ON THE MULTIPLICITY OF THE SECONDARIES PRODUCED IN COLLISIONS OF RELATIVISTIC NUCLEI

By Yu. M. Shabelsky

Leningrad Nuclear Physics Institute, Gatchina*

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The multiple scattering model without any corrections due to collective phenomena is shown to describe reasonably well the average numbers of particles produced and interacting nucleons in nucleus-nucleus collisions. Some predictions are given for inclusive spectrum ratios at very high energy.

1. Introduction

Inelastic hadron-nucleus interactions at high energy are the main source of information on the space-time structure of strong interactions. However, predictions of different models are sometimes similar making difficult the choice between them. The multiparticle production in nucleus-nucleus collisions may be in some cases more sensitive to these models. The purpose of this paper is to show that, contrary to a wide-spread opinion, such processes may be described quite satisfactorily within the framework of the multiple scattering model as a superposition of independent nucleon-nucleon collisions. Collective phenomena corrections do not seem to be necessary.

The main results of this paper are Eq. (9) for the average number of interacting nucleons and Eq. (14) for the ratio of the average multiplicities in nucleus-nucleus and nucleon--nucleon collisions. Both results are very clear from a geometrical point of view. Though the currently available energies are not high enough, nevertheless Eq. (9) should be useful even now. In the case of Eq. (14), only the ratio of the nucleus-nucleus and nucleon-nucleus average multiplicities may be calculated but not the nucleus-nucleus and nucleon-nucleon one which can be done however at a higher energy. Similarly, the predictions for inclusive spectrum ratios are expected to be valid only at the energies $\gtrsim 10^2$ GeV/nucleon.

2. Inclusive spectra of secondaries and number of interacting nucleons

Let us suppose at first that the energy of an incident nucleus is high enough (i.e., about a hundred GeV per nucleon), and secondary particles do not interact within the nuclei. By using the Abramovsky–Gribov–Kancheli rules [1] one may then express the inclusive

* Address: Leningrad Nuclear Physics Institute, Gatchina, Leningrad district 188350, USSR.

spectrum $\varrho_{AB}(x)$ of secondaries in the central region $|x| \ll 1$, for a collision of a nucleus, A, with a nucleus, B, in terms of the nucleon-nucleon one, $\varrho_{NN}(x)$

$$2E \frac{d^3\sigma^{AB}}{d^3p} = \varrho_{AB}(x) = A \cdot B \cdot \varrho_{NN}(x), \quad |x| \ll 1. \tag{1}$$

In nucleus-nucleus experiments, both the events with disintegration of one or two nuclei and those with secondary particle production are measured. This suggests the definition of differential multiplicity as

$$f_{AB}(x) = \varrho_{AB}(x)/\sigma^{AB}_{\text{inel}} \tag{2}$$

rather than the more popular definition, $f_{AB}(x) = \varrho_{AB}(x)/\sigma^{AB}_{\text{prod}}$, where $\sigma^{AB}_{\text{prod}}$ is the cross section for production of at least one secondary. The ratio of multiplicities in A—B and N—N collisions in the central region is then of the form

$$f_{AB}(x)/f_{NN}(x) = \frac{A \cdot B \cdot \sigma^{NN}_{\text{tot}}}{\sigma^{AB}_{\text{inel}}}, \quad |x| \ll 1, \tag{3}$$

where

$$f_{NN}(x) = \varrho_{NN}(x)/\sigma^{NN}_{\text{tot}}. \tag{4}$$

This formula may be obtained also directly from the optical model [2, 3].

In the case of hadron-nucleus collisions the multiplicity ratio in the central region is equal [4, 5] to the average number of interacting nucleons, $\langle \nu_A \rangle$:

$$f_{NA}(x)/f_{NN}(x) = \langle \nu_A \rangle = \frac{A \cdot \sigma^{NN}_{\text{tot}}}{\sigma^{NA}_{\text{inel}}}. \tag{5}$$

$\langle \nu_A \rangle$ is different from the well known quantity

$$\bar{\nu}_A = \frac{A \cdot \sigma^{NN}_{\text{inel}}}{\sigma^{NA}_{\text{prod}}} \tag{6}$$

due to our normalization of multiplicities, Eqs. (2), (4). By using Eq. (5) one may rewrite Eq. (3) in the form

$$f_{AB}(x)/f_{NN}(x) = \langle \nu_A \rangle \cdot N_B = \langle \nu_B \rangle \cdot N_A = \langle \nu_A \rangle \cdot \langle \nu_B \rangle \cdot F(A, B), \tag{7}$$

$$F(A, B) = \frac{\sigma^{NA}_{\text{inel}} \cdot \sigma^{NB}_{\text{inel}}}{\sigma^{NN}_{\text{tot}} \cdot \sigma^{AB}_{\text{inel}}}. \tag{8}$$

The quantity in the right-hand side of Eq. (7) denotes average number of the nucleon--nucleon interactions when nuclei A and B collide. The quantity

$$N_A = A \cdot \sigma^{NB}_{\text{inel}}/\sigma^{AB}_{\text{inel}} \tag{9}$$

is the average number of the nucleons of the nucleus, A, participating in the interaction. Each of them collide, on the average, with $\langle \nu_B \rangle$ nucleons of the nucleus, B, and vice versa when A is interchanged with B. This expression for N_A was obtained earlier in Ref. [6]

Eq. (7) permits a simple geometric interpretation. $\langle \nu_A \rangle$ is an average number of nucleons in the tube of the nucleus A in which an incident nucleon travels with cross section σ_{tot}^{NN}, $\langle \nu_B \rangle$ is a similar quantity for the nucleus B. The geometrical factor $F(A, B)$ in Eq. (8) is therefore the average number of the colliding tubes. Naturally, the same factor appears also in the collective tube model [7].

The magnitude of N_A can be measured experimentally as a difference between the atomic weight of the projectile nucleus and the average number of stripping nucleons. The prediction (9) shows a very weak energy dependence of N_A, so we may compare Eq. (9) with experimental data at an energy not too great (several GeV per nucleon). For this purpose, we accept for $\sigma_{\text{inel}}^{NA}$ the parametrization

$$\sigma_{\text{inel}}^{NA} = 39 \text{ mb} \cdot A^{0.72} \tag{10}$$

that is consistent with the data of Refs. [8, 9]. The inelastic nucleus-nucleus cross sections are given by Bradt–Peters formula [10]

$$\sigma_{\text{inel}}^{AB} = \pi R_0^2 [A^{1/3} + B^{1/3} - C]^2, \quad R_0 = 1.52 \text{ fm}, \quad C = 1.35. \tag{11}$$

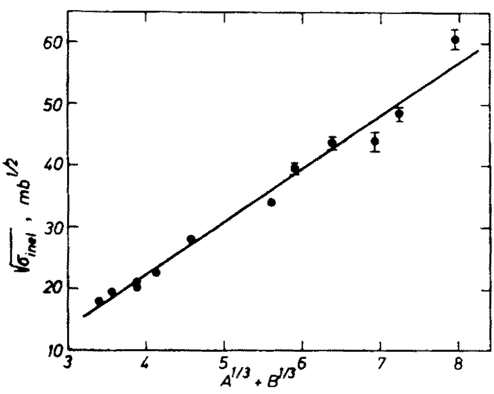

Fig. 1

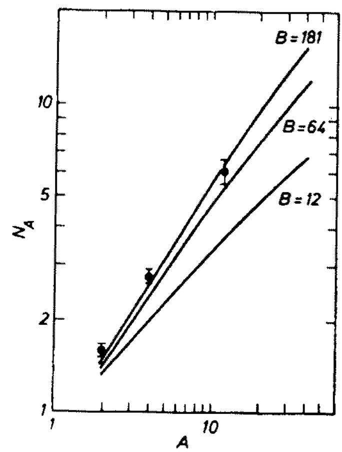

Fig. 2

Fig. 1. Inelastic nucleus-nucleus cross sections Refs. [9, 11–13] as a function of atomic weights. The results of calculations using Eq. (11) are shown by the solid line

Fig. 2. Average number of interacting nucleons, N_A within the incident nucleus, A vs its atomic weight, A, for different targets, B. The data of Ref. [9] are shown for d-Ta, α-Ta and C-Ta collisions

This choice of parameters R_0 and C, as shown in Fig. 1, is also in reasonable agreement with experimental data [9, 11–13]. In Fig. 2 N_A is shown as a function of A, calculated by using Eqs. (9)–(11) for three target nuclei B. The experimental data of Ref. [9] have been obtained on the target nucleus ^{181}Ta. The number of stripping nucleons in the deuteron and ^{4}He interactions with photoemulsion, obtained in Ref. [14], is also compatible with Eq. (9).

In the nucleon-nucleus collisions the multiplicity ratio $f_{NA}(x)/f_{NN}(x)$ in the fragmentation region ($x \gtrsim 0.1$) becomes less than unity (e.g. Ref. [15]). Such behaviour is described quantitatively [16] by the composite quark model. From the viewpoint of the multiple scattering model it is a consequence of the energy conservation [5]. A similar phenomenon must occur in the nucleus-nucleus collisions as well. At high energy we expect that in the fragmentation region of the projectile nucleus, A,

$$f_{AB}(x)/f_{NN}(x) \sim N_A \cdot B^{-\gamma(x)}, \quad x \gtrsim 0.1, \tag{12}$$

and for the target, B, fragmentation

$$f_{AB}(x)/f_{NN}(x) \sim N_B \cdot A^{-\gamma(x)}, \quad -x \gtrsim 0.1, \tag{13}$$

where the function $\gamma(x) > 0$ can be obtained from the nucleon-nucleus and nucleon--nucleon data. It increases slowly with $|x|$. Our predictions (12), (13) disagree with those of Winbow [17]. On the other hand, near the edges of the spectrum the $f_{AB}(x)/f_{NN}(x)$ ratio may increase because of the nucleon Fermi motion. It is possible to distinguish between these two phenomena by comparing the collisions of nuclei with different atomic weights but the same Fermi-momenta.

TABLE I

Predicted ratios of the secondary particle production cross sections in nucleus-nucleus collisions at high energies ($\gamma \sim 0.2$ at $x \sim 0.5$)

Inclusive cross section ratio	Region of the nucleus B fragmentation $x < -0.1$	Central region $\lvert x\rvert < 0.1$	Region of the nucleus A fragmentation $x > 0.1$
$\frac{\sigma(A_1B \to hX)}{\sigma(A_2B \to hX)}$	$\left(\frac{A_1}{A_2}\right)^{-\gamma} \frac{\sigma_{\text{inel}}^{NA_1}}{\sigma_{\text{inel}}^{NA_2}}$	$\frac{A_1}{A_2}$	$\frac{A_1}{A_2}$
$\frac{\sigma(AB_1 \to hX)}{\sigma(AB_2 \to hX)}$	$\frac{B_1}{B_2}$	$\frac{B_1}{B_2}$	$\left(\frac{B_1}{B_2}\right)^{-\gamma} \frac{\sigma_{\text{inel}}^{NB_1}}{\sigma_{\text{inel}}^{NB_2}}$

The consequences of Eqs. (12) and (13) for the ratios of the production cross sections are given in Table I. We expect them to be valid at a high enough energy, simultaneously with Eq. (5) for hadron-hadron collisions. For intermediate energies, it is necessary to take into account that if the projectile collides with $\langle \nu \rangle$ nucleons, only the first interaction occurs with the total energy E_0. In the case of $A \ll B$ we expect the predictions of the first line of Table I to be valid even at existing energies. However, the ratios $\sigma(AB_1 \to \to hX)/\sigma(AB_2 \to hX)$ ($B_1, B_2 \gg 1$) in the second line for the target fragmentation and central regions may differ significantly from the expected values but become closer to them with increasing energy. At the present energies we may only expect for these ratios to be independent of A up to the Fermi motion corrections.

3. *Average multiplicities of secondaries*

Les us consider here the yields of negative particles in order to eliminate the contributions from relatively fast protons. In hadron-nucleus collisions the $R_{NA} = \langle n_- \rangle_{NA} / \langle n_- \rangle_{NN}$ ratio differs from $\langle \nu_A \rangle$ even at high energy due to the presence of the fragmentation contribution. A similar behaviour is expected to occur in nucleus-nucleus collisions. When two nuclear tubes collide, the number of nucleon-nucleon interactions is given by the $\langle \nu_A \rangle \langle \nu_B \rangle$ product. However the multiplicity of secondaries increases with A and B substantially slower, especially at intermediate energies. Neglecting this phenomena and using Eq. (7) for the ratio of average multiplicities in Ref. [3] would lead to a strong contradiction with the experimental data.

Multiplicity of secondaries at intermediate energies can be calculated in a rather simple way provided one of the two colliding nuclei is much heavier than another. Let us suppose for example $A \ll B$. At energies less than ~ 20 GeV per nucleon where the experimental data exist, the R_{NB} value are much smaller than $\langle \nu_B \rangle$. This means that most of the secondaries is being produced in the projectile interaction with only one of the target nucleons. Other target nucleons play a passive role in hadron production. The second nucleon from the $\langle \nu_A \rangle$ tube interacts with the $(\langle \nu_B \rangle - 1)$ target nucleons and produces the same number of secondaries as the first one. Thus we obtain the total multiplicity of secondaries for the nucleons from the $\langle \nu_A \rangle$ tube interacting with the nucleons from the $\langle \nu_B \rangle$ tube in the form $\langle \nu_A \rangle \cdot R_{NB} \cdot \langle n_- \rangle_{NN}$, and the ratio of the average multiplicities in A—B and N—N collisions accounting for (7) is

$$R_{AB} = \frac{\langle n_- \rangle_{AB}}{\langle n_- \rangle_{NN}} = \langle \nu_A \rangle \cdot R_{NB} \cdot F(A, B), \qquad A \ll B. \tag{14}$$

Though this expression is asymmetrical and therefore not valid for $A \approx B$, the numerical magnitudes of R_{AB} are close to the more accurate estimates.

It is worth noting once more that if one uses the experimental magnitude of R_{NB} in the right-hand side of Eq. (14), only the interactions of secondaries within the B nucleus but not within the A nucleus will be accounted for.

To compare Eq. (14) with the data we write R_{NB} as

$$R_{NB} = 1 + \delta(\langle \nu_B \rangle - 1). \tag{15}$$

In accordance with the results of Ref. [18], the values of δ are taken to be 0.1 at $p_A/A \sim 4.2$ GeV/c and 0.02 at $p_A/A \sim 2.6$ GeV/c. The calculated magnitudes of $\langle n_- \rangle_{AB} = R_{AB} \cdot \langle n_- \rangle_{NN}$ for interactions of the d, α, C and Ar nuclei at $p_A/A \sim 4.2$ GeV/c or C and Ar nuclei at $p_A/A \sim 2.6$ GeV/c with different targets are shown in Fig. 3. Agreement with the experimental data [9, 11, 19, 20] is seen to be quite reasonable. On the other hand, Eq. (11) seems to give too small cross sections $\sigma_{\rm inel}^{AB}$ when both nuclei are relatively heavy. This apparently happens for the C—Ta interaction in Fig. 1, and for the Ar—B interactions at $p_A/A = 2.6$ GeV/c in Fig. 3b.

In Fig. 4 we compare the calculated R_{AB} ratio with the data of Ref. [21] on the cosmic nuclei interactions with photoemulsion at an average energy of 19 GeV per nucleon. The

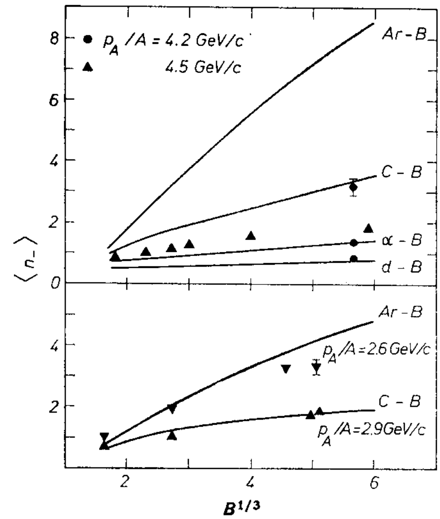

Fig. 3. Average negative particle multiplicities in A—B collisions at a) $p_A/A = 4.2$ GeV/c (● and curves), $p_A/A = 4.5$ GeV/c (▲) and b) $p_A/A = 2.6$–2.9 GeV/c as a function of the target, B, atomic weight

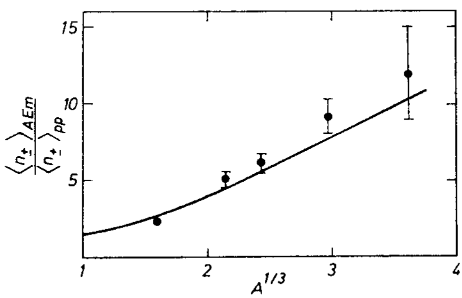

Fig. 4. Ratio of the charged meson multiplicities in A-Em and pp collisions at an average energy of 19 GeV per nucleon, Ref. [21]

sampling with at least one secondary particle was used, so all multiplicities must be normalized to $\sigma_{\text{inel}}^{NN}$, $\sigma_{\text{prod}}^{NA}$ and $\sigma_{\text{prod}}^{AB}$, respectively. Let $\tilde{R}_{AB}$ and $\tilde{R}_{NA}$ be the ratios of the multiplicities. Taking into account that these cross sections were used instead of σ_{tot}^{NN}, $\sigma_{\text{inel}}^{NA}$ and $\sigma_{\text{inel}}^{AB}$, and that at $p_A/A \lesssim 10$ GeV/c per nucleon the magnitudes of $\sigma_{\text{inel}}^{NA} \cdot \sigma_{\text{inel}}^{NB}/(\sigma_{\text{tot}}^{NN} \cdot \sigma_{\text{inel}}^{AB})$ and $\sigma_{\text{prod}}^{NA} \cdot \sigma_{\text{prod}}^{NB}/(\sigma_{\text{inel}}^{NN} \cdot \sigma_{\text{prod}}^{AB})$ are approximately equal we obtain

$$\tilde{R}_{AB} = \bar{\nu}_A \cdot \tilde{R}_{NB} \cdot F(A, B), \qquad A \ll B. \tag{16}$$

The values of $\tilde{R}_{AB}$ for the photoemulsion target have been calculated by assuming $\tilde{R}_{NB} = 1 + \delta(\bar{\nu}_B - 1)$ and $\delta = 0.35$ [5]. As shown in Fig. 4 they are in good agreement with the data [21].

The incident energy dependence of R_{AB} and R_{NB} (or $\tilde{R}_{AB}$ and $\tilde{R}_{NB}$), as follows from Eqs. (14), (16) is predicted to be almost the same. These quantities are expected to increase very slowly with energy.

Dependences of R_{AB} (or $\tilde{R}_{AB}$) on the projectile and target atomic weights also seem to be of interest. Consider at first the power-like parametrization

$$R_{AB} = C \cdot B^{\alpha(A)}, \quad A \lesssim B. \tag{17}$$

As shown in Fig. 3, $\alpha(A)$ increases rapidly with A. This effect is due to the geometrical factor $F(A, B)$. As follows from Eq. (8), $F(A, B) \sim A^{2/3} \cdot B^{\varepsilon}$, ($\varepsilon \ll 1$), for $A \ll B$ and $F(A, B) \sim A^{1/3}B^{1/3}$ for $A \sim B \gg 1$. Thus when A changes from $A \ll B$ to $A \sim B$ the $\alpha(A)-\alpha(1)$ difference must increase by $\sim 1/3$ independently of additional assumptions

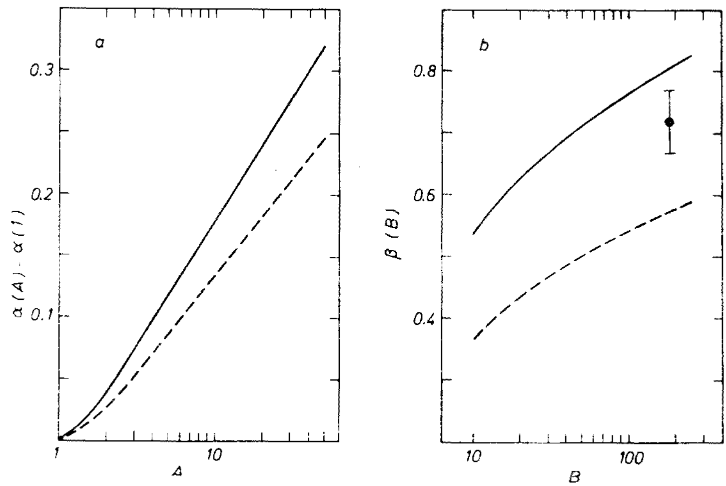

Fig. 5. a) A-dependence of the $\alpha(A)-\alpha(1)$ function in Eq. (17); b) B-dependence of $\beta(B)$ function in Eq. (18). The solid lines correspond to the multiple scattering model. The dashed lines have been calculated for the model of Ref. [6]. The data point for $B = 181$ was obtained from Ref. [9]

made when going from Eq. (7) to Eq. (14). The predicted values $\alpha(A)-\alpha(1)$ as a function of A in the range $12 < B < 200$ are shown in Fig. 5a.

Thus $\beta(B)$ function defined as

$$R_{AB} = C \cdot A^{\beta(B)}, \quad A \lesssim B \tag{18}$$

is shown in Fig. 5b for $2 \leqslant A \leqslant 12$. Also shown is the data point for $B = 181$ as taken from Ref. [9].

The $\alpha(A)$ and $\beta(B)$ functions are sensitive to the mechanism of the particle production on nuclei. For instance in the model of Ref. [6] the multiplicity of produced particles is proportional to the number of "wounded" nucleons, $R_{AB} = (N_N + N_B)/2$. In such a case $\tilde{R}_{hA} = (1+\bar{\nu}_A)/2$ for the hadron-nucleus collisions that is close to the multiple scattering model predictions. For the nucleus-nucleus interactions the distinction between these two models is clearer. The $\alpha(A)-\alpha(1)$ and $\beta(B)$ values predicted by the model of Ref. [6] are shown in Fig. 5 by the dashed lines[1]. They are lower than the multiple scattering model

[1] It is assumed that the cross sections $\sigma_{\text{inel}}^{NA}$ and $\sigma_{\text{prod}}^{NA}$ as well as $\sigma_{\text{inel}}^{AB}$ and $\sigma_{\text{prod}}^{AB}$ depend on A and B in the same way.

predictions. The $R_{AB} = R_{AB}(A)$ dependence for $A \lesssim B$ in the collective tube model [7, 22] is also rather weak. The $\beta(B)$ values in this model are found to be 0.25–0.3 smaller than in the multiple scattering model.

4. *Conclusion*

The consistency with the experiment which was demonstrated above shows that the multiple scattering model may appear to be applicable to the multiparticle production processes in nucleus-nucleus collisions. It describes reasonably well the main features of such processes. Apparently this means that the collective effect contributions are small and special sampling is needed to find them.

All the quantities in Eq. (9), (14), (16) may be measured experimentally, so it is possible and desirable to verify these predictions directly at existing energies. A measurement of the secondary multiplicities in a large range of A and B for different incident energies would be of much interest. As shown in Fig. 5, such data would enable one to distinguish between different models of multiparticle production on nuclei.

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