DEEP INELASTIC STRUCTURE FUNCTIONS AND THE QUARK PARTON MODEL

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We present calculations of deep inelastic electromagnetic and weak structure functions, both spin-averaged and spin-dependent, in the quark parton model. We use a framework of the parton model with arbitrary transverse momenta of nucleon constituents. The already known results are also included, in order to have a comprehensive review of the whole subject. The cross sections for neutrino induced scattering on polarized targets are also discussed.

1. Introduction

Deep inelastic lepton-nucleon scattering gives us important information about the internal structure of a target nucleon. This knowledge is contained in structure functions which may be extracted from measured cross sections. Bjorken hypothesis [1] that these functions scale (i.e. depend on one dimensionless variable only) was confirmed in many experiments, apart from rather small scaling violations (however, in this paper we shall ignore them). The structure functions were studied in different frameworks, e.g. of the quark parton model [2, 3] and the MIT bag model [4]. The light-cone analysis[1] [5, 6, 7] and the double spectral viewpoint [8, 9] were also used in these investigations (we have just mentioned a few of the methods and authors). Such different techniques give in many cases the same results, e.g. scaling, several relations among structure functions and certain sum rules are common features of all approaches.

We choose the quark parton model (QPM, for short) with arbitrary transverse momenta of nucleon constituents as a framework. We concentrate upon the structure functions measured in a scattering of high-energetic (in the laboratory frame) neutrinos on polarized nucleons. Although it is very difficult to build up the polarized target for neutrino projectile, we hope that in not very far future this will be possible, and one will be able to compare predictions given here with the experimental data. In this paper we give also formulae for other structure functions, both in spin-averaged and spin-dependent case. Those expres-

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[1] For a review see H. Fritzsch and M. Gell-Mann [5].

sions were already given in the framework of ordinary QPM (with neglect of the transverse momentum of partons) [2, 3, 10] and in our framework for spin-dependent electromagnetic structure functions [11].

We begin our discussion by first writing a form of the hadronic tensor and defining the structure functions with their scaling forms (Section 2). The consideration of helicity amplitudes for forward virtual photon (or intermediate vector boson W)–nucleon scattering follows (Section 3). Then, we derive the expressions for the structure functions in the QPM (Section 4) and give relations among them (Section 5). Finally, we present the cross sections for νN scattering (Section 6) and a summary of the whole work, which includes a comparison with other papers (Section 7). The Appendix contains less important formulae, as well as a review of the sum rules (six new ones are presented) for deep inelastic structure functions.

2. *The hadronic tensor*

In the scattering of interest, viewed in our framework as virtual photon (W)–nucleon constituent scattering, the intermediate particle (γ or W) of four-momentum $q^\mu(-q^2 \equiv Q^2 > 0)$ probes the structure of a nucleon (of four-momentum P^μ, polarization four--vector s^μ, and mass M). We define

$$(P \cdot q) \equiv M\nu, \tag{1}$$

so ν is a virtual photon (W) energy in the laboratory frame.

The hadronic tensor $W^{\mu\nu}$ (which describes the above mentioned process), multiplied by the leptonic one, is proportional to the differential cross section for lepton-nucleon scattering. The leptonic tensor is well-known, whereas one writes $W^{\mu\nu}$ as a linear combination of second-rank basis tensors. The unknown coefficients, which multiply those tensors, are structure functions. To build up $W^{\mu\nu}$ we have the following vectors (pseudovectors) and tensors (pseudotensors) at our disposal: $P^\mu, q^\mu, s^\mu, g^{\mu\nu}$, and $\varepsilon^{\mu\nu\alpha\beta}$. However, the hadronic tensor $W^{\mu\nu}(P, q, s)$ must be at most linear in s (since such s-dependence has the spin density matrix for a spin 1/2 particle), so we can only use s once. With this constraint we can construct 22 (6 independent of s and 16 linear in s) second-rank tensors, of which 20 (of them 14 linear in s) are independent because the following two identities hold (see Ref. [12])

$$Q^2\varepsilon^{\mu\nu}(Ps) = (q \cdot s)\varepsilon^{\mu\nu}(qP)-(P \cdot q)\varepsilon^{\mu\nu}(qs)-q^{[\mu}\varepsilon^{\nu]}(Pqs), \tag{2a}$$

$$Q^2MR^{[\mu}\varepsilon^{\nu]}(Pqs) = (P \cdot q)\{(q \cdot s)\varepsilon^{\mu\nu}(qP)-(1+Q^2/\nu^2)(P \cdot q)\varepsilon^{\mu\nu}(qs)\}, \tag{2b}$$

where $\varepsilon^{\mu\nu}(ab) \equiv \varepsilon^{\mu\nu\alpha\beta}a_\alpha b_\beta$ (similarly $\varepsilon^\mu(abc)$), R is defined in (5b), and $A^{[\alpha}B^{\beta]} \equiv A^\alpha B^\beta - A^\beta B^\alpha$ means the antisymmetrization in the Lorentz indices ($A^{\{\alpha}B^{\beta\}} \equiv A^\alpha B^\beta + A^\beta B^\alpha$ stands for symmetrization).

Among 20 structure functions we deal with ,10 give negligible contribution to the differential cross sections (i.e. zero for the electromagnetic interaction and of order m_l^2/Q^2 for the weak case; here m_l is the final lepton mass). These structure functions are coefficients of the gauge non-invariant tensors (i.e. such tensors that: $T^{\mu\nu}q_\mu \neq 0$, $T^{\mu\nu}q_\nu \neq 0$). Hence, we can write the following decomposition of the hadronic tensor:

$$MW^{\mu\nu}(P,q,s) = -G^{\mu\nu}MW_1 + R^\mu R^\nu MW_2 + i\varepsilon^{\mu\nu}(qP)W_3/2M + iM^2\varepsilon^{\mu\nu}(qs)G_1$$
$$+ i[(P\cdot q)\varepsilon^{\mu\nu}(qs) - (q\cdot s)\varepsilon^{\mu\nu}(qP)]G_2 + G^{\mu\nu}(q\cdot s)G_3 + R^\mu R^\nu(q\cdot s)G_4 + R^{\{\mu}T^{\nu\}}MG_5/2$$
$$+ MR^{\{\mu}\varepsilon^{\nu\}}(Pqs)G_{10}/2 + iR^{[\mu}T^{\nu]}MG_{11}/2 + \ldots, \tag{3}$$

where we include the gauge invariant terms only. The rest of the hadronic tensor is

$$MW^{\mu\nu}(P,q,s) = \ldots + q^\mu q^\nu W_4/M + P^{\{\mu}q^{\nu\}}W_5/2M + iP^{[\mu}q^{\nu]}W_6/2M$$
$$+ q^\mu q^\nu(q\cdot s)G_6/M^2 + P^{\{\mu}q^{\nu\}}(q\cdot s)G_7/2M^2 + q^{\{\mu}s^{\nu\}}G_8/2 + iq^{[\mu}\varepsilon^{\nu]}(Pqs)G_9/2$$
$$+ q^{\{\mu}\varepsilon^{\nu\}}(Pqs)G_{12}/2 + iP^{[\mu}q^{\nu]}(q\cdot s)G_{13}/2M^2 + iq^{[\mu}s^{\nu]}G_{14}/2. \tag{4}$$

Note that the i-factor stands before antisymmetric basis tensors. It is there, so that the hermicity condition holds: $W^*_{\mu\nu} = W_{\nu\mu}$. All structure functions: spin-averaged F_i ($i = 1, 2, \ldots, 6$) and spin-dependent G_j($j = 1, 2, \ldots, 14$) are *real* functions of two Lorentz invariants: ν and Q^2. In the above expressions we do not use tensors which are on the left-hand side in the formulae (2). The abbreviations $G^{\mu\nu}$, R^μ and T^μ are used for gauge--invariant tensors and vectors

$$G^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu/q^2, \tag{5a}$$

$$R^\mu = [P^\mu - (P\cdot q)q^\mu/q^2]/M, \tag{5b}$$

$$T^\mu = s^\mu - (q\cdot s)q^\mu/q^2. \tag{5c}$$

If we assume time reversal invariance, the structure functions G_{10}, G_{11} in Eq. (3) and W_6, G_{12}, G_{13}, G_{14} in Eq. (4) are absent. Taking this into account we get that the time reversal and gauge invariant part of our hadronic tensor is similar to one given by Dicus [7], apart from several constant factors and other subtleties (the most important common feature: one deals with eight structure functions), but differ from basis used by Kaur [10] (nine structure functions) or Nash [13] (six structure functions).

Defining

$$MW_1 \equiv F_1, \quad \nu W_2 \equiv F_2, \quad \nu W_3 \equiv F_3, \quad M^2\nu G_1 \equiv g_1,$$
$$M\nu^2 G_2 \equiv g_2, \quad \nu G_3 \equiv g_3, \quad \nu^2 G_4/M \equiv g_4, \quad \nu G_5 \equiv g_5, \tag{6}$$

we can easily give the scaling predictions for structure functions. The QPM or light-cone technique give that in the Bjorken limit (ν, $Q^2 \to \infty$, $x = Q^2/2M\nu$ finite) F_i and g_j are functions of the variable x only. The scaling predictions for the other structure functions are written in the Appendix A.

The functions F_2, g_3, g_4 and g_5 are absent in parity conserving interactions, hence we have only four (F_1, F_2, g_1 and g_2) electromagnetic structure functions for polarized deep inelastic electron (muon)-nucleon scattering.

We can split up the hadronic tensor as follows

$$W_{\mu\nu}(P, q, s) = W_{\mu\nu}^{[S]} + iW_{\mu\nu}^{[A]}, \tag{7}$$

i.e. we have decomposed it into parts symmetric ($W_{\mu\nu}^{[S]}$) and antisymmetric ($W_{\mu\nu}^{[A]}$) under interchange of the Lorentz indices μ, ν. Suitable contractions allow us to extract structure functions from $W_{\mu\nu}$. We have, for example

$$2[F_1 - (\varrho \cdot s)g_3] = MW_{\mu\nu}^{[S]}\{R^\mu R^\nu/(1+\nu^2/Q^2) - G^{\mu\nu}\}, \tag{8}$$

and

$$F_3 - 2(\varrho \cdot s)g_2 = M(\nu^2/Q^2)W_{\mu\nu}^{[A]}\{(1-\Delta)\varepsilon^{\mu\nu}(\pi\varrho) + (\varrho \cdot s)\varepsilon^{\mu\nu}(\varrho s)\}/\Delta, \tag{9}$$

where $\Delta \equiv (1+Q^2/\nu^2) - (\varrho \cdot s)^2$, and π^μ (ϱ^μ) are dimensionless versions of P^μ (q^μ) defined by

$$\pi^\mu = P^\mu/M, \qquad \varrho^\mu = q^\mu/\nu. \tag{10}$$

Other contractions, which give the expressions for further structure functions, are given in the Appendix A.

3. *Helicity amplitudes versus structure functions*

The number of structure functions is related to the number of helicity amplitudes for forward virtual Compton scattering. We have ten gauge-invariant terms in a decomposition of the hadronic tensor and ten a priori independent helicity amplitudes. The parity and time reversal invariance reduces the number of electromagnetic structure functions to four, and this is also the number of helicity amplitudes for forward virtual photon--nucleon scattering.

The s-channel helicity amplitudes at $t = 0$ (here s, t are usual Mandelstam variables) may be written as

$$T(\lambda, s \to \lambda', s') = \varepsilon_\mu^*(\lambda')T^{\mu\nu}(s', s)\varepsilon_\nu(\lambda), \tag{11}$$

where λ (λ'), s (s') are the s-channel helicities of an initial (final) virtual photon (or W) and nucleon, respectively. The absorptive part of $T^{\mu\nu}$ is just $W^{\mu\nu}$ and $\varepsilon_\mu(\lambda)$ is a polarization four-vector of a virtual γ(W) with the helicity λ. The proper linear combinations of invariant amplitudes T_i ($i = 1, 2, 3$) and S_j ($j = 1, 2, 3, 4, 5, 10, 11$), whose imaginary parts are W_i and G_j respectively, give us the helicity amplitudes. Thus for transverse (in this case transverse stands for the direction of the magnetic field) photon (or W) (helicity ± 1)-nucleon (helicity $\pm 1/2$) elastic scattering we get (calculations are presented in the Appendix B)

$$\begin{aligned} MT(\pm 1, \pm\{\mp\}\tfrac{1}{2} \to \pm 1, \pm\{\mp\}\tfrac{1}{2}) &= MT_1 \pm (1+Q^2/\nu^2)^{1/2}\nu T_3/2 \\ &+\{-\}[M^2\nu S_1 - MQ^2 S_2] \mp \{\pm\}(1+Q^2/\nu^2)^{1/2}\nu S_3. \end{aligned} \tag{12}$$

Taking that the parity changes the signs of helicities (s^μ is a pseudovector) we get that T_3 and S_3 are parity non-invariant amplitudes.

The optical theorem relates those amplitudes to the total cross sections. For example, the total cross section for $\gamma(W)-N$ scattering with spins of the initial particles parallel (antiparallel), which is denoted by $\sigma^{\uparrow\uparrow}(\sigma^{\uparrow\downarrow})$, read

$$\sigma^{\uparrow\uparrow(\uparrow\downarrow)} \propto F_1 \mp [g_1 - (Q^2/\nu^2)g_2]. \tag{13}$$

For scalar (sometimes called longitudinal) polarization of a virtual intermediate particle ($\lambda = 0$) we have the following formulae

$$T(0, \pm\tfrac{1}{2} \to 0, \pm\tfrac{1}{2}) = T_L \mp (1+Q^2/\nu^2)^{1/2} S_L, \tag{14}$$

where we use abbreviations: $T_L \equiv (1+\nu^2/Q^2)T_2 - T_1$, and $S_L \equiv -\nu[S_3 + (1+\nu^2/Q^2)S_4 + S_5/2x]/M$. Imaginary parts of T_L and S_L are W_L and G_L respectively, so called longitudinal structure functions

$$MW_L = (1+Q^2/\nu^2)F_2/2x - F_1, \tag{15a}$$

$$MG_L = -[g_3 + (1+Q^2/\nu^2)g_4/2x + g_5/2x]. \tag{15b}$$

Parton model or light-cone ideas give that in the Bjorken limit: $MW_L = MG_L = 0$, whereas $\nu W_L \equiv F_L$ and $\nu G_L \equiv g_L$ are finite functions of a single variable x.

For the helicity-flip amplitudes we obtain

$$QT(0, \pm\tfrac{1}{2} \to \mp 1, \mp\tfrac{1}{2}) = 2^{-1/2}(1+Q^2/\nu^2)^{1/2}\{4x(M^2\nu S_1 + M\nu^2 S_2)/(1+Q^2/\nu^2)^{1/2} \mp S_5 - iM\nu^2 S_{10} \pm i\nu S_{11}\}, \tag{16}$$

where Q is defined: $Q \equiv (Q^2)^{1/2}$. For a $T(\mp 1, \mp\frac{1}{2}, \to 0, \pm\frac{1}{2})$ amplitude we have a similar expression, only the third and fourth term change the sign. The time reversal interchanges the final and initial helicities, parity changes their signs, so looking at Eq. (16) we see that the amplitudes S_5 and S_{11} violate the parity, whereas S_{10} and S_{11} do not occur when time reversal holds.

Most results given in this section are already known. They were written in many papers (e.g. [6], [10], [12]). We present them, since we want to compare in the next section the QPM results for the structure functions with the imaginary parts of helicity amplitudes for different γ (W)-helicity transitions (transverse-transverse, transverse-scalar, scalar--scalar).

4. *Deep inelastic structure functions in the quark parton model*

The QPM expression for the hadronic tensor reads [11]

$$MW^{\mu\nu}(P, q, s) = \sum_{\alpha} \sum_{\eta=\pm 1} \int d\mu_\alpha^\eta(\kappa)\xi^{-1} w_\alpha^{\mu\nu}(\kappa, \sigma), \tag{17}$$

where we sum over different quark flavours (index α) and two polarization of a spin 1/2 parton (denoted by $\eta = \pm 1$). The ξ factor ($\xi \simeq k^0/P^0$) is responsible for different normalizations of hadronic and partonic tensors [3]. The α-parton tensor $w_\alpha^{\mu\nu}(\kappa, \sigma)$ depends on four-momentum k^μ ($k^\mu = M\kappa^\mu$) and polarization four-vector σ^μ of a quark (of mass m). In the above integral we integrate over parton four-momentum with a weight given by the momentum probability distribution $H^{\eta}_{\alpha}(\kappa)$ (the integral measure $d\mu^{\eta}_{\alpha}(\kappa)$ is proportional to $H^{\eta}_{\alpha}(\kappa)$).

We define the following integral

$$I[A] = \sum_{\alpha} \sum_{\eta} e_{\alpha}^{2} \int d\mu^{\eta}_{\alpha}(\kappa)\xi^{-1}\delta((\kappa \cdot \varrho)-x+\omega)A(\kappa, \sigma), \tag{18}$$

where the symbol ω is explained in the Appendix C. If we replace in the expression (18) the electric charge e_{α} by the weak charge g_{α} we get the integral denoted below by $J[A]$. The weak charge g_{α} is a coupling constant of a quark to the weak charged intermediate boson W.

Contracting Eq. (17) with suitable tensors (see Eqs. (8), (9) in Section 2 and (A4), (A5), (A6) in the Appendix A) and making use of the definition (18) we get, e.g. for F_1 and g_1 electromagnetic structure functions

$$2F_1 = I[(\kappa \cdot \varrho)]-\{Q^2/2x(\nu^2+Q^2)\}I[\mu^2(1+Q^2/\nu^2)+(\kappa \cdot \varrho)^2-2(\kappa \cdot \varrho)(\kappa \cdot \pi) -(Q^2/\nu^2)(\kappa \cdot \pi)^2], \tag{19a}$$

$$2(\varrho \cdot s)g_1 = I[(\tilde{\sigma} \cdot \varrho)]+(Q^2/\nu^2)I[(\tilde{\sigma} \cdot v)], \tag{19b}$$

where $\mu \equiv m/M$, $\tilde{\sigma} \equiv \eta\mu\sigma$, and $v^{\mu} \equiv [\pi^{\mu}-\varrho^{\mu}-(\varrho \cdot s)s^{\mu}]/\Delta$. For the weak F_3 and g_3 structure functions we obtain

$$(1+Q^2/\nu^2)F_3 = -2J[\zeta(\kappa \cdot \varrho)]+2(Q^2/\nu^2)J[\zeta(\kappa \cdot \pi)], \tag{20a}$$

$$(\varrho \cdot s)g_3 = J[\zeta(\tilde{\sigma} \cdot \varrho)]+(Q^2/2x\nu^2)J[\zeta\{(\tilde{\sigma} \cdot \varrho)[(\kappa \cdot \pi)-(\kappa \cdot \varrho)]/(1+Q^2/\nu^2) +(\tilde{\sigma} \cdot \pi)[(\kappa \cdot \varrho)+(Q^2/\nu^2)(\kappa \cdot \pi)]\}], \tag{20b}$$

where ζ distinguishes between quarks ($\zeta = +1$) and antiquarks ($\zeta = -1$). In the Bjorken limit ($Q^2/\nu^2 \to 0$) only the first terms on the right-hand side in (19) and (20) give non-zero contribution. For other structure functions one may write similar expressions.

In order to compute the Lorentz scalar products in Eqs. (19) and (20) we must specify the four-vectors κ^{μ} and σ^{μ}. The form of the polarization four-vector of a quark (calculated in the infinite momentum frame) is given in the Appendix C, whereas we decompose κ^{μ} as follows [11]:

$$\kappa^{\mu} = \chi\pi^{\mu}+\varepsilon\varrho^{\mu}+\kappa_{\perp}n^{\mu}, \tag{21}$$

where $(\pi \cdot n) = (\varrho \cdot n) = 0$ and $n^2 = -1$. The transverse four-vector n^{μ} (we choose $\vec{\pi}$ and $\vec{\varrho}$ along the z-axis) gives the direction of transverse momentum of a parton, whereas $\kappa_{\perp}$ is the absolute value of this momentum. The Sudakov parameters χ and ε are related to the energy and z-component of the quark momentum (see Ref. [11]).

Thus the results for electromagnetic spin-averaged structure functions read (we use Eqs. (21) and (C8)) in the Bjorken limit

$$2F_1 = I[\chi], \quad F_2 = I[\chi^2]; \quad F_L = I[\kappa_{\perp}^2+\mu^2]. \tag{22}$$

For spin-dependent functions we get

$$2g_1 = I[\eta\chi], \quad 2(g_1+g_2) = I[\eta\mu]. \tag{23}$$

For neutrino induced reactions the structure functions are given by the following integrals

$$F_1 = J[\chi], \quad F_2 = 2J[\chi^2], \quad F_3 = -2J[\zeta\chi]; \quad F_L = J[2\kappa_\perp^2+\mu^2] \tag{24}$$

and

$$g_1 = J[\eta\chi], \quad 2(g_1+g_2) = J[\eta\mu], \quad g_3 = J[\eta\zeta\chi], \quad g_4+g_5 = -2J[\eta\zeta\chi^2],$$
$$g_5 = -2J[\eta\zeta\chi\mu]; \quad g_L = 2J[\eta\zeta\kappa_\perp^2]. \tag{25}$$

The results for gauge non-invariant functions (i.e. those which stand in front of the gauge non-invariant basis tensors) can be found in the Appendix D.

Comparing Eqs. (22), (23), (24) and (25) with the ones given in Section 3 we notice the following correspondence in the QPM. Those structure functions which occur in the decomposition of the imaginary part of the helicity amplitude $T(\pm 1 \to \pm 1)$ (transverse photon (W)-transverse photon (W) transition), i.e. F_1, F_3, g_1 and g_3 are proportional to χ (under I or J integral). The functions F_L and g_L which give the Im $T(0 \to 0)$ (scalar--scalar transition) are proportional to the linear combinations of $\kappa_\perp^2$ and μ^2. For the helicity-flip amplitude (Im $T(0 \to 1)$) we expect the proportionality to μ (helicity flip only takes place for massive particles). And this is the case, since g_1+g_2 and g_5 structure functions are proportional to μ or $\mu\chi$, respectively.

We define the function $f_\alpha^\eta(x)$

$$f_\alpha^\eta(x) = \int d^4\kappa H_\alpha^\eta(\kappa)\delta(\chi-x) \tag{26}$$

and similarly

$$\langle\kappa_\perp^2(x)\rangle_\alpha f_\alpha^\eta(x) = \int d^4\kappa H_\alpha^\eta(\kappa)\kappa_\perp^2\delta(\chi-x). \tag{27}$$

Using above definitions (and $\xi \simeq \chi$) we are able to recover the well-known results for the electromagnetic structure functions. For F_1, F_2, and F_L we get (see e.g. [3])

$$2F_1(x) = F_2(x)/x = \sum_\alpha e_\alpha^2 f_\alpha(x), \tag{28a}$$

$$xF_L(x) = \sum_\alpha e_\alpha^2[\langle\kappa_\perp^2(x)\rangle_\alpha+\mu_\alpha^2]f_\alpha(x), \tag{28b}$$

where $f_\alpha(x) = f_\alpha^\uparrow(x)+f_\alpha^\downarrow(x)$ ($f_\alpha^{\uparrow(\downarrow)}(x)$ is the distribution of quarks with spin up (down)). The expressions for g_1 (Ref. [3]) and g_1+g_2 (Ref. [11]) can be written

$$2g_1(x) = \sum_\alpha e_\alpha^2 \Delta f_\alpha(x), \tag{29a}$$

$$2x[g_1(x)+g_2(x)] = \sum_\alpha e_\alpha^2\mu_\alpha\Delta f_\alpha(x), \tag{29b}$$

where $\Delta f_\alpha(x) = f_\alpha^\uparrow(x)-f_\alpha^\downarrow(x)$. The formulae for spin-averaged structure functions in neutrino induced reactions are

$$F_1(x) = F_2(x)/2x = \sum_\alpha g_\alpha^2 f_\alpha(x), \tag{30a}$$

$$F_3(x) = -2\sum_\alpha g_\alpha^2\zeta_\alpha f_\alpha(x), \tag{30b}$$

$$xF_L(x) = \sum_\alpha g_\alpha^2[2\langle\kappa_\perp^2(x)\rangle_\alpha+\mu_\alpha^2]f_\alpha(x), \tag{30c}$$

and for spin-dependent functions we have

$$g_1(x) = \sum_\alpha g_\alpha^2 \Delta f_\alpha(x), \tag{31a}$$

$$2x[g_1(x)+g_2(x)] = \sum_\alpha g_\alpha^2 \mu_\alpha \Delta f_\alpha(x), \tag{31b}$$

$$g_3(x) = \sum_\alpha g_\alpha^2 \zeta_\alpha \Delta f_\alpha(x), \tag{31c}$$

$$g_4(x)+g_5(x) = -2x \sum_\alpha g_\alpha^2 \zeta_\alpha \Delta f_\alpha(x), \tag{31d}$$

$$g_5(x) = -2 \sum_\alpha g_\alpha^2 \zeta_\alpha \mu_\alpha \Delta f_\alpha(x), \tag{31e}$$

$$x g_L(x) = 2 \sum_\alpha g_\alpha^2 \zeta_\alpha \langle \kappa_\perp^2(x) \rangle_\alpha \Delta f_\alpha(x). \tag{31f}$$

If partons are on the mass-shells we have to add in Eqs. (26) and (27) the mass-shell delta function $\delta(\kappa^2-\mu^2)$ under the integrals. Neglecting transverse momenta of such partons, i.e. assuming $\kappa^\mu \simeq \chi\pi^\mu$ instead of Eq. (21) (we put ε, $\kappa_\perp \simeq 0$), we get that the polarization vectors of a parton and parent nucleon are equal: $\sigma^\mu \simeq s^\mu$ (see calculations in the Appendix C), and hence several structure functions (or their linear combinations) vanish in such approximation. These functions are: $g_2 = 0$ in the electromagnetic, and $g_4 = 0$ and $g_1 + 2g_2 = 0$ in the weak interactions case. It is also obvious that the QPM gives zero result for the time reversal non-invariant structure functions (see Appendix D).

5. *Relations among structure functions*

The QPM gives several relations among structure functions. The best known is probably Callan-Gross relation [14]:

$$F_2(x) = 2xF_1(x), \tag{32}$$

written here in Eqs. (29a) and (30a), and which holds for electron and neutrino scattering. The parton model confirms also the relation for weak spin-dependent structure functions obtained by Dicus [7] and De Raad, Milton, and Tsai [9] with help of different approaches. In our notation it reads (compare Eqs. (31c, d))

$$-2xg_3(x) = g_4(x)+g_5(x). \tag{33}$$

We can get additional equalities if we specify the weak charged current of a parton. Using the GIM current, we have for quark coupling constant to $W^\pm$ (neutrino (antineutrino) induced scattering):

$$g_\alpha^2(\pm) = \mp I_\alpha^{(3)} \mp \tfrac{1}{2} F_\alpha + \tfrac{1}{2}, \tag{34}$$

where $I_\alpha^{(3)}$ is the third component of α-quark isospin and F_α is the flavour number (defined as hypercharge minus baryonic number: $F \equiv Y-B$). For the electric charge squared we have [15]

$$e_\alpha^2 = \zeta[\tfrac{1}{3} I_\alpha^{(3)} + \tfrac{5}{6} B_\alpha + \tfrac{1}{6} F_\alpha]. \tag{35}$$

Taking a difference of structure functions for proton and neutron target (isospin symmetry changes $I_\alpha^{(3)}$, but not B_α and F_α) we get (using Eqs. (28), (29), (30) and (31))

$$12(F_1^{ep}-F_1^{en}) = F_3^{\nu p}-F_3^{\nu n}, \tag{36a}$$

$$6(g_1^{ep}-g_1^{en}) = -(g_3^{\nu p}-g_3^{\nu n}), \tag{36b}$$

$$12x[(g_1+g_2)^{ep}-(g_1+g_2)^{en}] = g_5^{\nu p}-g_5^{\nu n}. \tag{36c}$$

The first equality is the best known one [16], third was written in the covariant parton model framework by Nash [13] (the second and the third one were first derived, using light-cone algebra, by Dicus [7]). Above relations are consequences of quark weak charged current assumed here, for another choice, e.g. the Cabibbo one, we obtain similar but distinct equalities.

For several structure functions discussed in this paper one can write certain sum rules. Those sum rules are considered in the Appendix E.

6. *The cross sections for neutrino-nucleon scattering*

The double differential cross section for neutrino-nucleon scattering is equal to

$$\frac{d^2\sigma}{dxdy} = \frac{G^2 y}{16\pi} l_{\mu\nu}(MW^{\mu\nu}), \tag{37}$$

where $y = (P \cdot q)/(P \cdot l) = \nu/E$ (l^μ is the four-momentum of incident neutrino, E its energy in the laboratory frame) and leptonic tensor $l_{\mu\nu}$ is written in the Appendix F. The form of this cross section in terms of scaling structure functions is also given in this appendix.

Taking the difference of such cross section for scattering on target polarized in the s-direction and in the opposite one we get (in the scaling limit)

$$\frac{d^2\sigma}{dxdy}(s)-\frac{d^2\sigma}{dxdy}(-s) = \frac{2G^2ME}{\pi}\{\tau y[y(\varrho\cdot s)-2(\lambda\cdot s)]xg_1$$
$$+2\tau y[(\varrho\cdot s)-(\lambda\cdot s)]xg_2-[1+(1-y)^2](\varrho\cdot s)xg_3-\tfrac{1}{2}(2-y)[(\varrho\cdot s)-(\lambda\cdot s)]g_5\}, \tag{38}$$

where we have used the relation (33) to eliminate g_4 structure function. Here $\tau = \pm 1$ differentiates between cross section for neutrinos ($\tau = +1$) and antineutrinos ($\tau = -1$), whereas $\lambda^\mu = l^\mu/E$. For nucleons polarized in the beam direction, i.e. $s_{\mathrm{L}}^\mu = \lambda^\mu-\pi^\mu$ (L $\equiv$ longitudinal polarization; $(\varrho\cdot s_{\mathrm{L}}) \simeq (\lambda\cdot s_{\mathrm{L}}) \simeq -1$ in the scaling limit), we obtain the difference of cross sections which rises linearly with the energy E

$$\frac{d^2\sigma}{dxdy}(s_{\mathrm{L}})-\frac{d^2\sigma}{dxdy}(-s_{\mathrm{L}}) = \frac{2G^2MEx}{\pi}\{[1+(1-y)^2]g_3+\tau y(2-y)g_1\}. \tag{39}$$

The QPM enables one to predict y-dependence of such cross section. If we denote the quark (antiquark) contribution to $2xg_1(x)$ by $\Delta q(x)(\Delta\bar{q}(x))$ (from Eq. (31a) we have

$\Delta q(x) = 2x \sum_\alpha g_\alpha^2 \Delta f_\alpha(x)$, where we sum quark contributions only), then we get

$$2xg_1(x) = q(x)+\bar{q}(x), \tag{40a}$$

$$2xg_3(x) = q(x)-\bar{q}(x). \tag{40b}$$

Hence we obtain for neutrino-nucleon scattering

$$\frac{1}{2}\left[\frac{d^2\sigma^\nu}{dxdy}(\uparrow\downarrow)-\frac{d^2\sigma^\nu}{dxdy}(\uparrow\uparrow)\right] = \frac{G^2ME}{\pi}[\Delta q^\nu(x)-(1-y)^2\Delta\bar{q}^\nu(x)], \tag{41}$$

whereas for antineutrino induced reactions

$$\frac{1}{2}\left[\frac{d^2\sigma^{\bar{\nu}}}{dxdy}(\uparrow\uparrow)-\frac{d^2\sigma^{\bar{\nu}}}{dxdy}(\uparrow\downarrow)\right] = \frac{G^2ME}{\pi}[(1-y)^2\Delta q^{\bar{\nu}}(x)-\Delta\bar{q}^{\bar{\nu}}(x)], \tag{42}$$

where we mark in brackets whether spins of neutrino (or antineutrino) and nucleon are parallel ($\uparrow\uparrow$) or antiparallel ($\uparrow\downarrow$). This y-dependence is similar to one obtained for spin--averaged scattering, the only difference is that antiquark contributions have opposite signs.

For nucleons polarized in the plane transverse to the beam direction, i.e. $(\lambda \cdot s_T) = 0$ (T $\equiv$ transverse polarization of a nucleon), we obtain in the Bjorken limit

$$\frac{1}{2}\left[\frac{d^2\sigma}{dxdy}(s_T)-\frac{d^2\sigma}{dxdy}(-s_T)\right] = \frac{G^2MQ}{\pi y}\,\mathrm{sign}\,(\varrho \cdot s_T)\,\{\tau y^2 xg_1+2\tau yxg_2$$
$$-[1+(1-y)^2]xg_3-(2-y)g_5/2\}, \tag{43}$$

so the difference is rising linearly with Q, not E, in this case. The QPM result for this difference is discussed in the Appendix F. An interesting result is obtained for on mass-shell partons with negligible transverse momentum. In such approximation the following asymmetry for scattering on the isoscalar target

$$\frac{\sigma^{\nu N}(s_T)-\sigma^{\nu N}(-s_T)}{\sigma^{\bar{\nu}N}(s_T)-\sigma^{\bar{\nu}N}(-s_T)} = -\frac{\Delta q}{\Delta\bar{q}}$$

is rather small, since clearly quark contribution Δq dominates over antiquark one $\Delta\bar{q}$ $(\Delta q \equiv \int_0^1 \Delta q(x)dx$, and similarly for $\Delta\bar{q}$).Hence, the quark contribution to $\sigma^\nu(s_T)-\sigma^\nu(-s_T)$ is mainly of transverse momentum origin in the QPM with on mass-shell partons.

7. *Summary*

In this paper we have derived the forms of the spin-dependent structure functions for deep inelastic neutrino-nucleon scattering. To get the results we have used the QPM framework wtih arbitrary transverse momenta of partons. The last assumption is very important since for on mass-shell partons several structure functions, i.e. g_2^e, $g_1^\nu+2g_2^\nu$ and g_4^ν vanish if one neglects transverse momenta. Kaur [10] has written similar formulae, but with neglect of transverse momentum ($k^\mu = \chi P^\mu$ in this work) and also had one structure function more.

We have repeated here, in the QPM framework, relations among different polarized structure functions first given by Dicus [7], who has used the light-cone algebra technique.

We have showed that the structure functions may be divided in three different groups (with different functional forms), each connected with the corresponding forward amplitude for definite helicity transition.

The spin-dependent part of the νN cross section has been discussed here and the result that it rises linearly with E for longitudinally (with Q for transversely) polarized target has been obtained. We have also given the y-dependence of such cross section for both polarization cases.

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APPENDIX A

In this paper we deal with the structure functions which may be gauge (G), parity (P) or/and time reversal (T) invariant. Thus we have: for W_1, W_2, G_1 and G_2 (G, P, T); for W_3, G_3, G_4 and G_5 (G, T), whereas G_{10} (G, P) and G_{11} (G). After each group of functions we have marked their type of invariance. Among gauge non-invariant structure functions W_6(P), G_{12} (P), G_{13} and G_{14} are also time reversal non-invariant. For other functions the following invariance holds: for W_4, W_5, G_9 (P, T) whereas G_6, G_7, G_8 (T). Thus we have 10 parity invariant and 10 parity non-invariant functions, whereas 14 of them are time reversal invariant and 6 are not.

For neutrino scattering the following crossing properties hold

$$W_i^\nu(\nu, q^2) = -W_i^\nu(-\nu, q^2) \quad (i = 1, 2, 3, 4, 6), \tag{A1a}$$

$$W_5^\nu(\nu, q^2) = W_5^{\bar\nu}(-\nu, q^2), \tag{A1b}$$

and similarly for spin-dependent structure functions

$$G_i^\nu(\nu, q^2) = -G_i^{\bar\nu}(-\nu, q^2) \quad (i = 1, 5, 7, 12, 14), \tag{A2a}$$

$$G_i^\nu(\nu, q^2) = G_i^{\bar\nu}(-\nu, q^2) \quad (i = 2, 3, 4, 6, 8, 9, 10, 11, 13). \tag{A2b}$$

The scalling limits of the structure functions which are not written in Eq. (6) are:

$$\begin{gathered} \nu W_4 \to F_4, \quad \nu W_5 \to F_5, \quad \nu W_6 \to F_6, \\ \nu^2 G_6/M \to g_6, \quad \nu^2 G_7/M \to g_7, \quad \nu G_8 \to g_8, \\ M\nu^2 G_9 \to g_9, \quad M\nu^2 G_{10} \to g_{10}, \quad \nu G_{11} \to g_{11}, \\ M\nu^2 G_{12} \to g_{12}, \quad \nu^2 G_{13}/M \to g_{13}, \quad \nu G_{14} \to g_{14}. \end{gathered} \tag{A3}$$

On the right-hand side we have functions which depend on x only (ignoring scaling violations).

We present below the expressions for structure functions in terms of suitable contractions of the hadronic tensor. Thus we have for gauge invariant ones

$$(1+Q^2/\nu^2)F_2+(1+Q^2/\nu^2)(\varrho\cdot s)g_4+(\varrho\cdot s)g_5$$
$$= MxW_{\mu\nu}^{[S]}\{3R^\mu R^\nu/(1+\nu^2/Q^2)-G^{\mu\nu}\}, \tag{A4a}$$
$$g_1+g_2 = -M(\nu^2/Q^2)W_{\mu\nu}^{[A]}\{(\varrho\cdot s)\varepsilon^{\mu\nu}(\pi\varrho)+(1+Q^2/\nu^2)\varepsilon^{\mu\nu}(\varrho s)\}/2\Delta, \tag{A4b}$$
$$g_5 = 4xMW_{\mu\nu}^{[S]}\{(\varrho\cdot s)R^\mu R^\nu/(1+Q^2/\nu^2)-R^\mu T^\nu\}/\Delta, \tag{A4c}$$
$$F_L-(\varrho\cdot s)g_L = 2xMW_{\mu\nu}^{[S]}R^\mu R^\nu/(1+Q^2/\nu^2). \tag{A4d}$$

For other functions we can write

$$F_4+(\varrho\cdot s)g_6 = -MW_{\mu\nu}^{[S]}\{2\pi^\mu\varrho^\nu+[\Delta(\nu^2/Q^2)-2]\varrho^\mu\varrho^\nu-2(\varrho\cdot s)\varrho^\mu s^\nu\}/2x\Delta, \tag{A5a}$$
$$F_5+(\varrho\cdot s)g_7 = -2M(\nu^2/Q^2)W_{\mu\nu}^{[S]}\{(\Delta-1)\pi^\mu\varrho^\nu+\varrho^\mu\varrho^\nu+(\varrho\cdot s)\varrho^\mu s^\nu\}/\Delta, \tag{A5b}$$
$$g_8 = 2M(\nu^2/Q^2)W_{\mu\nu}^{[S]}\{-(\varrho\cdot s)\pi^\mu\varrho^\nu+(\varrho\cdot s)\varrho^\mu\varrho^\nu+(1+Q^2/\nu^2)\varrho^\mu s^\nu\}/\Delta, \tag{A5c}$$
$$g_9 = -MW_{\mu\nu}^{[A]}\varepsilon^{\mu\nu}(RT)/\Delta. \tag{A5d}$$

For time reversal non-invariant structure functions we obtain

$$F_6+(\varrho\cdot s)g_{13} = -2M(\nu^2/Q^2)W_{\mu\nu}^{[A]}\{(\Delta-1)\pi^\mu\varrho^\nu-(\varrho\cdot s)\varrho^\mu s^\nu\}/\Delta, \tag{A6a}$$
$$g_{10} = -2\nu W_{\mu\nu}^{[A]}\{\varrho^\mu+(Q^2/\nu^2)\pi^\mu\}\varepsilon^\nu(\pi\varrho s)/\Delta(1+Q^2/\nu^2), \tag{A6b}$$
$$g_{11} = -2\nu W_{\mu\nu}^{[A]}\{(\varrho\cdot s)\pi^\mu\varrho^\nu+(Q^2/\nu^2)\pi^\mu s^\nu/\Delta+\varrho^\mu s^\nu\}, \tag{A6c}$$
$$g_{12} = 2M(\nu^2/Q^2)W_{\mu\nu}^{[S]}\varrho^\mu\varepsilon^\nu(\pi\varrho s)/\Delta, \tag{A6d}$$
$$g_{14} = 2M(\nu^2/Q^2)W_{\mu\nu}^{[A]}\{(\varrho\cdot s)\pi^\mu\varrho^\nu+(1+Q^2/\nu^2)\varrho^\mu s^\nu\}/\Delta. \tag{A6e}$$

APPENDIX B

The s-channel helicity amplitudes in the forward direction ($t = 0$) are defined by

$$T(\lambda, s\to\lambda', s') = \varepsilon_\mu^*(\lambda')T^{\mu\nu}(P, q, s)\varepsilon_\nu(\lambda), \tag{B1}$$

where $\varepsilon_\mu(\lambda)$ ($\lambda = 0, \pm 1$) is the polarization four-vector of a virtual intermediate photon (or W), which satisfies: $\varepsilon(\lambda)\cdot q = 0$. The tensor $T^{\mu\nu}(P, q, s)$ has the same covariant form as $W^{\mu\nu}$ (apart from terms required by current algebra, see e.g. Ref. [7]), only the coefficients are now complex functions T_i and S_j, which imaginary parts are W_i and G_j, respectively. The spin four-vector s^μ of a nucleon is

$$s^\mu = \bar{u}_{s'}(P)\gamma^\mu\gamma_5 u_s(P), \tag{B2}$$

where the nucleon spinors are normalized: $\bar{u}_{s'}(P)u_s(P) = \delta_{ss'}$. The form of a polarization four-vector for right (left)-handed ($\lambda = +1(-1)$) boson is [2, 17]

$$\varepsilon_\mu(\pm) = (0, \pm 1, i, 0)/2^{1/2}, \tag{B3}$$

whereas for scalar polarization, i.e. for $\lambda = 0$ we have

$$\varepsilon^\mu(0) = (2xP^\mu + q^\mu)/Q(1+Q^2/\nu^2)^{1/2}, \tag{B4}$$

which satisfies $\varepsilon^2(0) = 1$ and where we choose $\vec{P}$ and $\vec{q}$ to lay along the z-axis (e.g. in the laboratory system we get: $q^\mu = (\nu, 0, 0, -(1+Q^2/\nu^2)^{1/2})$). The formulae for $\varepsilon^\mu(\lambda)$ given above are valid in all frames which can be reached applying the Lorentz boost along the z-axis.

APPENDIX C

In this appendix we give several useful formulae. The first expresses the hadronic tensor in terms of parton quantities (compare with Eq. (17))

$$MW^{\mu\nu}(P, q, s) = \sum_\alpha \sum_\eta \int d\mu_\alpha^\eta(k, u^2)\xi^{-1} w_\alpha^{\mu\nu}(k, \sigma; u^2), \tag{C1}$$

where most of the symbols are explained in Section 4, and u^2 is equal to the final parton four-momentum squared (for on mass-shell partons: $u^2 = m_f^2$). The integral measure is defined by

$$d\mu_\alpha^\eta(k, u^2) = d^4k du^2 H_\alpha^\eta(k, u^2), \tag{C2}$$

and normalized as follows

$$\int d\mu_\alpha^\eta(k, u^2) = n_\alpha^\eta, \tag{C3}$$

where n_α^η is a number of partons with flavour α and polarization η inside the nucleon. For partons on the mass-shells we have to add, in Eq. (C2), two delta functions: $\delta(k^2-m^2)$ and $\delta(u^2-m_f^2)$. The partonic tensor $w_\alpha^{\mu\nu}$ is obtained from Feynman diagram for virtual γ(W)–parton scattering. The result reads in the electromagnetic case

$$\begin{aligned} w_\alpha^{\mu\nu} &= \tfrac{1}{4}\, e_\alpha^2 \delta((k+q)^2-u^2)\,\mathrm{Tr}\,[(1+\eta\xi\gamma_5\sigma\cdot\gamma)\,(k\cdot\gamma+\zeta m)\gamma^\mu(k\cdot\gamma+q\cdot\gamma+\zeta m)\gamma^\nu] \\ &= e_\alpha^2\delta((k+q)^2-u^2)\,\{2k^\mu k^\nu + k^{\{\mu}q^{\nu\}} - g^{\mu\nu}[(k\cdot q)+k^2-m^2] + i\eta m\varepsilon^{\mu\nu}(q\sigma)\}, \end{aligned} \tag{C4}$$

whereas for the weak charged current we get

$$\begin{aligned} w_\alpha^{\mu\nu} &= \tfrac{1}{4}\, g_\alpha^2\delta((k+q)^2-u^2)\,\mathrm{Tr}\,[(1+\eta\zeta\gamma_5\sigma\cdot\gamma)\,(k\cdot\gamma+\zeta m)\gamma^\mu(1-\zeta\gamma_5) \\ &\times(k\cdot\gamma+q\cdot\gamma+\zeta m_f)\gamma^\nu(1-\zeta\gamma_5)] = 2g_\alpha^2\delta((k+q)^2-u^2)\,\{2k^\mu k^\nu + k^{\{\mu}q^{\nu\}} - g^{\mu\nu}[(k\cdot q)+k^2] \\ &+ i\eta m\varepsilon^{\mu\nu}(q\sigma) + i\eta m\varepsilon^{\mu\nu}(k\sigma) + i\zeta\varepsilon^{\mu\nu}(kq) - \eta\zeta m\sigma^{\{\mu}k^{\nu\}} + \eta\zeta m g^{\mu\nu}(\sigma\cdot q) - \eta\zeta m\sigma^{\{\mu}q^{\nu\}}\}. \end{aligned} \tag{C5}$$

The delta function in (C4) and (C5) can be written

$$\delta((k+q)^2-u^2) = \delta((\kappa\cdot\varrho)-x+\omega)/2M\nu, \tag{C6}$$

where

$$\omega = (k^2-u^2)/2M\nu. \tag{C7}$$

Our assumption is that $\omega \to 0$ in the Bjorken limit, and this enables to write Eqs. (28), (29), (30) and (31). Such assumption is satisfied for on mass-shell partons and for partons which are not very far from their mass-shells.

The polarization four-vector of a spin 1/2 quark was calculated by us [11] in the case of non-negligible transverse momentum (for simpler case see Ref. [3]). The result is

$$\mu\sigma^\alpha = \mu s^\alpha + (\varrho \cdot s)(\kappa^\alpha - \mu\pi^\alpha)/(1+Q^2/v^2)^{1/2}$$

$$-\mu\{(\kappa \cdot s) - (\varrho \cdot s)[(\kappa \cdot \pi) - \mu]/(1+Q^2/v^2)^{1/2}\}(\varrho^\alpha + a\pi^\alpha)/[(\kappa \cdot \varrho) + a(\kappa \cdot \pi)], \quad \text{(C8)}$$

where a stands for: $a \equiv (1+Q^2/v^2)^{1/2} - 1$, and hence vanishes in the scaling limit. If we neglect Fermi motion of quarks (i.e. put ε and $\kappa_\perp$ equal to zero in Eq. (21), so $\kappa^\alpha \simeq \chi\pi^\alpha$) we obtain

$$\sigma^\alpha \simeq s^\alpha + (\varrho \cdot s)(\chi - \mu)\{[\chi + a\mu/(1+Q^2/v^2)^{1/2}]\pi^\alpha + \varrho^\alpha/(1+Q^2/v^2)^{1/2}\}/\chi\mu. \quad \text{(C9)}$$

For on mass-shell partons ($\kappa^2 = \mu^2$) we have $\mu \simeq \chi$ (it comes out from the equality: $\kappa^2 \simeq \chi^2$ and hence $\chi^2 \simeq \mu^2$), and therefore we get the known result: $\sigma^\alpha \simeq s^\alpha$.

APPENDIX D

The appendix deals with calculations of certain weak structure functions in the QPM framework We present here results for the gauge non-invariant ones. The time reversal non-invariant functions are zero in our framework since we have no such terms in the decomposition of the partonic tensor. Thus we have: $F_6, g_{10}, g_{11}, g_{12}, g_{13}, g_{14} = 0$.

A typical example of a formula for the structure function is

$$g_9 = -J[(\varrho \cdot s)\{(\kappa \cdot \pi)(\tilde{\sigma} \cdot \varrho) - (\kappa \cdot \varrho)(\tilde{\sigma} \cdot \pi)\} + \{(\kappa \cdot \varrho)(\tilde{\sigma} \cdot s) - (\kappa \cdot s)(\tilde{\sigma} \cdot \varrho)\}$$

$$+(Q^2/v^2)\{(\kappa \cdot \pi)(\tilde{\sigma} \cdot s) - (\kappa \cdot s)(\tilde{\sigma} \cdot \pi)\}]/x\Delta. \quad \text{(D1)}$$

Some of the gauge non-invariant functions, i.e. F_4, F_5 and g_6 give zero in the Bjorken limit. However, if we define $\tilde{F}_4 = vF_4/M$ and $\tilde{g}_6 = vg_6/M$ we get for these functions finite results

$$\tilde{F}_4 = J[\mu^2]/2x, \quad \text{(D2a)}$$

$$\tilde{g}_6 = J[\eta\zeta\mu(\mu - \chi)]/x. \quad \text{(D2b)}$$

Using Eq. (26) we may write

$$2x^2\tilde{F}_4(x) = \sum_\alpha g_\alpha^2\mu_\alpha^2 f_\alpha(x), \quad \text{(D3a)}$$

$$x^2\tilde{g}_6(x) = \sum_\alpha g_\alpha^2\zeta_\alpha\mu_\alpha(\mu_\alpha - x)\Delta f_\alpha(x). \quad \text{(D3b)}$$

For other functions we obtain

$$g_7 = -g_8 = J[\eta\zeta\mu], \quad \text{(D4a)}$$

$$g_9 = J[\eta\chi\mu]/x, \quad \text{(D4b)}$$

and similarly to (D3) we have

$$xg_7(x) = -xg_8(x) = \sum_\alpha g_\alpha^2 \zeta_\alpha \mu_\alpha \Delta f_\alpha(x), \tag{D5a}$$

$$xg_9(x) = \sum_\alpha g_\alpha^2 \mu_\alpha \Delta f_\alpha(x). \tag{D5b}$$

Comparing (D5a, b) with (31e, b) we get the following identities

$$-2xg_7 = 2xg_8 = g_5, \tag{D6a}$$

$$g_9 = 2x(g_1+g_2). \tag{D6b}$$

APPENDIX E

In this appendix we recall known, and derive unknown sum rules for the structure functions. From the charge symmetry we have: $F_i^{\nu n} \equiv F_i^{\bar{\nu} p}$ and $g_j^{\nu n} \equiv g_j^{\bar{\nu} p}$, so we can always change the weak structure function under the integral according to these equalities.

First we write the known sum rules, true in the QPM, as for example the one given by Bjorken [18] (the similar for F_2^ν/x was obtained by Adler [19]),

$$\int_0^1 [F_1^{\nu p}(x) - F_1^{\bar{\nu} n}(x)]dx = -1, \tag{E1}$$

Gross and Llewellyn Smith [20]

$$\int_0^1 [F_3^{\nu p}(x) + F_3^{\bar{\nu} p}(x)]dx = -6, \tag{E2}$$

and once more Bjorken [21]

$$6\int_0^1 [g_1^{ep}(x) - g_1^{en}(x)]dx = (G_A/G_V). \tag{E3}$$

The scaling version [3] of Burkhardt–Cottingham sum rule [22] is

$$\int_0^1 [g_2^{ep}(x) - g_2^{en}(x)]dx = 0. \tag{E4}$$

Equations (E3) and (E4), together with relations (36b, c) and (33) give the new sum rules (assuming GIM weak charged current for quarks)

$$\int_0^1 [g_3^{\nu p}(x) - g_3^{\nu n}(x)]dx = -(G_A/G_V), \tag{E5a}$$

$$\int_0^1 [g_4^{\nu p}(x) - g_4^{\nu n}(x)]dx/x = 0, \tag{E5b}$$

$$\int_0^1 [g_5^{\nu p}(x) - g_5^{\nu n}(x)]dx/x = 2(G_A/G_V). \tag{E5c}$$

If we assume SU(2) symmetry for the "sea" of quark-antiquark pairs (i.e. $n_{\bar{u}} = n_{\bar{d}}$) we get in the QPM

$$\int_0^1 [F_2^{ep}(x) - F_2^{en}(x)]dx/x \simeq 1/3, \tag{E6a}$$

$$\int_0^1 [F_3^{\nu p}(x) - F_3^{\nu n}(x)]dx \simeq 2, \tag{E6b}$$

where the first formula is a scaling version [23] of Gottfried's sum rule [24], whereas the second one follows directly from Eqs. (32), (36a) and (E6a).

Assuming $n_{\bar{u}}^{\uparrow} = n_{\bar{d}}^{\uparrow}$, $n_{\bar{u}}^{\downarrow} = n_{\bar{d}}^{\downarrow}$ ($n_\alpha^{\uparrow}$ ($n_\alpha^{\downarrow}$) is the number of α-flavour quarks polarized parallelly (antiparallelly) to the nucleon spin, i.e. described by $\eta = +1(-1)$), or that the "sea" of quark–antiquark pairs is unpolarized (i.e. $n_{\bar{u}}^{\uparrow} = n_{\bar{u}}^{\downarrow}$, $n_{\bar{d}}^{\uparrow} = n_{\bar{d}}^{\downarrow}$) we get

$$\int_0^1 [g_1^{\nu p}(x) - g_1^{\nu n}(x)]dx \simeq -(G_A/G_V), \tag{E7a}$$

$$2\int_0^1 [g_2^{\nu p}(x) - g_2^{\nu n}(x)]dx \simeq (G_A/G_V), \tag{E7b}$$

where the first sum rule was written previously by Nash [13].

Denoting by S the quark *spin* contributions to the spin of the nucleon we have

$$\sum_\alpha \sum_\eta \eta n_\alpha^\eta = \sum_\alpha (n_\alpha^{\uparrow} - n_\alpha^{\downarrow}) = 2S. \tag{E8}$$

We get $2S = 1$, if gluons give no contribution to the spin of a nucleon and the orbital angular momentum of partons is zero (see discussion in Refs. [25] and [26]). Denoting by S_V the similar contribution which comes from valence quarks ($S_V = 1/2$ when valence quarks, with $L = 0$, give the whole spin structure of a nucleon) we may write the new sum rules

$$\int_0^1 [g_1^{\nu p}(x) + g_1^{\bar{\nu} p}(x)]dx \simeq 2S, \tag{E9a}$$

$$\int_0^1 [g_3^{\nu p}(x) + g_3^{\bar{\nu} p}(x)]dx \simeq 2S_V, \tag{E9b}$$

where we have used the normalization condition

$$\int_0^1 \Delta f_\alpha(x)dx = n_\alpha^{\uparrow} - n_\alpha^{\downarrow} \tag{E10}$$

to derive them. Those sum rules are written under the assumption, that $n_\alpha^{\uparrow} = n_{\bar{\alpha}}^{\uparrow}$ in the "sea" (for the first one it is enough to assume that for heavy quarks only).

We present here several sum rules but we are not able to repeat Nash sum rule [13], which in our notation reads

$$\int_0^1 [g_5^{\nu p}(x) + g_5^{\bar{\nu} p}(x)]dx/x = -6(G_A/G_V). \tag{E11}$$

Assuming for heavy quark contributions $n_s^{\uparrow}-n_s^{\downarrow} = n_{\bar{s}}^{\uparrow}-n_{\bar{s}}^{\downarrow}$, $n_c^{\uparrow}-n_c^{\downarrow} = n_{\bar{c}}^{\uparrow}-n_{\bar{c}}^{\downarrow}$ [25] we get

$$6\int_0^1 [g_1^{ep}(x)+g_1^{en}(x)]dx \simeq 10S/3, \tag{E12}$$

which combined with (E3) gives well-known expressions

$$2\int_0^1 g_1^{ep}(x)dx \simeq [10S+3(G_A/G_V)]/18 \simeq 0.37\pm 0.03, \tag{E13a}$$

$$2\int_0^1 g_1^{en}(x)dx \simeq [10S-3(G_A/G_V)]/18 \simeq -0.05\pm 0.03, \tag{E13b}$$

where the experimental figures for S and (G_A/G_V) are taken from Sehgal's paper [25]. Similar results were derived, using the light-cone algebra technique, by Ellis and Jaffe [27] and in the QPM by Sehgal [25] and Close [26]. Note that in the SU (6) symmetry limit ($S = 1/2$, $G_A/G_V = 5/3$) we get zero for the second integral.

APPENDIX F

In this appendix we give the neutrino (antineutrino)-nucleon scattering cross section. It is proportional to the leptonic tensor $l_{\mu\nu}$ given by

$$l_{\mu\nu} = 8\{2l_\mu l_\nu - l_{\{\mu}q_{\nu\}} - (q\cdot l)g_{\mu\nu} + \tau i\varepsilon_{\mu\nu}(ql)\}, \tag{F1}$$

where $\tau = +1(-1)$ for neutrino (antineutrino) induced reactions.

The differential cross section for scattering off polarized target (we sum over final particle polarizations) is

$$\frac{d^2\sigma}{dxdy} = \frac{G^2}{\pi}\{MEA+\tfrac{1}{2}m_l^2B+\tfrac{1}{2}MQ(s\cdot r)C\}, \tag{F2}$$

where m_l is final lepton mass, whereas spatial part of r^μ_{LAB} ($|\vec{r}|_{\text{LAB}} = 1$) gives the direction perpendicular to the scattering plane ($\vec{r} \propto \vec{q}\times\vec{l}$). The symbols A, B and C are defined below

$$\begin{aligned} A = {} & xy^2F_1+[(1-y)-xyM/2E]F_2-\tau xy(2-y)F_3/2+\tau xy[y(\varrho\cdot s)-2(\lambda\cdot s)]g_1 \\ & +2\tau xy[(\varrho\cdot s)-(\lambda\cdot s)]g_2-xy^2(\varrho\cdot s)g_3+[(1-y)-xyM/2E](\varrho\cdot s)g_4 \\ & +[(2-y)(\lambda\cdot s)-y(\varrho\cdot s)]g_5/2, \end{aligned} \tag{F3}$$

whereas

$$B = xyF_4-F_5+xy(\varrho\cdot s)g_6-(\varrho\cdot s)g_7-(\varrho\cdot s)g_8, \tag{F4}$$

and

$$C = \{[(1-y)-Mxy/2E]^{1/2}/y\}[-(2-y)g_{10}+\tau yg_{11}+(m_l^2/ME)g_{12}]. \tag{F5}$$

Note that the structure functions F_6, g_9, g_{13}, and g_{14} do not contribute to the cross section when the sum over final lepton polarizations is taken.

The QPM result for double differential cross section is

$$\frac{d^2\sigma}{dxdy} = \frac{G^2ME}{2\pi} J[2(\kappa\cdot\lambda)^2 - 2y(\kappa\cdot\lambda)(\kappa\cdot\varrho) + xy^2(\kappa\cdot\varrho) - y^2\mu^2Q^2/2\nu^2$$

$$+\tau\zeta xy\{2(\kappa\cdot\lambda) - y(\kappa\cdot\varrho)\} - \eta\zeta\{2(\kappa\cdot\lambda) - y(\kappa\cdot\varrho)\}(\tilde{\sigma}\cdot\lambda)$$

$$+\eta\zeta y\{(\kappa\cdot\lambda) - xy\}(\tilde{\sigma}\cdot\varrho) + \tau\eta y\{(\kappa\cdot\varrho) - 2x\}(\tilde{\sigma}\cdot\lambda) - \tau\eta y\{(\kappa\cdot\lambda) - xy\}(\tilde{\sigma}\cdot\varrho)]. \tag{F6}$$

The asymmetry for scattering off transversely polarized target reads in parton model

$$\frac{1}{2}\left[\frac{d^2\sigma}{dxdy}(s_T) - \overline{\frac{d^2\sigma}{dxdy}}(-s_T)\right] = \frac{G^2MQ}{\pi xy}(1-y)^{1/2}\,\mathrm{sign}\,(\varrho\cdot s_T)\left[-2x\begin{Bmatrix}1\\(1-y)^2\end{Bmatrix}\Delta q(x)\right.$$

$$\left.+2x\begin{Bmatrix}(1-y)^2\\1\end{Bmatrix}\Delta\bar{q}(x) + 2\langle\mu\rangle\begin{Bmatrix}1\\1-y\end{Bmatrix}\Delta q(x) - 2\langle\bar{\mu}\rangle\begin{Bmatrix}1-y\\1\end{Bmatrix}\Delta\bar{q}(x)\right], \tag{F7}$$

where

$$\langle\mu\rangle = \sum_\alpha g_\alpha^2\mu_\alpha\Delta f_\alpha(x)/\sum_\alpha g_\alpha^2\Delta f_\alpha(x), \tag{F8}$$

and similarly is defined $\langle\bar{\mu}\rangle$, the only difference is that we sum antiquark contributions instead of the quark ones.

Assuming $\langle\mu\rangle = \langle\bar{\mu}\rangle \simeq x$ (this holds for on mass-shell partons with tiny transverse momenta), we get for the asymmetry from Eq. (F7)

$$\frac{2G^2MQ}{\pi}(1-y)^{3/2}\,\mathrm{sign}\,(\varrho\cdot s_T)\left[\begin{Bmatrix}0\\1\end{Bmatrix}\Delta q(x) - \begin{Bmatrix}1\\0\end{Bmatrix}\Delta\bar{q}(x)\right], \tag{F9}$$

where in formulae (F7) and (F9) the upper (lower) expressions in curly brackets stand for neutrino (antineutrino) scattering. From Eq. (F9) we conclude that quarks in *transversely* polarized nucleon contribute to the difference of *neutrino* cross sections only when they have nonvanishing transverse momentum.

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