DISSIPATION IN THE HAGEDORN EARLY UNIVERSE

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The viscosity coefficients of the Hagedorn hadronic matter are obtained by solving the Boltzmann kinetic equation in the relaxation-time approximation. The upper limit for the increase of entropy per baryonic charge was found to be $< 10^{-4}$ in the "hadron era" of the Friedman universe and the model behaves almost like dust-filled models. There is no anisotropy damping in the Bianchi type I universe but a substantial growth of entropy is possible in that era when anisotropy has an extremely high value.

Introduction

The entropy per baryon Σ ($= s/kn$, where s is the entropy density, n — baryonic charge density, k — Boltzmann constant) is one of the observational features of the Universe. Its rather high value ($\sim 10^8$) can be interpreted either as a particular initial condition for the Universe, or as a result of cosmological evolution. The second point of view has no strong theoretical support (see e.g., [1]–[6]). The present work is intended as a study of a new possibility. It is the Universe filled, in the hadron era, with fluid obeying the Hagedorn equation of state [7], which allows particles to be non relativistic as the energy density approaches to infinity. This equation results from a bootstrap model for hadrons when assuming an exponential mass spectrum of hadrons (resonances), and consequently the universal maximum temperature T_0. We are attempting to answer the question how these properties can affect cosmological evolution and entropy production processes in Friedman and anisotropic Bianchi type I models near the initial singularity. We are dealing with this equation of state following Sisteró's argument [8] that it should be valid up to the quantum threshold.

The other important problem we are dealing with and which can be strictly connected with the first one, is the damping of anisotropies in the early evolution of cosmological models to the observed — from relic 3 K radiation — level ($< 10^{-3}$; see [9]). As suggested by Misner [10], the viscosity could be an efficient damping factor. To obtain viscosity

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coefficients, the relativistic Boltzmann kinetic equation is solved in the relaxation-time approximation as proposed by Anderson [11]. The CGS units are used. The signature of the metric is $(+---)$, Greek indices run 0, 1, 2, 3 values.

1. The viscosity of Hagedorn fluid

We apply the procedure of Appendix I for the Hagedorn fluid as referred to a local Lorentz reference frame. The equilibrium distribution function is

$$f_0(p^\mu, T)dmcd^3p = h^{-3}\left(\exp\left[\frac{\sqrt{m^2c^4+\bar{p}^2c^2}}{kT}\right]\pm 1\right)\varrho(m)dmcd^3p$$

$$= h^{-3}\sum_{n/1}^{\infty}\varrho(m,n)\exp\left[-n\frac{\sqrt{m^2c^4+p^2c^2}}{kT}\right]dmcd^3p, \tag{1.1}$$

where h is the Planck constant, c — speed of light, $\varrho(m)$ — density of hadron states, the sign + for fermions (f) and the sign − for bosons (b), $\varrho(m,n) = \varrho_b-(-1)^n\varrho_f$. A non--equilibrium correction to f_0 is τ_{rel} (= a relaxation time) times

$$f_1(p^\mu, T) = h^{-3}\frac{p^\mu p^\gamma}{\sqrt{m^2c^2+p^2}}(\beta u_\mu)_{,\gamma}\sum_{n/1}^{\infty}n\varrho(m,n)\exp\left[-n\beta\sqrt{m^2c^2+p^2}\right],$$

where $\beta = c/kT$. It is a good approximation if we take only the first term of the above series. Then $\varrho(m,1) = \varrho_b+\varrho_f = \varrho_{\mathrm{Hagedorn}} \equiv \varrho_H$ and

$$f_1(p^\mu, T) = h^{-3}\frac{p^\mu p^\gamma}{\sqrt{m^2c^2+p^2}}(\beta u_\mu)_{,\gamma}\varrho_H(m). \tag{1.2}$$

A non-equilibrium part of the energy momentum tensor is

$$T_1^{\mu\nu} = \frac{\tau_{rel}}{h^3}(\beta\mu_\gamma)_{,\sigma}\int\frac{p^\mu p^\nu p^\gamma p^\sigma}{m\sqrt{m^2c^2+p^2}}\exp\left(-\beta\sqrt{m^2c^2+p^2}\right)\varrho_H(m)dmcd^3p, \tag{1.3}$$

where

$$\varrho_H(m) = \frac{a\exp[mc^2/kT_0]}{(m_0^2c^2+m^2c^2)^{5/4}} \underset{m\gg m_0}{\cong} \frac{a\exp[\beta_0 mc]}{(mc)^{5/2}} \equiv \varrho_A(m), \tag{1.4}$$

$a = 2\cdot 10^{-9}\,\mathrm{erg}^{3/2}c^{-3/2}$, $kT_0 = 160$ MeV $= 2.6\cdot 10^{-4}$ erg, $m_0 = 500$ MeV. Decomposition of $T_1^{\mu\nu} = X_{\gamma,\sigma}C^{\mu\nu\gamma\sigma}$, $X_\gamma \equiv \frac{\tau_{rel}}{h^3}\beta u_\gamma$, taking into account the asymptotic behaviour (1.4) ϱ_A, allows one to find viscous terms in $T^{\mu\nu}$ (see Appendix I). We shall use the following abbreviation

$$A_{(i,k)}(\beta) \equiv 4\pi ac\beta^{-7/2}\int_{\beta m_* c}^{\infty}dzz^{5/2-k}\exp[z\beta_0/\beta]K_i(z), \tag{1.5}$$

where $m_* c^2 \gg kT$ is a limiting value of mass arising from the substitution of the asymptotic formula for ϱ_H. The approximation used here is physically irrelevant in the limit $T \to T_0$. $\beta_0 = c/kT_0$ and $K_i(z)$ denotes the modified Bessel function of the second kind. With these abbreviations we get viscosity coefficients in the form

$$\chi = \frac{\tau_{\mathrm{rel}}}{h^3}\beta^2[40\Lambda_{(1,5)}+20\Lambda_{(0,4)}+7\Lambda_{(1,3)}+\Lambda_{(0,2)}], \tag{1.6a}$$

$$\zeta = \frac{\tau_{\mathrm{rel}}}{h^3}\beta\left[\tfrac{5}{8}(8\Lambda_{(1,5)}+4\Lambda_{(0,4)}+\Lambda_{(1,3)})-\left(\frac{\partial p}{\partial \varepsilon}\right)_n \right.$$
$$\left.\times(40\Lambda_{(1,5)}+20\Lambda_{(0,4)}+7\Lambda_{(1,3)}+\Lambda_{(0,2)})\right], \tag{1.6b}$$

$$\eta = \frac{\tau_{\mathrm{rel}}}{h^3}\beta[8\Lambda_{(1,5)}+4\Lambda_{(0,4)}+\Lambda_{(1,3)}]. \tag{1.6c}$$

It must be pointed out that the δ coefficient in $T^{\mu\nu}$ does not vanish and it is of the magnitude order of χ, when $T \to T_0$. The Weinberg procedure (Appendix II) enables one to put $\delta = 0$ with a newly defined temperature $\tilde{T}$, but the resulting $\chi(\tilde{T})$, $\zeta(\tilde{T})$, $\eta(\tilde{T})$ are qualitatively the same (as $T \to T_0$) as in the above ones. It seems that in our rough approximation, it is not necessary to put $T^{\mu\nu}$ into the Eckart form. Therefore, in the following we will use relations (1.6a, b, c).

From Hagedorn paper we borrow the asymptotic form $\partial p/\partial\varepsilon = (T_0 - T)/T_0$ and then using (1.6), we get the asymptotic form of viscosity coefficients $\Bigl($all terms $f(T)\colon f(T)/\ln\dfrac{T_0-T}{T_0} \to 0$ $(T \to T_0)$ are omitted. The asymptotic from of $K_i(z) = \sqrt{\dfrac{\pi}{2z}}\,e^{-z}$ is used because in (1.5) the lower limit of integration $\beta m_* c \gg 1\Bigr)$:

$$\chi_A = \Omega\left[\frac{T_0}{T_0 - T} + 7\ln\frac{T_0}{T_0 - T}\right], \tag{1.7a}$$

$$\zeta_A = \tfrac{5}{3}\,\Omega\beta^{-1}\ln\frac{T_0}{T_0 - T}, \tag{1.7b}$$

$$\eta_A = \Omega\beta^{-1}\ln\frac{T_0}{T_0 - T} \tag{1.7c}$$

where $\Omega \equiv \tau_{\mathrm{rel}}ac/h^3(2\pi\beta)^{3/2}$.

2. *The viscous dissipation in the Friedman universe*

In the universe with the Robertson–Walker metric, Einstein equations take the Friedman form (with $\Lambda = 0$):

$$\dot{R}^2 = \tfrac{1}{3}\kappa\varepsilon R^2 - kc^2, \tag{2.1}$$

$$(\varepsilon R^3)^{\cdot} = -p(R^3)^{\cdot} + 9\zeta R\dot{R}^2. \tag{2.2}$$

Here $\kappa = 8\pi G/c^2$. In equation (2.2) the first term on the right hand side describes the adiabatic expansion and the second term is a dissipative one (only the bulk viscosity does not vanish). $(\ )^{\cdot}$ indicates a derivative with respect to time.

There is a thermodynamic relation [12]:

$$\frac{dp}{dT} = \frac{1}{T}(p+\varepsilon).$$

A curvature term in (2.1) is negligible near the singularity, therefore, using (2.2) and (2.1) one obtains the equation describing an influence of bulk viscosity on the early evolution of Friedman models:

$$\left[\frac{\varepsilon+p}{T}R^3\right]^{\cdot} = 3\zeta\frac{\kappa\varepsilon}{T}R^3. \tag{2.3}$$

Two cases should be considered:

i. The adiabatic case $\zeta = 0$. There is no entropy production. Thus, the entropy per baryon does not change with time. The quantity $(\varepsilon+p)/T$ is proportional to the entropy density.

ii. The viscous case $\zeta = \zeta_A$. For two time instances, given by their temperatures T_* and T, from (2.3) one obtains

$$\Sigma/\Sigma_* = \frac{\varepsilon+p}{T}R^3 \Big/ \left(\frac{\varepsilon_*+p_*}{T_*}R_*^3\right).$$

Using (2.1) and the abbreviations $(\varepsilon+p)/T = x$, $3\zeta(\kappa\varepsilon)^{1/2}/T = f(x)$, it is easy to obtain:

$$\ln(\Sigma/\Sigma_*) = \int\limits_{x(T)}^{x(T_*)} \frac{f(x)}{x(x-f(x))}\,dx, \tag{2.4}$$

where $*$ denotes the initial value of any quantity. From Hagedorn's paper we have $\varepsilon/c^2 = 1.26 \cdot 10^{14}\,T_0/(T_0-T)\ \mathrm{g\cdot cm^{-3}}$ and $p = \varepsilon_0 \ln\dfrac{\varepsilon}{\varepsilon_0}$ in the asymptotic region $T \to T_0$. To find the upper limit on ζ, one has to take τ_{rel} of the order of the age of the universe, t, but not shorter than a characteristic time of strong interactions $\sim 10^{-23}$ s (approximation limit). Then $\zeta \cdot \theta < 10^{-2}p$, and the condition $T^{\mu\nu}_{\mathrm{visc}} \ll p$ is satisfied in the whole range of validity of our approximation $1-10^{-38} > T/T_0 > \frac{1}{2}$, where the upper limit follows from the inequality $t > 10^{-23}$ s, the lower limit is a limit of the validity of the Hagedorn description of the cosmological matter. In that range the entropy production is negligible $(\Sigma-\Sigma_*)/\Sigma_* < 10^{-4}$.

3. The role of shear viscosity in a simple anisotropic (Bianchi type I) model

Among the spatially homogeneous anisotropic models, Bianchi type I models which have a simple Euclidean homogeneity group possess the simplest structure. The line element has the form:

$$ds^2 = dt^2 - R^2(t)\,(e^{2\beta_1}dx^2 + e^{2\beta_2}dy^2 + e^{2\beta_3}dz^2), \tag{3.1}$$

where $\beta_1+\beta_2+\beta_3 = 0$ and β's are functions of time only. We omit the bulk viscosity term in the energy momentum tensor. Then,

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p h^{\mu\nu} + 2\eta h^{\mu\varrho} h^{\nu\gamma} \sigma_{\varrho\gamma} \tag{3.2}$$

and Einstein equations take the form:

$$\kappa\varepsilon = 3\frac{\dot{R}^2}{R^2} - \frac{\sigma^2}{2}, \tag{3.3a}$$

$$\kappa p = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{\sigma^2}{2}, \tag{3.3b}$$

$$\left(3\frac{\dot{R}}{R} + 2\kappa\eta\right)\dot{\beta}_i + \ddot{\beta}_i = 0; \qquad i = 1, 2, 3, \tag{3.3c}$$

where $\sigma = \sqrt{\dot{\beta}_1^2+\dot{\beta}_2^2+\dot{\beta}_3^2}$ describes the anisotropic part of the expansion. This set of equations can be solved when the equation of state is given. If one takes Hagedorn's equation, then, in the asymptotic limit $T \to T_0, p \ll \varepsilon$ always holds and it is a very good approximation to put

$$p = 0. \tag{3.4}$$

In that particular case it is easy to obtain solutions from (3.3a, b, c):

$$\kappa\varepsilon R^3 = Q + \int 2\kappa\eta\sigma^2 R^3 dt, \qquad Q = \text{const.}, \tag{3.5}$$

$$\dot{\beta}_i R^3 = C_i \exp\left[-\int 2\kappa\eta dt\right]; \qquad C_i = \text{const.}, \qquad i = 1, 2, 3, \tag{3.6}$$

$$R = [At(t+B)]^{1/3}, \qquad A, B = \text{const.} \tag{3.7}$$

We can follow (3.6) to derive

$$\sigma = C \exp\left[-\int 2\kappa\eta dt\right]. \tag{3.8}$$

The strongly anisotropic $B \gg t$ and the nearly isotropic $B \ll t$ cases of (3.7) have the simple form $R_a = R_1 t^{1/3}$ and $R_i = R_2 t^{2/3}$ respectively, R_1, R_2 = const. As in the end of [2], we estimate in (3.6), (3.8) the value $\int_{10^{-23}}^{10^{-4}} \kappa\eta dt < 10^{-6}$. The conclusion is that there is not anisotropy damping in the hadron era. On the other hand, energy εR^3 (and entropy $\varepsilon/T \cdot R^3$) can rise substantially as can be seen from (3.5), but it is possible only for very high anisotropies (i.e., "anisotropy energy density" $\sigma^2/2$ by many orders of magnitude exceeds the matter energy density $\kappa\varepsilon$). It is not clear whether our approximation can be applied in that extreme case.

4. *Conclusions*

The energy momentum tensor for a homogeneous, viscous hadron liquid described by the Hagedorn equation of state has a dust-like form for very high energy densities ($T \to T_0$). There is negligible entropy production in isotropic Friedman models as in anisotropic Bianchi type I models except for extremely high anisotropies ($\sigma^2/2 \gg \kappa\varepsilon$). No viscous anisotropy damping appears.

Because of strong anisotropy damping that takes place in the lepton era and because the ratio $\sigma^2/\kappa\varepsilon \sim R^{-3}$ (adiabatic case) grows very quickly when $R \to 0$, it is possible to reconcile the observed high degree of isotropy of the microwave background with the extremely high anisotropies in the hadron era only if the mentioned anisotropy is allowed by the "matter creation" processes.

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APPENDIX I

The relaxation-time approximation procedure

The Boltzmann relativistic kinetic equation in the relaxation-time approximation with vanishing external and self-consistent forces has the form:

$$p^\mu f_{;\mu} = -\frac{f-f_0}{\tau_{\mathrm{rel}}}\, p^\mu u_\mu$$

where f, f_0 are one particle distribution functions: near equilibrium and in equilibrium, respectively; τ_{rel} is the relaxation time for the system considered. Decomposition $f = f_0 + \tau_{\mathrm{rel}} f_1 + \ldots$ allows one, when $O(\tau_{\mathrm{rel}}^2)$ terms and gradients of f_1 are neglected, to obtain

$$f_1 = -p^\mu f_{0;\mu}(p^\nu u_\nu)^{-1}.$$

The energy momentum tensor

$$T_1^{\mu\nu} = \int p^\mu p^\nu m^{-1} f \sqrt{-g}\, d^4p$$

can be split into an "equilibrium part" and a "dissipative part". The last part is

$$T_1^{\mu\nu} = \tau_{\mathrm{rel}} \int p^\mu p^\nu m^{-1} f_1 \sqrt{-g}\, d^4p.$$

If the only vector in f_1 is u^μ, then we can decompose it as follows

$$T_1^{\mu\nu} = X_{\gamma;\sigma} C^{\gamma\sigma\mu\nu},$$

where X_γ is a function of u^μ and T only, and

$$C^{\gamma\sigma\mu\nu} = \alpha u^\mu u^\nu u^\gamma u^\sigma + \beta(u^\gamma u^\sigma g^{\mu\nu} + u^\mu u^\nu g^{\gamma\sigma} + u^\gamma u^\mu g^{\nu\sigma} + u^\gamma u^\nu g^{\mu\sigma} + u^\sigma u^\mu g^{\nu\gamma} + u^\sigma u^\nu g^{\mu\gamma})$$
$$+\gamma(g^{\gamma\sigma}g^{\mu\nu} + g^{\gamma\mu}g^{\nu\sigma} + g^{\gamma\nu}g^{\mu\sigma}).$$

It is easy to find α, β, γ coefficients. With abbreviations

$$C_1 = u_\mu u_\nu u_\gamma u_\sigma C^{\mu\nu\gamma\sigma}, \quad C_2 = u_\mu u_\nu g_{\gamma\sigma} C^{\mu\nu\gamma\sigma}, \quad C_3 = g_{\mu\nu} g_{\gamma\sigma} C^{\mu\nu\gamma\sigma}$$

they are

$$\alpha = \tfrac{1}{15}(48C_1 - 36C_2 + 3C_3), \quad \beta = \tfrac{1}{15}(-6C_1 + 7C_2 - C_3),$$
$$\gamma = \tfrac{1}{15}(C_1 - 2C_2 + C_3).$$

The energy momentum tensor, for a viscous fluid not far from the equilibrium can be decomposed as follows (we omit an "ideal fluid part")

$$T_1^{\mu\nu} = (\delta+\tau)u^\mu u^\nu - \tau g^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \pi^{\mu\nu},$$

where $\delta = T_1^{\mu\nu} u_\mu u_\nu$; $\tau = -\frac{1}{3}(T_1^{\mu\nu} g_{\mu\nu} - \delta)$; $q^\mu = T_1^{\mu\nu} u_\nu - \delta u^\mu$ and $q^\mu u_\mu = 0$; $\pi^{\mu\nu} = T_1^{\mu\nu} - (\delta+\tau)u^\mu u^\nu + \tau g^{\mu\nu} - q^\mu u^\nu - q^\nu u^\mu$ and $\pi^{\mu\nu} g_{\mu\nu} = \pi^{\mu\nu} u_\nu = 0$. To get $T^{\mu\nu}$ in the Eckart form (i.e., $T^{\mu\nu} u_\mu u_\nu$ = energy density = ε) one must put $\delta = 0$ (see Appendix II).

With the condition of non-decreasing entropy, it is easy to obtain

$$\tau = -\zeta\theta, \quad q^\mu = \chi h^{\mu\nu}(T_{;\nu} - T\dot{u}_\nu), \quad \pi^{\mu\nu} = -2\eta h^{\mu\gamma} h^{\nu\varrho} \sigma_{\gamma\varrho},$$

where ζ, η, χ are bulk viscosity, shear viscosity and heat conduction coefficients respectively, and: $\theta = u^\gamma{}_{;\gamma}$, $\dot{u}^\gamma = u^\gamma{}_{;\varrho} u^\varrho$, $h^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$, $\sigma_{\mu\nu} = u_{(\mu;\nu)} - \frac{1}{3} h_{\mu\nu}\theta - \dot{u}_{(\mu} u_{\nu)}$. There are a few problems connected with the substitution method of the viscosity coefficients calculation which have not been mentioned here. The reader is advised to consult the original references.

APPENDIX II

The Weinberg procedure to convert $T^{\mu\nu}$ into the Eckart form

Let

$$T^{\mu\nu} = (\varepsilon+p)u^\mu u^\nu - p g^{\mu\nu} + (\delta+\tau)u^\mu u^\nu - \tau g^{\mu\nu} + \ldots$$

(δ and τ are defined in Appendix I). We define a new temperature $\tilde{\beta} : \beta = \tilde{\beta} + \alpha$, where $\alpha = O(\tau_{\rm rel})$. Then,

$$\varepsilon(\beta, \varrho) = \varepsilon(\tilde{\beta}, \varrho) + \left(\frac{\partial \varepsilon}{\partial \tilde{\beta}}\right)_\varrho \alpha + \ldots$$

$$p(\beta, \varrho) = p(\tilde{\beta}, \varrho) + \left(\frac{\partial p}{\partial \tilde{\beta}}\right)_\varrho \alpha + \ldots$$

and

$$T^{\mu\nu} = (\tilde{\varepsilon}+\tilde{p})u^{\mu}u^{\nu}-\tilde{p}g^{\mu\nu}+(\tilde{\delta}+\tilde{\tau})u^{\mu}u^{\nu}-\tilde{\tau}g^{\mu\nu}+ \dots$$

where

$$\tilde{\varepsilon} \equiv \varepsilon(\tilde{\beta}, \varrho), \quad \tilde{p} \equiv p(\tilde{\beta}, \varrho), \quad \tilde{\delta} \equiv \delta+\left(\frac{\partial\varepsilon}{\partial\tilde{\beta}}\right)_{\varrho}\alpha, \quad \tilde{\tau} \equiv \tau+\left(\frac{\partial p}{\partial\tilde{\beta}}\right)_{\varrho}\alpha.$$

To obtain the Eckart form ($T^{\mu\nu}u_{\mu}u_{\nu}$ = energy density = ε), for $T^{\mu\nu}$, we now require $\tilde{\delta} = 0$, hence

$$\alpha = -\left(\frac{\partial\varepsilon}{\partial\tilde{\beta}}\right)_{\varrho}^{-1}\cdot\delta$$

and

$$\tilde{\tau} = \tau-\left(\frac{\partial p}{\partial\tilde{\beta}}\right)_{\varrho}\left(\frac{\partial\varepsilon}{\partial\tilde{\beta}}\right)_{\varrho}^{-1}\delta.$$

This procedure converts $T^{\mu\nu}$ into the form needed but it provides a new temperature $\tilde{\beta}$ which has no clear physical meaning. It is valid only for small deviations from the equilibrium state, where τ_{rel} is very short.

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