscalar, vector and tensor

TABLE VI

Comparison of the predicted values of the charged average multiplicity $\langle n_{ch} \rangle$ the relative weights of the 2, 4, 6, 8 prong events (assuming $w_2+w_4+w_6+w_8 = 100\%$) and the average multiplicity of photons at fixed prong number $\langle \gamma \rangle_{n_{ch}}+2\langle \pi^0 \rangle_{n_{ch}}$ in the mesonic decays of J/ψ with the experimentally measured data taken from Ref. [19]

n_{ch}	$w_{ch}(\%)$		$\langle \gamma \rangle_{n_{ch}}+2\langle \pi^0 \rangle_{n_{ch}}$	
	model	exp.	model	exp.
2	30.8	32 ± 5	6.4	7.2 ± 1.8
4	45.7	$49+8$	3.9	6.2 ± 1.4
6	19.7	18 ± 3	3.1	4.6 ± 1.2
8	3.8	1 ± 0.6	4.6	6.2 ± 1.6
$\langle n_{ch} \rangle$	3.74	3.8 ± 0.3		

mesons and average multiplicities of the particles π^0, π^+, K^+, K_{0L}, γ arising from the decay of resonances, respectively. Here only the J/ψ decay and the e^+e^- annihilation are included. In Table VI we compared with experiment the predicted charged average multiplicity, the relative weight of 2, 4, 6 and 8 prong events and the average multiplicity of the photons at fixed number of charged tracks in the mesonic final states of J/ψ decay. The data were

TABLE VII

Total branching ratios of classes of final states. The data are taken from Ref. [1] and [20]

Final state / Initial state	Purely mesonic % (model)	e^+e^-, $\mu^+\mu^-$ % (experiment)	$\gamma\psi$ % (experiment)	Left over %
J/ψ	42.7 = 34.4 (strong) + 8.3 (elect.)	14	—	43.3
e^+e^-	29.4	44.6	—	26.0
χ(3.415)	96.1	—	3	0.9
χ(3.505)	50.8	—	35	14.2
χ(3.550)	75.6	—	14	10.4

taken from Ref. [19]. We should like to note, however, that these quantities were measured in Ref. [19] assuming that the non-pionic final states can be neglected.

We can compute in our model (using Eq. (3.6), (3.7) and taking the parameter values from Table I) the sum of the branching ratios of all the channels containing only meson resonances. In such a way we can also predict the sum of the branching ratios of all the remaining channels (e. g. baryonic or radiative channels). These results are collected in Table VII.

4. *Conclusions*

In the previous Section the presently available data on J/ψ(3095) mesonic decays were fitted by the parameters B (effective cross-section or volume), x and y (weight factors of vector and tensor meson resonances, respectively), φ (octet-singlet mixing angle in the Zweig-rule violating strong decay), λ (proportional to the total branching ratio of purely mesonic channels) and $g_2, g_3, \ldots, g_6$ (dimensionless n-body amplitudes squared). The results and predictions of the fit are contained in Tables I–VII. The obtained value of $B = 3.21\ \mathrm{GeV}^{-2}$ is more than an order of magnitude larger than the current algebra inspired guess of Ref. [6]: $(2\pi)^{-3} f_\pi^{-2} = 0.23\ \mathrm{GeV}^{-2}$. (This implies in general larger multiplicities.) As a cross-section $B = 1.25$ mb is, in fact, rather reasonably hadronic (and psionic). (So is the corresponding length $B = 0.35$ fm.) It is about the same as the value ($2.32\ \mathrm{GeV}^{-2}$) of the corresponding parameter in Ref. [11]. In summary, such a value can be expected from the beginning for heavy particle decays, and it can be taken (at least roughly) universal.

The parameters $x = 0.36$ and $y = 1.33$ give the relative weights of vector and tensor mesons, respectively (in dimensionless amplitudes) compared to pseudoscalar mesons. On purely statistical grounds one would have $x = \sqrt{3}$ and $y = \sqrt{5}$ corresponding to the number of different spin states, hence the values obtained are lower than expected. The channels with $n = 2, 3, 6$ resonances make up about 15% each, $n = 4$ gives more, namely 40% (the rest is for $n = 5$ and $n \geqslant 7$). The average multiplicity of the "direct" mesons is 4.19. Out of this 74%, 20% and 6% are pseudoscalar, vector and tensor meson, respectively. For instance, the "direct" pions ($\langle n_{\pi^+} \rangle = 0.78$) are more copious than the "direct" ϱ-mesons ($\langle n_{\varrho^0} \rangle = 0.15$). The relative number of vector mesons is therefore considerably less than in hadronic collisions [15]. The contribution of tensor mesons is not large. In fact we also tried a fit to J/ψ decays without tensor mesons ($y = 0$). The results turned out almost as good as those in Table IIB (of course, except the channels containing explicitly tensor mesons) and the values of the other parameters are essentially unchanged. No acceptable fit can be obtained however with the pseudoscalar mesons only ($x = y = 0$). Concerning other resonance multiplets (axialvector, scalar, etc.) the stability of the fit at the omission of tensor mesons suggests that they are even less important in influencing the branching ratios observed up to now. Such higher multiplets are, in principle, possible to include at a later stage if there will be some experimental data on them.

The value of octet-singlet mixing angle $\varphi = 71^0$ means that the J/ψ(3095) is, in fact, far enough from being pure SU(3)-singlet (corresponding to $\varphi = 54.7°$): there are less strange quarks in it than in an SU(3) singlet. This gives a suppression for K-mesonic channels. Apart from this there is still another stronger suppression due to the larger kaon mass. The final result is, for instance, the large difference in $\langle n_{\pi^+} \rangle = 1.73$ and $\langle n_{K^+} \rangle = 0.17$. We want to emphasize, however, that no other SU(3)-breaking factor is needed for the suppression of the production of strange quarks as it is usually believed in naive quark models. Our amplitude is exactly SU(3)-symmetric, the SU(3)-breaking is due to the mass differences only. Another interesting SU(3)-feature in the J/ψ(3095) mesonic final states is the relative abundance of η and ω mesons: $\langle n_\eta \rangle = 0.26$, $\langle n_\omega \rangle = 0.26$. Comparing to the number of kaons or ϱ^0 mesons, respectively (nearby in masses) we can see that the initial SU(3) quantum numbers of the hadronic state manifest themselves rather strongly in the multimesonic final state. This is one of the interesting questions which can be studied in multiparticle final states of heavy particle decays [14].

An interesting feature of our fit is that the sum of all purely mesonic channels for J/ψ(3095) is only 43%. Adding another 14% for lepton pair decays we are left with 43% for other channels. This seems to be at the first sight surprisingly large. Among the candidates for the left out channels there are the ones with at least one pair of baryons or at least one photon. This is, after all, an experimental question (and we not know any data which would contradict such a number) therefore we do not wish to comment very much on this point. We only note that in large transverse momentum multiparticle final states there are measurements [21] indicating a quite large (20–30%) "direct" γ production compared to pion production and in e^+e^- annihilation at $\sqrt{s} = 3.8$ GeV the measured inclusive antinucleon cross-section ($\sigma_{\bar{N}} \simeq 2\sigma_{\bar{p}}$) is about 10% of the total hadronic annihilation cross-section [1, 22]. The direct γ production can also contribute (besides the η mesons) to the so called "energy crises" (more neutral than half of the charged energy in the final state) observed in e^+e^- annihilation. From this point of view the numbers for $\langle n_{\pi^0}\rangle$ and $\langle n_{\pi^+}\rangle$ in Table V are quite remarkable showing the predominance of neutral particles, especially in the e^+e^- case. (This is another place where the initial SU(3) quantum numbers seem to play an unexpectedly large role.)

Besides the application of the constant matrix element quark model to other charmonium states (χ's and X, as we did, and possibly others) there is also the possibility to apply it to other processes like, for instance: the opening up of the multiparticle channels above charm-threshold in e^+e^- annihilation (e. g. $D\bar{D}\pi K\bar{K}\eta$ in competition with $D\bar{D}$) or, perhaps, also to the decay of heavier quarkonium states. It seems possible to extend the model to calculate baryonic and radiative channels as well.

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APPENDIX

Hilbert-space element, density operator, branching ratios

The Hilbert-space element in the space of outgoing particles corresponding to the initial state $|I\rangle$ is $S|I\rangle$. Using the resolution of identity in Eq. (2.2) and the definition of the transition amplitude Eq. (2.3) we have:

$$S|I\rangle = -i(2\pi)^4 \sum_{n=2}^{\infty} \frac{1}{n!} \sum_{P_1 \dots P_n} \int \frac{d^3p_1 \cdots d^3p_n}{2p_{10}N \cdots 2p_{n0}N} \delta^4(p-p_1- \cdots -p_n)$$

$$\times T_I(p_1P_1, \cdots, p_nP_n) a^\dagger(p_nP_n) \cdots a^\dagger(p_1P_1)\,|0\rangle. \qquad \text{(A1)}$$

In the constant matrix element quark model the transition amplitude is given by Eq. (2.5) therefore, as it was given in Ref. [13]:

$$S|I\rangle = -i(2\pi)^4 \sum_{n=2}^{\infty} c_n \sqrt{\frac{B^{n-3}}{n!}} \int \frac{d^3p_1 \cdots d^3p_n}{2p_{10}N \cdots 2p_{n0}N} \delta^4(p-p_1- \cdots -p_n)$$

$$\times \operatorname{Tr} \{M_I A^\dagger(p_n) \cdots A^\dagger(p_1)\} |0\rangle, \tag{A2}$$

where the creation operator matrix $A^\dagger(p)$ is defined by

$$A^\dagger(p) = \sum_{\mathrm{P}} \overline{M}(\mathrm{P}) a^\dagger(p, \mathrm{P}). \tag{A3}$$

The Hilbert-space element in Eq. (A2) is equivalent to the following density operator (with $M_I^\dagger = \overline{M}_I$):

$$R_I(p, p') = S|I\rangle \langle I'|S^\dagger. \tag{A4}$$

Here, for later convenience, the four momentum of the state is taken to be different (p and p') on the two sides of the vacuum. The measurable quantity is, of course:

$$R_I(p, p) = S|I\rangle \langle I|S^\dagger. \tag{A5}$$

The distribution of the particles in the final state can be calculated with the help of the operator functional $I[\Phi(\cdot\cdot)]$ depending on the functions $\{\Phi(p, \mathrm{P})\} \equiv \Phi(\cdot\cdot)$:

$$I[\Phi(\cdot\cdot)] = \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{\mathrm{P}_1 \dots \mathrm{P}_n} \int \frac{d^3p_1 \cdots d^3p_n}{2p_{10}N \cdots 2p_{n0}N} \Phi(p_1\mathrm{P}_1) \cdots \Phi(p_n\mathrm{P}_n)$$

$$\times a^\dagger(p_n\mathrm{P}_n) \cdots a^\dagger(p_1\mathrm{P}_1) |0\rangle \langle 0| a(p_1\mathrm{P}_1) \cdots a(p_n\mathrm{P}_n). \tag{A6}$$

The generating functional of the exclusive and inclusive distributions is

$$V_I[p; \Phi(\cdot\cdot)] = \frac{\langle I|S^\dagger I[\Phi(\cdot\cdot)]S|I\rangle}{\langle I|S^\dagger S|I\rangle}. \tag{A7}$$

The exclusive distributions can be obtained from here by taking functional derivatives at $\Phi = 0$ whereas the inclusive ones by taking functional derivatives at $\Phi = 1$. In particular, the generating function of the multiplicity distribution is obtained if the functions $\Phi(p, \mathrm{P})$ are replaced by constants ζ_{P}.

Due to the use of plane wave states the matrix elements on the right hand side of Eq. (A7) contain a factor $\delta^4(0)$ which is usually identified as $(2\pi)^{-4} VT$ where V is the "whole volume" and T is the "whole time interval" of the transition. This infinite factor has to be omitted during the calculation therefore it is convenient to introduce another (well normalized) density operator $R_I(p)$ instead of $R_I(p, p')$. In general, we can write

$$\langle I'|S^\dagger I[\Phi(\cdot\cdot)]S|I\rangle = \operatorname{Tr} \{R_I(p, p') I[\Phi(\cdot\cdot)]\} \equiv \delta^4(p-p') \operatorname{Tr} \{R_I(p) I[\Phi(\cdot\cdot)]\}. \tag{A8}$$

This relation does not uniquely define $R_I(p)$ because in the trace only diagonal elements enter. The non-uniqueness is, however, irrelevant for the distributions we are interested in (the non-diagonal terms correspond to interferences among states with different particle numbers etc.), therefore any choice satisfying Eq. (A8) is right. According to Ref. [12] we can choose $R_I(p)$ as

$$R_I[p;B] \equiv (2\pi)^8 \sum_{n=2}^{\infty} |c_n|^2 \frac{B^{n-3}}{n!} \int \frac{d^3p_1 \cdots d^3p_n}{2p_{10}N \cdots 2p_{n0}N} \frac{d^3p'_1 \cdots d^3p'_n}{2p'_{10}N \cdots 2p'_{n0}N}$$

$$\delta^4[p - \tfrac{1}{2}\sum_{i=1}^{n}(p_i + p'_i)] \operatorname{Tr}\{M_I A^\dagger(p_n) \cdots A^\dagger(p_1)\} |0\rangle \langle 0| \operatorname{Tr}\{\overline{M}_I A(p'_1) \cdots A(p'_n)\}. \tag{A9}$$

Combining Eqs. (A7), (A8) the generating functional of the final state distributions becomes:

$$V_I[p; \Phi(\cdot\cdot)] = \frac{\operatorname{Tr}\{R_I[p;B] I[\Phi(\cdot\cdot)]\}}{\operatorname{Tr}\{R_I[p;B]\}}. \tag{A10}$$

In order to explicitly calculate the branching ratios into different final state channels it is useful to introduce the states with fixed occupation numbers in SU(3)-states $\{n(\mathrm{P})\} \equiv n(\cdot)$. The resolution of the identity in Eq. (2.2) is in such a notation:

$$I = \sum_{n(\cdot)=0}^{\infty} \left[\prod_{\mathrm{P}} n(\mathrm{P})!\right]^{-1} \int \prod_{\mathrm{P}} \prod_{i(\mathrm{P})=1}^{n(\mathrm{P})} \frac{d^3p\{\mathrm{P}, i(\mathrm{P})\}}{2p\{\mathrm{P}, i(\mathrm{P})\}_0 N}$$
$$\times \prod_{\mathrm{P}, i(\mathrm{P})} a^\dagger(p\{\mathrm{P}, i(\mathrm{P})\}, \mathrm{P}) |0\rangle \langle 0| \prod_{\mathrm{P}, i(\mathrm{P})} a(p\{\mathrm{P}, i(\mathrm{P})\}, \mathrm{P}). \tag{A11}$$

If the state $\{\mathrm{P}_1 \mathrm{P}_2 \ldots \mathrm{P}_n\}$ corresponds to the occupation numbers $n(\cdot)$ then let us denote the transition amplitude in Eq. (2.5) by

$$T_I[n(\cdot)] \equiv \sqrt{\frac{B^{n-3}}{n!}}\, G_I[n(\cdot)] \prod_{\mathrm{P}} n(\mathrm{P})! \tag{A12}$$

For a function F depending only on the occupation numbers $n(\cdot)$ we generally have

$$\sum_n \frac{1}{n!} \sum_{\mathrm{P}_1 \ldots \mathrm{P}_n} F = \sum_{n(\cdot)} \left[\prod_{\mathrm{P}} n(\mathrm{P})!\right]^{-1} F, \tag{A13}$$

therefore

$$\sum_n \frac{c_n}{n!} \sum_{\mathrm{P}_1 \ldots \mathrm{P}_n} x(\mathrm{P}_1) \cdots x(\mathrm{P}_n) \sum_{\pi(!)n} \operatorname{Tr}\{M_I \overline{M}(\mathrm{P}_{\pi(1)}) \cdots \overline{M}(\mathrm{P}_{\pi(n)})\})$$

$$= \sum_{n(\cdot)} [\prod_{\mathrm{P}} x(\mathrm{P})^{n(\mathrm{P})}] G_I[n(\cdot)] = \sum_{n=0}^{\infty} c_n \operatorname{Tr}\{M_I (\sum_{\mathrm{P}} x(\mathrm{P}) \overline{M}(\mathrm{P}))^n\}. \tag{A14}$$

This is the generating function of the factors $G_I[n(\cdot)]$ in Eq. (A12) also given in Eq. (2.7). From Eqs. (A11)–(A12) in the constant matrix element quark model it follows:

$$S|I\rangle = -i(2\pi)^4 \sum_{n(\cdot)} \sqrt{\frac{B^{n-3}}{n!}}\, G_I[n(\cdot)] \int \prod_{\mathrm{P},i(\mathrm{P})} \frac{d^3p\{\mathrm{P}, i(\mathrm{P})\}}{2p\{\mathrm{P}, i(\mathrm{P})\}_0 N}$$

$$\times \delta^4(p - \sum_{\mathrm{P},i(\mathrm{P})} p\{\mathrm{P}, i(\mathrm{P})\}) \prod_{\mathrm{P},i(\mathrm{P})} a^{\dagger}(p\{\mathrm{P}, i(\mathrm{P})\}, \mathrm{P})\, |0\rangle. \tag{A15}$$

Using this expression together with Eq. (A10) or (A7) the generating functional V_I can be obtained in the following form:

$$V_I[p; \Phi(\cdot\cdot)] = \frac{\sum_{n(\cdot)} \frac{\prod_{\mathrm{P}} n(\mathrm{P})!}{n!} |G_I[n(\cdot)]|^2 \varrho[n(\cdot); p; B\Phi(\cdot\cdot)]}{\sum_{n(\cdot)} \frac{\prod_{\mathrm{P}} n(\mathrm{P})!}{n!} |G_I[n(\cdot)]|^2 \varrho[n(\cdot); p; B]}. \tag{A16}$$

Here $\varrho[n(\cdot); p; B\Phi(\cdot\cdot)]$ is the same as the phase space integral defined in Eq. (2.8) only B in it is replaced by $B\Phi(p\{\mathrm{P}, i(\mathrm{P})\}, \mathrm{P})$. The expression in Eq. (2.6) for the branching ratio into the final state with occupation numbers $n(\cdot)$ is an immediate consequence of Eq. (A16).

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