HARTREE FOCK CALCULATIONS FOR EVEN-EVEN $N=Z$ NUCLEI IN THE $2s$–$1d$ SHELL USING VELOCITY-DEPENDENT EFFECTIVE POTENTIAL OF s-WAVE INTERACTION

By M. Y. M. Hassan and A. Sh. Ghazal

Physics Department, Faculty of Science, Cairo University*

(Received May 15, 1978)

Single-major shell Hartree-Fock calculations of the doubly even $N = Z$ nuclei in the $2s-1d$ shell have been performed using velocity-dependent effective potential of s-wave interaction. The results are compared with previous calculations as well as with the experimental data. Good agreement is obtained between the calculated binding energies and the experimental ones.

1. Introduction

In the last few years there has been an extensive interest in the use of effective interactions in the Hartree-Fock (HF) approximation to calculate the nuclear properties. In most cases the parameters of the effective interaction are determined by studying the many-body problem, particularly from the requirement that the first order of the perturbation theory yields the correct value of the binding energy (BE) and the density of nuclear matter, and the BE of light nuclei. However these parameters are not related to the parameters of the free nucleon-nucleon force. An attempt in this direction was made by Dzhibuti and Mamasakhlisov [1] who have derived an effective interaction which is directly related to the free nucleon-nucleon force. This effective interaction was modified by Dzhibuti and Sallam [2] (DS) who applied it to calculate the BE of large group of nuclei ranging from ^{4}He to ^{208}Pb using shell model techniques. In this work, single-major shell HF calculation has been performed to even-even, $N = Z$ nuclei in the $2s-1d$ shell using the modified form of the effective interaction introduced by Dzhibuti and Sallam.

Many HF calculations with single major shell and multi-shells have been performed to $2s-1d$ shell nuclei. The first explicit application of single-major shell HF calculation to deformed nuclei of the $2s-1d$ shell was made by Kelson [3]. Ripka [4] has studied in detail single-major shell HF calculations to $2s-1d$ shell nuclei. On the other hand, many multi-shell HF calculations have been performed to $2s-1d$ shell nuclei such as

* Address: Physics Dept., Faculty of Science, Cairo University, Cairo, Egypt.

those of Krieger [5] (K), Muthukrishnan [6] (M), Zofka and Ripka [7] (LINGE) and Lassey et al. [8] (MDIK). We have compared our results with those of K, M, LINGE and MDIK.

In the next Section, a description of the potential and the method of calculation is given. The results and the discussion are given in Section 3.

2. *Description of the potential and method of calculation*

The effective potential used has the following form [2]:

$$V_{\text{eff}}(\vec{r}) = \tfrac{1}{2}\left[V_{\text{real}}(\vec{r})\exp\left(-\vec{a}\cdot\frac{\partial}{\partial\vec{r}}\right)+\exp\left(\vec{a}\cdot\frac{\partial}{\partial\vec{r}}\right)V_{\text{real}}(\vec{r})\right]_{\vec{a}\to\vec{r}}$$

$$-\frac{\lambda(A)\hbar^2}{M}[\delta(\vec{r})\nabla^2+\nabla^2\delta(\vec{r})], \tag{1}$$

where $V_{\text{real}}(\vec{r})$ is the realistic potential parametrized in accordance with the two-nucleon problem in vacuum; it has the form:

$$V_{\text{real}}(\vec{r}) = [a_\tau(\vec{\tau}_1\cdot\vec{\tau}_2)+a_{\sigma\tau}(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2)]\exp(-r^2/r_c^2), \tag{2}$$

where $\vec{\sigma}_i$, $\vec{\tau}_i$ are the spin and isotopic spin operators and the parameters a_τ, $a_{\sigma\tau}$ and r_c are given by:

$$a_\tau = 2.096\ \text{MeV}, \quad a_{\sigma\tau} = 7.767\ \text{MeV}, \quad r_c = 2.18\ \text{fm}. \tag{3}$$

In equation (1) $\vec{a}\to\vec{r}$ denotes that after applying the operators $\exp\left(-\vec{a}\cdot\frac{\partial}{\partial\vec{r}}\right)$ and $\exp\left(\vec{a}\cdot\frac{\partial}{\partial\vec{r}}\right)$ to the two similar wave functions from the RHS and LHS respectively in the two body matrix element it is necessary to make the substitution $\vec{a}\to\vec{r}$. The second term in equation (1) describes the multiparticle effect; this part of the effective potential depends on the parameter $\lambda(A)$ which is a function of the mass number A. The method of single-major shell HF calculations is described in detail by Ripka [4] and is applied in the present work. The HF calculations did not include Coulomb effects. To include such effect we used the formula [2]

$$E_c = 0.71Z^2/A^{1/3}. \tag{4}$$

3. *Results and discussion*

The values of the parameter $\lambda(A)$ were taken from the calculation of DS for ^{28}Si, ^{32}S and ^{40}Ca. With respect to ^{20}Ne and ^{24}Mg these two nuclei were not considered by DS. We have varied λ and the oscillator parameter $\alpha = \sqrt{\frac{m\omega}{\hbar}}$ until a minimum HF energy

is obtained. The parameter α, for ^{28}Si, ^{32}S, and ^{40}Ca was chosen to give the experimental [9] mean square radius R [10],

$$R = (\langle r^2 \rangle / Z)^{1/2},$$

where

$$\langle r^2 \rangle = \frac{1}{\alpha^2}\left[18 + \frac{7M_p}{2}\right].$$

M_p is the number of proton particle orbits outside the core ^{16}O. The values of λ, α, R (theoretical) and R (experimental) are shown in Table I.

TABLE I

The values of the oscillator parameter α (fm^{-1}), the experimental (Ref. [9a, b]) and the theoretical values of the root mean square radius

Nucleus	α(fm^{-1})	R_{th}(fm)	R_{exp}(fm)	λ(fm^3)
^{20}Ne	0.581	2.72	$2.91^{(a)}$	0.18
^{24}Mg	0.561	2.91	$3.03 \pm 0.03^{(a)}$, $3.01 \pm 0.03^{(b)}$	0.5
^{28}Si	0.549	3.04	$3.11 \pm 0.03^{(a)}$, $3.09 \pm 0.02^{(b)}$	2.5
^{32}S	0.531	3.193	$3.25 \pm 0.03^{(a)}$, $3.24 \pm 0.02^{(b)}$	2.8
^{40}Ca	0.501	3.457	$3.48 \pm 0.02^{(a)}$, $3.47 \pm 0.06^{(b)}$	3.9

Table II contains the binding energies per particle of the HF states calculated with various forces as well as with the experimental values [11]. We see from columns 2 and 3 that the binding energy per particle calculated by K and M are considerably smaller than

TABLE II

A comparison between the experimental binding energy per particle (MeV) and the theoretical values

Nucleus	Exp.	Present work	K	M	LINGE	MDIK
^{20}Ne	−8.00	−8.29	−4.76	−4.31	−7.15	−7.50
^{24}Mg	−8.20	−7.69	−4.88	—	−7.20	−7.53
^{28}Si$_{ob}$	−8.42	−8.28	—	−4.66	−7.48	−7.88
^{28}Si$_{pr}$	—	−8.25	—	−4.65	−7.37	−7.69
^{32}S	−8.46	−8.22	−5.94	—	−7.57	−7.77
^{40}Ca	−8.55	−7.95	−7.07	−5.71	−8.17	−8.40

the experimental values. The values obtained by K increase faster with particle number than either the experimental values or those of the other calculations which may be due to a too strongly repulsive surface energy, while the values given by Zofka and Ripka (LINGE) are in better agreement with the experiment but are still smaller than the experi-

mental data. The calculations of Lassey et al. (MDIK) are slightly greater than those of LINGE and so are nearer to the experimental values. The values found in the present work are in good agreement with experiment as shown in Table II.

Ripka [4] has calculated the charge quadrupole moment using single major shell HF approximation. The results of his calculation show that the values obtained are very much smaller than the experimental data. The values we have obtained for the charge quadrupole moment for ^{20}Ne and ^{24}Mg are 21.57 and 16.27 fm^2 compared to the experimental values 54 ± 3 [12] and 67 ± 4 fm^2 [13], respectively.

Table III gives the values of the energy gaps calculated in the present work as well as those obtained by LINGE and MDIK forces. Our values are less than those obtained using LINGE and MDIK forces. This may be attributed to the absence of a *p*-state term in the interaction as pointed out by Khadkikar [14]. Also Ripka [4] has shown that the gap is due to the large Majorana exchange component of the force. In this work the second part of the potential does not contain any Majorana exchange component which may explain the small values obtained for the energy gaps. Indeed, all the calculated values of the energy gap are less than the experimental ones as can be shown from Table III.

TABLE III

A comparison between the experimental energy gaps (MeV) (Ref. [10]) and the theoretical calculations [7, 8]

Nucleus	Exp.	Present work	LINGE	MDIK
^{20}Ne	10.11	2.0	6.9	6.2
^{24}Mg	9.21	2.9	4.8	2.8
$^{28}Si_{ob}$	8.69	4.7	6.4	7.5
$^{28}Si_{pr}$		3.2	8.4	4.5
$^{32}S_{pr}$	6.43	8.1	5.1	3.7

In conclusion one may hope to obtain better results by modifying the potential in such a way as to contain a *p*-state term. Also the assumption of an inert core (which leads to small quadrupole moments) must be released. Such calculations are now in progress.

REFERENCES

[1] R. I. Dzhibuti, V. I. Mamasakhlisov, *Soobschenie* (Communications), *Akad. Nauk. Gruz. SSR* **54**, 57 (1969).

[2] R. I. Dzhibuti, H. M. Sallam, *Sov. J. Nucl. Phys.* **19**, 40 (1974).

[3] I. Kelson, *Phys. Rev.* **132**, 2189 (1963).

[4] G. Ripka, *Advances in Nuclear Physics*, Vol. 1, Plenum Press, New York 1968.

[5] S. J. Krieger, *Phys. Rev.* **C1**, 76 (1970).

[6] R. Muthukrishnan, *Nucl. Phys.* **A93**, 417 (1967).

[7] J. Zofka, G. Ripka, *Nucl. Phys.* **A168**, 65 (1971).

[8] K. R. Lassey, M. R. Manning, A. B. Volkov, *Can. J. Phys.* **51**, 2522 (1973).

[9] a) A. Nakada, Y. Torizuka, Y. Horikawa, *Phys. Rev. Lett.* **27**, 745 (1971); I. Sick, J. S. Mc Carthy, *Nucl. Phys.* **A150**, 631 (1970); Y. Horikawa, Y. Torizawa, A. Nakada, S. Mitsunobu, Y. Kojima, M. Kimura, *Phys. Lett.* **B36**, 9 (1970); G. C. Li, I. Sick, M. R. Yearian, *Phys. Lett.* **35B**, 217 (1971); B. B. P. Sinha, G. A. Peterson, I. Sick, J. S. Mc Carthy, *Phys. Lett.* **35B**, 217 (1971). b) C. S. Wu, L. Wilets, *Ann. Rev. Nucl. Sci.* **19**, 527 (1969); H. L. Acker, G. Backenstross, C. Daum, J. C. Sens, S. A. De Witt, *Nucl. Phys.* **87**, 1 (1966).

[10] G. Ripka, *Fundamentals in Nuclear Theory*, IAEA, Vienna 1967, STI/PUB/145.

[11] Robert A. Howard, *Nuclear Physics*, Wadsworth Publishing Company, INC. Belmont, California 1963.

[12] A. E. Litherland et al., *Can. J. Phys.* **45**, 1901 (1967).

[13] H. Hultzsch, P. Junk, G. Mulhaupt, Ph. D. Thesis at Naturwiss. Fak. Johannes Gutenberg University, Mainz, Germany 1969.

[14] S. B. Khadkikar, *Phys. Lett.* **36B**, 451 (1971).