HADRONIC MATTER DISTRIBUTION OF PARTICLES AND NUCLEI

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(*Received June 13, 1978*)

Similarities in the high-energy behaviour, elastic and inelastic collisions, of hadrons and nuclei are interpreted as due to the existence of a universal interacting hadronic matter distribution function. Differences arise from the different geometrical dimensions which are themselves determined by the quark content. We formulate the idea in an eikonal model, the only parameters being geometrical factors, the transverse radius and the opacity parameter. The model describes forward elastic scattering (hadron-hadron, hadron-nucleus, nucleus-nucleus interactions) and particle production at high energies. We discuss the relation of our model to the Glauber model as well as to the quark model and to the geometrical ideas in QCD.

1. *Introduction*

Recently data have been accumulating on high energy inelastic production and elastic scattering in reactions of hadrons with hadrons, hadrons with nuclei, and nuclei with nuclei. In general we shall refer to these data and processes as hadronic data and hadronic processes, respectively. The global features of the hadronic data show some intriguing similarities. If we look at inelastic collisions, we observe that average multiplicities have similar energy dependences and are of the same order in all hadronic processes; the multiplicity distributions show remarkable KNO universality; the fast particle inclusive density is roughly the same in all hadron induced reactions [1]. On the other hand at high energy from hadron-hadron and proton-deuteron data the elastic and inelastic cross-sections show similar rises with the energy, the ratio σ^{el}/σ^{tot} being quite constant [2, 3] and the rise being peripheral in the impact parameter [2, 4]; the differential cross-sections have sharp shrinking peaks in the forward direction followed by a large $|t|$ flattening with or without diffraction zeros [2].

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We believe that these global similarities of the data reflect global similarities of the geometrical distribution of the hadronic matter in hadronic processes: hadron-hadron, hadron-nucleus, and nucleus-nucleus collisions. Leaving for a moment the discussion of the content of our idea we give first its precise formulation in the framework of an eikonal model. Let Im $T_{AB}(s, t)$ be the imaginary part of the high-energy diffractive amplitude in an hadronic AB collision (A, B = hadron, nucleus). We neglect the real part and write Im $T_{AB}(s, t)$ in the impact parameter representation as:

$$\mathrm{Im}\, T_{AB}(s, t) = \bar{R}_{AB}^2 \int_0^\infty \beta d\beta J_0(\beta \bar{R}_{AB} \sqrt{-t}) \left[1 - e^{-\lambda_{AB} G(\beta)}\right], \tag{1}$$

where $\beta \equiv b/\bar{R}_{AB}$ is a scaled impact parameter variable, $G(\beta)$ is an universal function of β, the universal eikonal, and λ_{AB}, $\bar{R}_{AB}$ are reaction dependent parameters. G, λ and $\bar{R}$ may depend on energy. The observed similarities in the data are interpreted as arising from the universality of the function $G(\beta)$. The parameters λ and $\bar{R}$ are parameters characterizing the interacting hadronic region: λ, the opacity parameter, is related to the matter density averaged over the longitudinal range of the interacting region while $\bar{R}$ measures the transverse hadronic size.

Equation (1) is the equation for the scattering of infinitely composed extended objects with localized elementary interactions in the Chou-Yang sense [5]. Universality of $G(\beta)$ arises here from the assumption that these elementary interactions and the matter distribution inside the hadrons (particles and nuclei) are universal. λ and $\bar{R}$ are not universal and this happens because in our picture the amount of interacting matter varies from reaction to reaction. As a consequence of the internal symmetry structure of strong interactions, the matter clusters and the number of clusters is given by the number of valence quarks. So, it is convenient to refer all quantities to the cluster-cluster interaction which we simply call the quark-quark interaction. The integral over the eikonal $\chi(= \lambda G)$

$$I_{AB} = \int \chi_{AB}(\beta) b db = \lambda_{AB} R_{AB}^2 I_{qq}, \tag{2}$$

where I_{qq} is the corresponding integral for the quark-quark interaction, with $\lambda_{qq} \equiv 1$ and $R_{AB} = \bar{R}_{AB}/\bar{R}_{qq}$, if the constituents are not correlated, gives the amount of quark interacting centres in an AB collision. More precisely:

$$\lambda_{AB} R_{AB}^2 = N_A N_B, \tag{3}$$

where $N_A(N_B)$ is the number of quarks in A(B). It will turn out that this equation is a very strong constraint in our approach.

It is perhaps already clear at this stage that we are suggesting to describe high-energy interacting nuclei without making use of nuclear physics. In other words, particles and nuclei are, in our approach, considered as single hadrons ("atomic" bags of quarks) the expectation being that nuclear effects ("molecular" effects) should be negligible. We shall discuss later the extent of the validity of this limit. We would like just to mention now that the hadronic matter we consider here is interacting and highly energetic hadronic matter with properties which may be very different from the properties of the usual static nuclear matter.

The main content of this paper is the discussion of Eqs (1) and (3) (Section 2) with applications to hadron-hadron, hadron-nucleus and nucleus-nucleus collisions (Section 3). In Section 4, we present some conclusions.

2. General discussion of the eikonal formula

Let us now further discuss some properties of our Eq. (1). For the inelastic and total cross-sections we obtain, from Eq. (1),

$$\sigma_{AB}^{in} = \tfrac{1}{2} R_{AB}^2 \sigma_{qq} \Phi(2\lambda_{AB}), \tag{4}$$

and

$$\sigma_{AB}^{tot} = R_{AB}^2 \sigma_{qq} \Phi(\lambda_{AB}), \tag{5}$$

where

$$\Phi(\lambda) = \gamma \int \beta d\beta [1 - e^{-\lambda G(\beta)}], \tag{6}$$

γ being such that $\Phi(1) = 1$ and σ_{qq} being a reaction independent normalization quantity which might be interpreted as the quark-quark total cross-section. This function and all its derivatives are monotonic functions of λ since G is positive. Φ has the limits[1]

$$\Phi(\lambda) \sim \lambda \qquad \text{for } \lambda \to 0, \tag{7}$$

and

$$\Phi(\lambda) \to \infty \qquad \text{for } \lambda \to \infty. \tag{8}$$

The ratio σ^{in}/σ^{tot} is, in the two limits, 1 and $\frac{1}{2}$, respectively, and this tells us that as λ increases (as more quarks are involved) one approaches the black disc limit as experimentally observed for heavy nuclei.

Another parameter of interest in reactions with nuclei is the effective number of collisions $\bar{\nu}_{AP}$ for the reaction of particle P with the nucleus of mass number $A (N_A = 3A)$

$$\bar{\nu}_{AP} \equiv A \frac{\sigma_{pP}^{in}}{\sigma_{AP}^{in}}. \tag{9}$$

Using Eqs (3) and (4) we obtain for $\bar{\nu}_{AP}$,

$$\bar{\nu}_{AP} = \frac{\psi(2\lambda_{pP})}{\psi(2\lambda_{AP})}, \tag{10}$$

where

$$\psi(\lambda) = \frac{1}{\lambda} \Phi(\lambda), \tag{11}$$

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[1] In fact the expansion parameter should by $\lambda G(0)$ instead of simply λ as it is clear from Eq. (1).

is a monotonically decreasing function of λ varying between the constant $\gamma \int \beta d\beta G$ and zero. For light nuclei λ_{AP} is of the order of λ_{pP} and the effective number of collisions is of the order of 1. As $\lambda_{AP} \to \infty$ $\bar{\nu}_{AP}$ tends to infinity but not as fast as λ_{AP}. As we shall see later, this fact explains the weak A dependences of $\bar{\nu}_{AP}$ experimentally observed ($\bar{\nu}_{AP} \sim A^{0.22-0.26}$). Equation (3) suggests that $\lambda_{AP}/\lambda_{pP}$ and R^2_{AP}/R^2_{pP} should be independent of P and be only properties of the nucleus A. In this case we see from Eq. (10) that $\bar{\nu}_{AP}$ increases as the number of quarks in the beam particle P increases. Shadowing is stronger in reactions with protons than in reactions with pions, as is well known.

Finally, we note that $\bar{\nu}_{AP}$ is smaller than 1 of $\lambda_{AP} < \lambda_{pP}$.

3. Application

We try next to test our Eq. (1) and check the constraint (3). For the input $G(\beta)$ function we use the eikonal obtained from the analysis [6] of the ISR pp data [7] at $\sqrt{s} = 53$ GeV. In order to fix λ and R^2 one needs σ^{tot} and $d\sigma/dt$ at small $|t|$ (the slope parameter) or σ^{tot} and σ^{in}. As far as hadron-hadron scatterings are concerned, Eq. (1) was in fact already

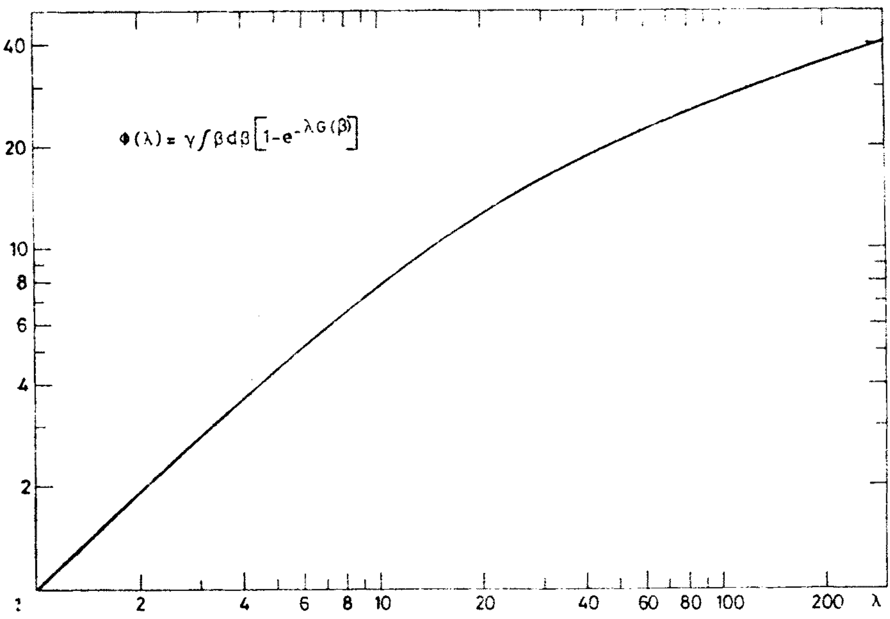

Fig. 1. The function $\Phi(\lambda)$ as defined in Eq. (6)

successfully tested before with the results $\lambda_{pp}/\lambda_{\pi p} \simeq N_p/N_\pi = \frac{3}{2}$, $R_{pp}/R_{\pi p} \simeq 1$ and Eq. (3) being satisfied [6]. For reactions with light nuclei we use pd data [3, 8, 9] (from FNAL and ISR), pHe data [10] (slope and σ^{tot}) and dd ISR preliminary data [9]. Since the various data are not measured at the same energy, we cannot avoid any longer the question of energy dependence. With rising cross-sections some of the parameters or functions in Eq. (1) must have energy dependence. To take that into account we make the simple geometrical scaling approximation [6] of considering λ a constant, $G(\beta)$ a scaling function and σ_{qq} an energy-dependent quantity. In this limit $\Phi(\lambda)$, Eq. (6), is strictly energy-independent and $\sigma^{in}_{AB}/\sigma^{tot}_{AB} = $ const. and $\bar{\nu}_A = $ const. as experimentally observed at high energies.

For later use, we show the function $\Phi(\lambda)$ properly normalized in Fig. 1. From the pp data we can fix σ_{qq}. We arbitrarily choose $R_{pp} = 1$ and find $\sigma_{qq}(\sqrt{s} = 53\text{ GeV}) = 5.92\text{ mb}$[2]. The energy dependence of σ_{qq} is within our model that of σ^{in}_{pp} and can be parametrized as

$$\sigma_{qq} = 4.92\text{ mb}\,(1+0.017\ln\hat{s})^2, \tag{12}$$

($\sqrt{\hat{s}}$, the centre-of-mass energy of the qq system, in GeV).

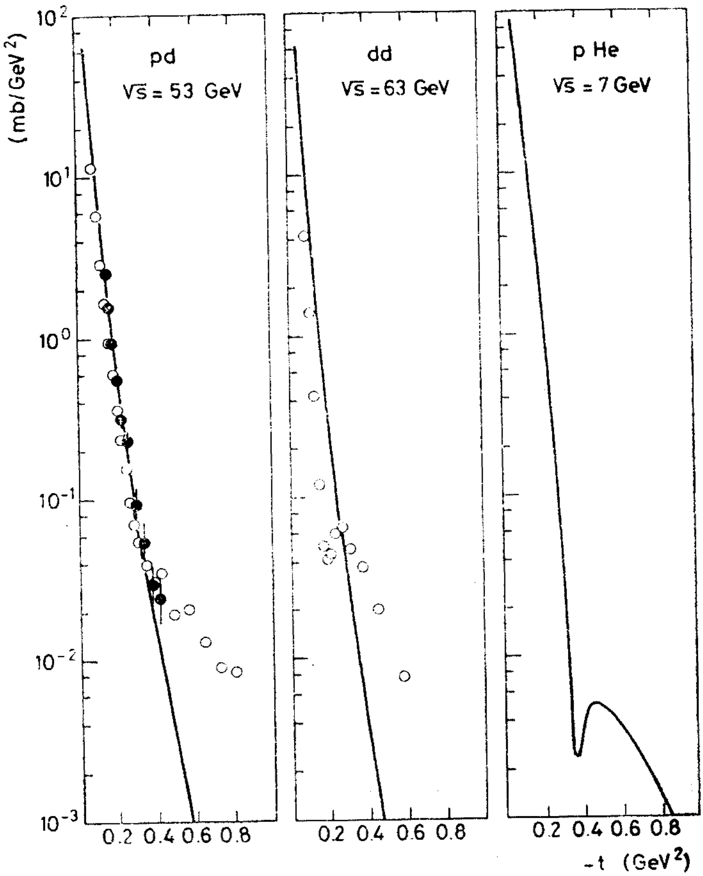

Fig. 2. The differential cross-sections for pd, dd and pHe. Data are taken from Refs. [8] and [9]. The solid lines represent our results. The dip in $d\sigma/dt$(pHe) has been partially filled in by the geometrical scaling real part [6], assuming that the ratio of real to imaginary part in pHe is equal to the corresponding ratio in pp scattering

[2] Within our model only the ratio $\sigma_{qq}/\bar{R}^2_{qq}$ is fixed. So, for instance, if the true quark-quark radius is much larger than $\bar{R}_{pp}$ the quark-quark cross-section is correspondingly larger than our value.

The fits to pd and dd data are presented in Fig. 2 and for comparison we have also shown the predictions for $d\sigma/dt$ of pHe at an energy rather low for our model. The parameters λ and R in this case are fixed from the slope of $d\sigma/dt$ and σ^{tot} of pHe scattering at that energy [10]. The fits are satisfactory in the small $|t|$ region but become rather poor at larger $|t|$ and relation (3) is not well satisfied (see the Table). Note that our model

TABLE

The parameters λ and R^2 and the product λR^2

Reaction	λ	R^2	λR^2
pd	4.3 ± 0.1	3.30 ± 0.04	14.2 ± 0.4
dd	5.2 ± 0.2	5.33 ± 0.06	27.8 ± 1.0
pHe	11.7 ± 0.9	3.07 ± 0.10	35.9 ± 2.8
HeHe	27.0 ± 1.8	5.18 ± 0.30	139.5 ± 9.0
NBe	10.5 ± 0.9	6.24 ± 0.40	65.7 ± 9.0
NC	15.0 ± 2.0	6.10 ± 0.50	91.5 ± 15.5
NAl	28.3 ± 4.5	7.79 ± 0.80	220.4 ± 38.9
NCu	59.4 ± 17.1	10.2 ± 1.5	603 ± 135
NPb	81.0 ± 27.0	21.6 ± 4.0	1755 ± 540
NU	324.0 ± 180.0	15.0 ± 5.0	4860 ± 2700

produces no dip in pd and dd scatterings but in pHe. The number of dips depends on the value of λ. Numerical calculations have shown that for the given function $G(\beta)$ dips appear only if $\lambda \gtrsim 8$.

From the available σ^{in} and σ^{tot} data [10, 11] for heavy nuclei we have calculated λ and R, solving Eqs (4) and (5) numerically with the aid of the function $\Phi(\lambda)$ shown in Fig. 1. The results are also presented in the Table. In Fig. 3 we show a test of relation (3). We have here included our earlier results for hadron-hadron scattering[3]. SU(3) breaking, that is the fact that a strange quark interacts weaker than a non-strange one, is taken into account by counting a strange quark not as 1 but as a smaller number. This number is fixed from the KN data and has the value 0.31.

It is striking that Eq. (3) is rather well satisfied for particles and nuclei. From this point of view it seems reasonable to say that at high energy, interacting hadrons and interacting nuclei behave similarly and both can be considered as single bags of quarks. In the case of light nuclei, however, it seems that care must be taken of nuclear effects. From the derived values for λ and R in the case of the deuteron, we see that the deuteron behaves as a relatively transparent object ($\lambda_{pd} \simeq \lambda_{\phi p} < \lambda_{pp}$) but is very large ($R_{pd} \sim 2R_{pp}$) as expected for a loosely bound system[4]. It is quite obvious that in loosely bound systems

[3] We now add the results for $\Sigma^- p$ ($\lambda_{\Sigma p} = 9.63 \pm 0.9$, $R^2_{\Sigma p} = 0.78 \pm 0.05$) calculated from the data of Blaising et al. [12] at 17.8 GeV.

[4] It is amusing to note that the obtained values for λ_{pd} and R_{pd} are close to the values found in a two-dimensional model of the deuteron as made up of two touching discs (the nucleons): $\lambda_{pd} = \lambda_{pp}/2$, $R_{pd} = 2R_{pp}$.

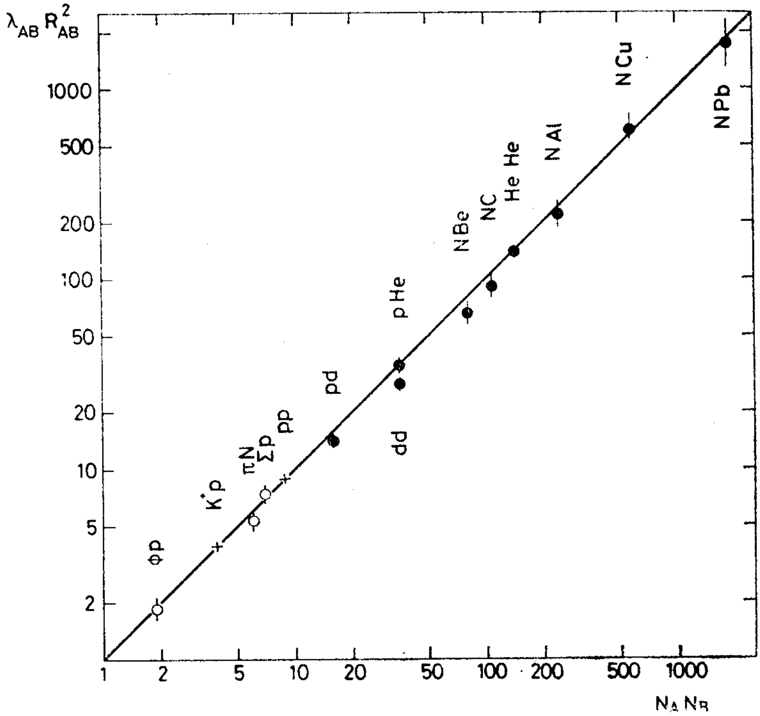

Fig. 3. Test of Eq. (3). The λR^2 values are taken from the Table and from Ref. [6]. pp scattering is used as normalization and K^+p to estimate the effective value of a strange quark

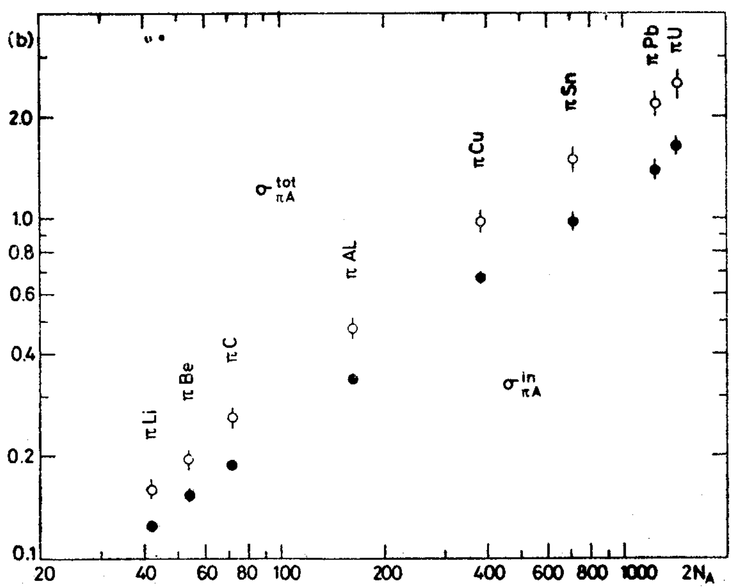

Fig. 4. The total and inelastic cross-sections for pion-nucleus scattering. The $\sigma^{in}_{\pi A}$ data (●) are taken from Denisov et al. [11]. The total cross-sections (○) are our predictions

the nuclear effects should be important. The deuteron cannot be well described as a six--quark bag because it is rather, as well shown by the large radius, a two-nucleon system[5]. As a consequence in interactions with the deuteron and light nuclei the nuclear structure manifests itself in the large $|t|$ region (compare Fig. 2) and the effective number of interacting centres is smaller than what is expected using Eq. (3).

For πA scattering, no σ^{tot} data for heavy nuclei are available but only $\sigma^{in}_{\pi A}$ has been measured [11]. So, we cannot calculate λ and R and check relation (3) further. However, we can turn around our argumentation and use our model to estimate $\sigma^{tot}_{\pi A}$. We assume that Eq. (3) holds and find $\lambda_{\pi A}$ from

$$\sigma^{in}_{\pi A} = N_A \sigma_{qq} \Phi(2\lambda_{\pi A})/\lambda_{\pi A}, \tag{13}$$

making again use of Fig. 1. $\sigma^{tot}_{\pi A}$ is then calculated from Eq. (5). The results are shown in Fig. 4 and are subject to experimental confirmation.

We turn now to inelastic production. In the framework of geometrical models the features of the data mentioned in the beginning and many other properties of the data can be easily reproduced by extending to hadron-nucleus and nucleus-nucleus collisions the geometrical ideas developed for hadron-hadron collisions [14]. As it is well known in collisions of extended objects the number of produced particles is related to the matter overlap. As the distribution function $G(\beta)$ in Eq. (1) is assumed the same in all processes this guarantees that the KNO distribution function is, at least approximately, universal [14][6]. In order to estimate the average multiplicities we note that in an AB collision, each quark in A excites $\bar{\nu}_B$ quarks in B ($\bar{\nu}_B$ is the effective number of collisions in B) and similarly each quark in B excites $\bar{\nu}_A$ quarks in A[7]. In general we thus have

$$\bar{n}_{AB} = \tfrac{1}{2}(\bar{\nu}_A + \bar{\nu}_B)\bar{n}, \tag{14}$$

where $\bar{n}$ is the average multiplicity for the basic quark-quark process. In hadron-hadron processes, shadowing effects are small and $\bar{\nu}_A \simeq \bar{\nu}_B \simeq 1$ or $\bar{n}_{\pi\pi} \simeq \bar{n}_{\pi p} \simeq \bar{n}_{pp} = \bar{n}$. In hadron-nucleus collisions, we recover, from (14), the usual fragmentation formula:

$$\bar{n}_A = \tfrac{1}{2}(1 + \bar{\nu}_A)\bar{n}. \tag{15}$$

For nucleus-nucleus processes, A, B $\equiv$ nucleus, Eq. (14), combined with (15), may be written as

$$\bar{n}_{AB} = \bar{n}_A + \bar{n}_B - \bar{n}. \tag{16}$$

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[5] Matveev and Sorba [13], for example, estimated the probability for the deuteron to be in a six-quark bag. They found a value of the order of 10%.

[6] The KNO universality is not much affected by shadowing because in the definition of KNO function and of the moments of the KNO distribution, all quantities are normalized in such a way that shadow effects in numerator and denominator tend to cancel.

[7] Note that $\bar{\nu}_A$ is here treated in an approximate way being indifferently interpreted as the number of collisions with a quark or with a proton because in collisions with elementary hadrons, shadow effects are small.

Equation (14) in general describes a very slow increase of the multiplicity as the atomic weight increases. In the case of nucleus-nucleus collisions it should be interesting to check the validity of Eqs (15) and (16).

4. Conclusions

We should like now to make some concluding remarks.

4.1. Comparison with the Glauber model

In the Glauber model [15], in particular for low A processes, apart from the knowledge of the basic pp or πp amplitude detailed knowledge is required of the nuclear form factor. The slope of $d\sigma/dt$ at small $|t|$ is essentially an energy-independent quantity and the appearance or not of dips is related, in pd and πd scattering, to the amount of d wave contributions to the wave function. In our case the rough features of the data are fixed by the eikonal and the two parameters λ and R. Shrinkage is expected to occur in a way similar to pp scattering. The possibility of dips is controlled only by the value of λ: for $\lambda \gtrsim 8$ dips must occur at high energies; the number of dips increases with λ.

In applications of large A the Glauber model depends crucially on the geometrical assumption normally made, $R_A \sim A^{1/3}$ and the model in good approximation eikonalizes. For large A our Eq. (1) is then, apart from the energy dependence we easily incorporate, equivalent to the Glauber formula: both Eq. (1) and the Glauber model fit σ^{in} and σ^{tot} successfully.

However, there is one important difference: our eikonal deviates drastically from the Glauber eikonal given by the integral over the nucleon density (e.g., the Wood-Saxon distribution) multiplied by the elementary inelastic cross-section. Also the usual nuclear radius ($\sim A^{1/3}$) is not equal to our parameter R (– which behaves as $A^{0.23}$ –) but rather equal to the radius $\sqrt{\langle b^2 \rangle}$ of the absorption region defined by

$$\langle b^2 \rangle = R^2 \frac{\int \beta^3 d\beta(1-e^{-2\lambda G(\beta)})}{\int \beta d\beta(1-e^{-2\lambda G(\beta)})} . \tag{17}$$

Notice that only for small λ and approximately Gaussian behaviour of G is $\langle b^2 \rangle \simeq R^2$.

4.2. Inclusive longitudinal distribution

Our approach is concentrated on impact parameter and one, so to say, averages over the longitudinal distributions of the hadronic matter. Information on rapidity distributions is then lost. However, one prediction, typical of geometrical models, we can still make, namely we expect strong positive forward-backward correlations. In particular, in a hadron-nucleus h–A collision, we expect

$$\bar{n}_{\text{Back}} \simeq \bar{\nu}_{\text{Ah}} \bar{n}_{\text{For}}, \tag{18}$$

(compare Eq. (15)).

4.3. Geometry and long-range correlations

In the conventional existing models for strong interactions the universality of the eikonal and the determining rôle of the parameters (λ, R) are properties very difficult to understand. Most of the ideas and models in strong interaction high-energy physics (the multiperipheral model, for instance) are based on the dominace of short-range correlations in impact parameter (and in rapidity). Data have been trying to tell us for quite some time that long-range correlations in impact parameter (and in rapidity) may be an essential property of strong interactions. The geometry of strong interactions is not an empty space geometry where the short-range correlation processes occur without much interaction with the boundaries, but rather a dynamical geometry in which the geometrical size in b (and in y) fixes the scale of the dynamics. It is interesting to note that in QCD the dominant high-energy amplitude is only non-vanishing in the presence of physical transverse dimensions and the strength of the effective coupling is somehow related to the transverse hadronic size [16]. The dynamics is then "determined" by the geometry and the geometry "determined" by the quark content — as Eq. (3) and Fig. 3 suggest.

4.4. The unitarity equation

It is well known that elastic and inelastic scatterings are not unrelated processes but, on the contrary, they are connected by the unitarity equation. Most of the models on high energy strong processes involving nuclei are unable to include *both* the elastic and inelastic effects. We think that our geometrical approach being rather naive makes, however, the connection between elastic scattering and inelastic production in a quite natural way. Our results on elastic scattering, Eq. (1) and Fig. 3, as well as our results on particle production, Eqs. (16) and (18) are fairly reasonable.

One of us (P.K.) would like to thank the hospitality of the CERN Theoretical Physics Division. We also would like to thank B. and F. Schrempp for careful reading of the manuscript.

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