THE KLEIN PARADOX AND THE MASS SPECTRA OF THE NEUTRAL VECTOR MESONS

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We use Dirac's equation with a long range harmonic potential to obtain the mass spectra of the neutral vector mesons ρ^0, ω, Φ, K^{0*} and ψ. Our predictions are in fairly good agreement with the experimental results.

1. *Introduction*

In numerous previous works [1] it has been proposed that mesons are formed by pairs of quarks. With this physical picture in mind several simple models of these bound states have been developed in order to calculate their structures and other related properties. Masses and decay widths of these resonances have been calculated with relativistic and non relativistic approaches. Several long range forces of quark confinement have been considered with particular emphasis on gluon exchange, analogous to linear and harmonic potentials.

In the present work we will show that reasonable values are found for the masses of the mesons, using Dirac's equation to describe the quark dynamics and considering the Klein Paradox as the main mechanism for the decay. Our intention is to obtain only the general features of the process and not to find an exact solution of the problem. As will be seen in what follows, the existence of a finite amplitude at infinity for the wavefunction of the two-quark system will be used to evaluate the masses of the resonances.

To treat, in a unified way, systems composed of heavy or light quarks it would be necessary to use a relativistic approach. In the absence of a satisfactory field theoretic scheme [2–5] we suppose Dirac's equation to be a good tool for the study of the two quark systems with equal or unequal masses. We assume also that the quarks interact via a static neutral vector gluon field $V_\mu(x)$ where $\vec{V} = 0$ and $V_4 = V(r) = Kr^2/2 - \Delta$; K being the harmonic constant and Δ another constant which subsumes, in a simple way, the remaining interaction between quarks [6, 7]. However, in a relativistic formalism, the long range character of the interaction gives rise to the Klein Paradox or its manifestations:

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when the potential energy becomes larger and larger with the distance, creation and annihilation processes become increasingly important. One verifies that, instead of bound states, one has "resonances" [2–5] for the quark-antiquark system and this probably occurs due to the instabilities generated by the vacuum polarization processes. In these processes, decay channels appear, for which pairs of quarks are created in order to prevent free quark escape. In the framework of our calculations (first quantization) it is not possible to study in detail the mechanism of pair creation.

2. *Solutions of Dirac's equation*

We use the reduced mass Dirac equation [8], that we interpret as an equation for a single particle of mass $\mu = m_1 m_2/(m_1+m_2)$ and spin 1/2 in an external potential $V(r)$:

$$\frac{df(r)}{dr} = \frac{\chi}{r} f(r) + \left[\frac{\mu c^2 - E^{(\mu)}}{\hbar c} + \frac{V(r)}{\hbar c}\right] g(r), \tag{1}$$

$$\frac{dg(r)}{dr} = -\frac{\chi}{r} g(r) + \left[\frac{\mu c^2 + E^{(\mu)}}{\hbar c} - \frac{V(r)}{\hbar c}\right] f(r), \tag{2}$$

where $g(r)$ and $f(r)$ are the large and small components, respectively, $\chi = -(l+1)$ if $j = l+1/2$ and $\chi = l$ if $j = l-1/2$, and $E^{(\mu)}$ is the energy eigenvalue of the reduced mass system. Note that the total angular momentum J of the meson states is obtained in our scheme by coupling the 1/2 unit of spin to the angular momentum $j = l+1/2$. Putting $K = \mu\omega^2$ and defining $\xi = (\mu\omega/\hbar)^{1/2} r$ and $E^{(\mu)} = \eta\hbar\omega + \mu c^2 - \Delta$, Eqs (1) and (2) become:

$$\frac{df(\xi)}{d\xi} = \frac{\chi}{\xi} f(\xi) + (\varepsilon_- + A\xi^2) g(\xi), \tag{3}$$

$$\frac{dg(\xi)}{d\xi} = -\frac{\chi}{\xi} g(\xi) + (\varepsilon_+ - A\xi^2) f(\xi), \tag{4}$$

where $\varepsilon_- = -\eta\varepsilon$, $\varepsilon_+ = \eta\varepsilon + 2/\varepsilon$, $A = \varepsilon/2$ and $\varepsilon = (\hbar\omega/\mu c^2)^{1/2}$.

We solve Dirac's equation by expanding the large component $g(\xi)$ and the small component $f(\xi)$ into power series [9] that are summed numerically. For large values of ξ, let us say, larger than a critical value ξ_c, we verify that $g(\xi) = if(\xi) = \varphi \exp[i(\varepsilon\xi^2/6+\theta)]$ meaning that there is no bound state for the quark-antiquark system (Klein Paradox) and, consequently, the energy spectrum $E^{(\mu)}$ is continuous. The total energy E of the system is given by $E = Mc^2 = E^{(\mu)} - \mu c^2 + (m_1+m_2)\,c^2 = (n\varepsilon^2/2 + (m_1+m_2)/2\mu)2\mu c^2 - \Delta$. If μ and the harmonic constant K are known, ε is determined (ε is assumed to be the same for all states of the meson).

So, for a given value of ε and for a given angular momentum l, we observe that, in general, $|\varphi|^2$ is larger than or of the same order of $|\psi|^2$, where ψ is the wavefunction of the system for $\xi \lesssim \xi_c$. Only for a few particular values of η, named $\eta^*_{(n,l)}$, $|\psi|^2 \gg |\varphi|^2$. For these particular values $\eta^*_{(n,l)}$ the system presents "resonances". In Fig. 1 are shown the values of $\eta^*_{(n,l)}$ as a function of ε for the particular case $l = 0$. For $l = 1, 2, \ldots$, there are similar curves.

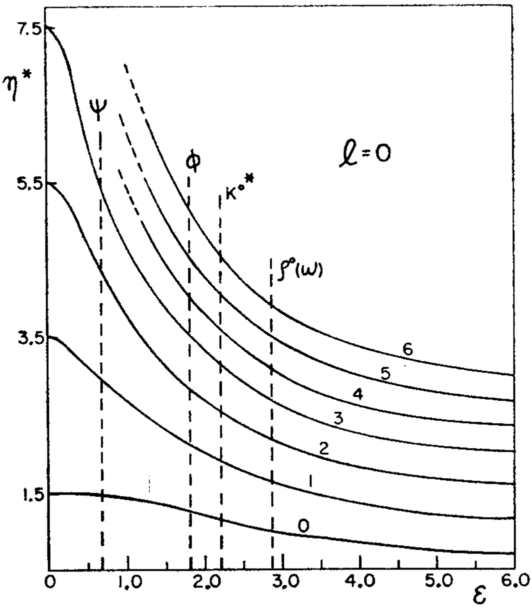

Fig. 1. The parameter $\eta^*_{(n,l)}$ as a function of ε for $l = 0$. The labels 0, 1, 2, ..., correspond to the fundamental, first, second excited state and so on, respectively. The vertical dashed lines correspond to $\rho^0(\omega)$, ϕ, K^{0*} and ψ systems

The mass $M^{(n,l)}$ of the n^{th} resonance, with angular momentum l, is given by

$$M^{(n,l)} = [\eta^*_{(n,l)}\varepsilon^2/2 + (m_1+m_2)/2\mu]2\mu - \Delta/c^2 \tag{5}$$

($n = 0, 1, 2, \ldots$, corresponds, respectively, to the fundamental, first, second excited state, and so on). The value $n^*_{(n,l)}$ for the n^{th} resonance is determined by the intersection of a straight line (passing by a given ε) parallel to the n^* axis with the n^{th} curve $n^*_{(n,l)}(\varepsilon)$ (see, for instance, Fig. 1). We interpret these $M^{(n,l)}$ as being the observed resonant masses of the mesons. When ε tends to zero (non relativistic limit [9]) we see from Fig. 1 that $\eta^*_{(n,l)} \to 2n+l+3/2$, as expected, and consequently, $M^{(n,l)}$ is given by $M^{(n,l)} = \hbar\omega(2n+l+3/2)/c^2+(m_1+m_2)-\Delta/c^2$.

Note that in our scheme it is not possible to distinguish the triplet state ^3l_J from the singlet state ^1l_J. If the calculations are performed in the spirit of the shell model, i.e., introducing separately the quarks 1 and 2 into a fixed potential, similar numerical results are obtained. Of course, a reduced mass Dirac equation for interacting particles with "equal masses" is far from being satisfactory [10], but for our purposes, it is quite sufficient to bring out the essential physical behaviour of the system.

We apply now the formalism developed above to calculate the masses of the neutral particles ρ^0, ω, ϕ, K^{0*} and ψ, with their radial excitations. We consider only neutral particles since the charged states are not relevant in our scheme. Taking into account the experimental values obtained for the mixing angle in the vector meson multiplet we can, to a fairly good approximation, write the physical states of the mesons, with the usual quark model notation, as: $\varrho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\omega = (d\bar{d} + u\bar{u})/\sqrt{2}$, $\phi = s\bar{s}$, $K^{0*} = d\bar{s}$ and $\psi = c\bar{c}$, in a spin triplet configuration.

In our simple model the quarks u and d have equal masses $m_u = m_d = 0.340$ GeV [6], so it is not possible to distinguish ϱ^0 from ω. We put $m_s = 0.540$ GeV [6] and $m_c = 1.640$ GeV [6].

An alternative treatment for the relativistic radial equations for two spin $-1/2$ particles with a static interaction has been developed recently by Królikowski and Rzewuski [11] and by Kluźniak, Królikowski and Rzewuski [12]. We have used the reduced mass equations, since they are more simple than those suggested by these authors and because we verified that, in the case of positronium, both treatments give similar results.

3. *Resulting spectra and comments*

a) ϱ^0, ω mesons

We assume that the ϱ^0, ω mass at the fundamental state is $M_\omega^{(0,0)} = M_\varrho^{(0,0)} = 0.778$ GeV [13]. There is a well established excitation at 1.600 GeV [13]. If we consider that the resonance near to 1.250 GeV [13] is a possible excited state of ϱ^0 we can try to put

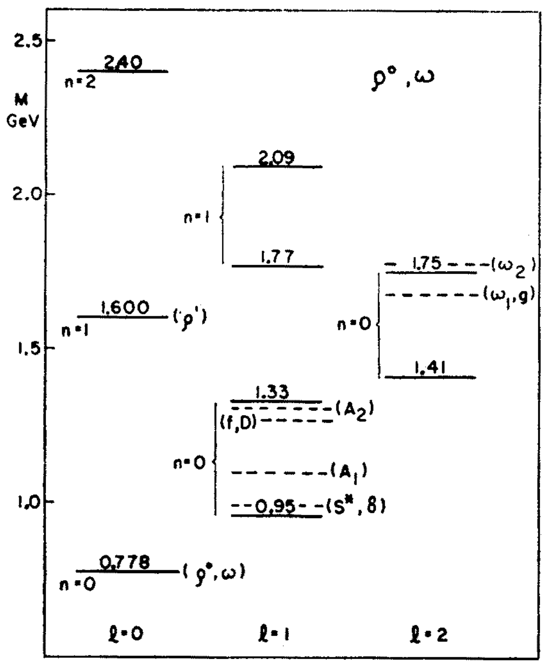

Fig. 2. The mass spectrum for ρ^0, ω mesons. Our theoretical predictions are indicated by (———) and the experimental results by (— — —). In our notation ω_1 refers to $\omega(1.675)$ and ω_2 refers to $\omega(1.780)$ [17] which we interpret as having $l = 2$

$M_{\varrho,\omega}^{(2,0)} = 1.600$ GeV. With this choice we obtain for $l = 1$ and $l = 2$ several resonant masses smaller than 1.300 GeV that are not observed experimentally. This seems to corroborate the hypothesis that the resonance at 1.250 GeV does not correspond to an excited state of ϱ^0 [14, 15].

The next possibility is to take $M_{\varrho,\omega}^{(1,0)} = 1.600$ GeV. In this case we obtain $\varepsilon = 2.86$ and $\Delta = 1.327$ GeV, as unique solutions. The predicted masses for the ϱ^0, ω and experimental results are shown in Fig. 2.

b) ϕ mesons

Here, assuming that $M_{\phi}^{(0,0)} = 1.020$ GeV [13] and that $M_{\phi}^{(1,0)} = 1.820$ GeV [16] we get $\varepsilon = 1.82$ and $\Delta = 1.164$ GeV as unique solutions. The predicted masses for the resonances and experimental results are seen in Fig. 3.

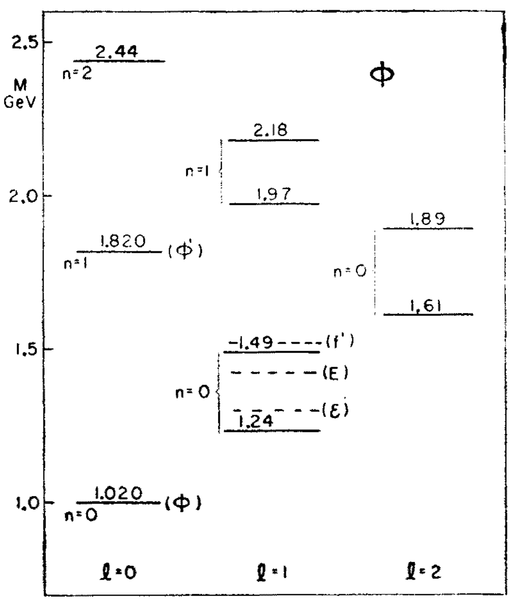

Fig. 3. The mass spectrum for ϕ mesons (conventions as in Fig. 2)

We note that if the resonance at 1.820 GeV were taken as the second excitation, several masses not observed experimentally would appear.

c) K^{0*} mesons

We take $M_{K}^{(0,0)} = 0.890$ GeV [13] and $M_{K}^{(1,0)} = 1.650$ GeV [13] obtaining $\varepsilon = 2.20$ and $\Delta = 1.155$ GeV, as unique solutions. The mass spectrum and the experimental results [13, 18] are seen in Fig. 4.

Also in this case the choice $M_{K}^{(2,0)} = 1.650$ GeV would give spurious resonances.

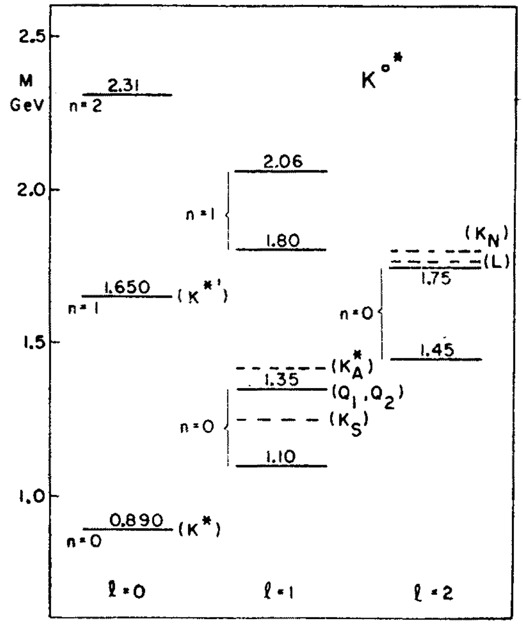

Fig. 4. The mass spectrum for K^{0*} mesons (conventions as in Fig. 2). In our notation Q_1 refers to Q(1.200) and Q_2 refers to Q(1.400)

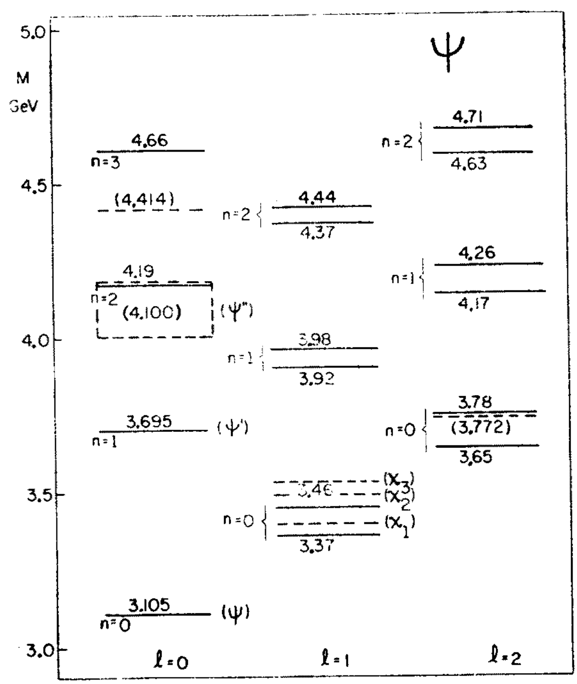

Fig. 5. The mass spectrum for ψ mesons (conventions as in Fig. 2). In our notation χ_1 refers to χ(3.415), χ_2 refers to χ(3.510) and χ_3 refers to χ(3.550)

a) ψ mesons

We assume that the resonance at 3.105 GeV corresponds to the fundamental state and that at 3.695 GeV corresponds to the first excited state, both with $l = 0$. So, using equation (5) and figure 1 we obtain $\varepsilon = 0.71$ and $\Delta = 0.780$ GeV, as unique solutions. The resonances for $l = 1, 2, \ldots$, are determined using these values for ε and Δ since they are taken as independent of the angular momentum l. The predicted masses for the charmonia and experimental results [19, 20] are shown in Fig. 5. We can conclude, from Figs 2–5, that there is a reasonable agreement between theory and experiment.

Our results for $\varrho^0(\omega)$ and ϕ, where relativistic effects are significant, are somewhat different from those obtained by Kang and Schnitzer [2]. They have not calculated the mass spectrum for K^{0*} since their formalism is unable to treat meson systems with unequal mass quarks. Our predicted masses for charmonia are numerically similar to those given, for instance, by Eichten et al. [21], by Harrington et al. [22] and by Kang and Schnitzer [2]. However, in these works, for a given excited state labeled by n, with $l \neq 0$, there appears only one energy level instead of the observed triplets. This occurs since Eichten et al. [21] and Harrington et al. [22] have used Schrödinger's equation and Kang and Schnitzer [2] have considered a simplified single-time two body equation based on classical considerations. In our approach, due to the spin-orbit coupling, we obtain doublets with energy splittings of the order of the experimental values. This permit us to obtain a better agreement with the experimental results.

As a final remark, if we make an analogy with the theory of alpha-decay of nuclei [23], we can estimate the lifetimes τ of the resonances assuming that the probability to find the system in an unbound state is proportional to $|\varphi|^2$. Following a similar procedure used in that theory, with relativistic currents and probability densities, τ is given by $\tau \sim \sqrt{\eta}\, \hbar/(\mu c \varepsilon |\varphi|^2)$. The resulting values for τ range from 10^{-23} s up to 10^{-20} s, which we consider as good estimates for the observed lifetimes, given the crudeness of the calculations.

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