QUARK AND GLUON TRANSVERSE MOMENTA AND QCD PREDICTIONS FOR LARGE TRANSVERSE MOMENTUM PARTICLE PRODUCTION

By J. Ranft and Gisela Ranft

Sektion Physik, Karl-Marx-Universität, Leipzig*

(*Received June 26, 1978*)

Large $p_\perp$ single particle and single jet cross sections according to QCD perturbation theory are calculated with parton transverse momenta taken into account. Good agreement is found with data at transverse momenta from 3 GeV/c up to 16 GeV/c. The inclusion of parton transverse momenta is essential to obtain agreement with data at intermediate energies $\sqrt{s} \simeq 20$ to 30 GeV.

1. *Introduction*

Large $p_\perp$ single particle distributions according to quark quark, quark gluon, and gluon gluon scattering as predicted by QCD were computed in Refs [1] and [2]. In Ref. [1] also the deviations from scaling of quark and gluon distributions were used as predicted by asymptotic freedom [3, 4]. Recent calculations of higher order contributions suggest that such an approach can indeed be justified from QCD [5]. In this approach it was found that no artificial suppression of the quark and gluon scattering was needed to understand the data. The Q^2 dependence of asymptotic freedom of the structure functions and of the coupling constant for the single particle distribution leads to the behaviour

$$E\frac{d^3\sigma}{d^3p} \sim \frac{1}{p_\perp^{n_{\mathrm{eff}}(x_\perp, \sqrt{s})}} f(x_\perp). \tag{1}$$

The power $n_{\mathrm{eff}}(x_\perp, \sqrt{s})$ at present energies is in the range 5 to 6 depending on $\sqrt{s}$ and $x_\perp$ in a complicated manner (see figure 1). At ISR energies of $\sqrt{s} \simeq 50$ to 60 GeV rather good agreement with the data was found in Refs [1] and [2]. However, at lower energy, $\sqrt{s} \simeq 20$ to 30 GeV, calculated single particle distributions [1] were below the data by a factor 5 to 10.

---

* Address: Sektion Physik, Karl-Marx-Universität, 701 Leipzig, Linnéstrasse 5, DDR.

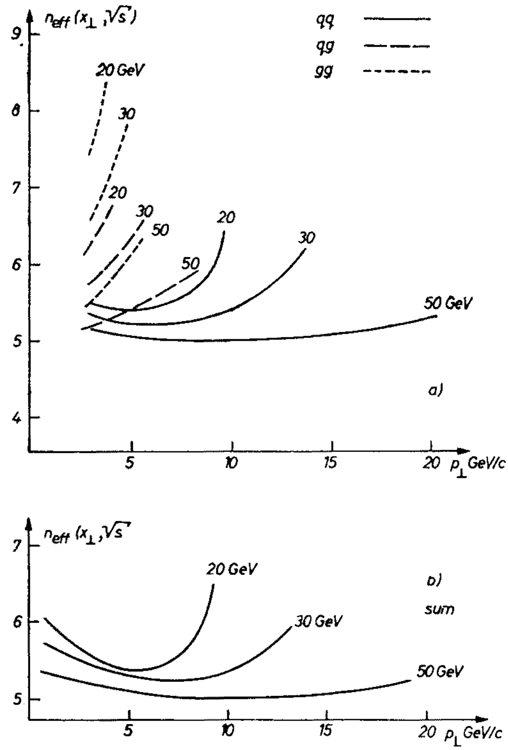

Fig. 1. The effective power $n_{eff}(p_\perp, \sqrt{s})$ for three different energies $\sqrt{s} = 20, 30$ and 50 GeV (a) plotted separately for the three dominant mechanisms contributing to the QCD predictions of large $p_\perp$ jet production quark-quark, quark-gluon, and gluon-gluon scattering, (b) plotted for their sum. The curves are only given in the $p_\perp$ range where a given mechanism contributes significantly to the total cross section

2. Approximate analytical calculation of the influence of parton transverse momenta on single jet distributions

The influence of parton transverse momenta on large $p_\perp$ spectra was studied last year in a couple of papers [6–8]. Most of them did indicate that the parton transverse momentum does not change the effective power significantly. Therefore the conclusion was that starting from hard scattering models with $\sim p_\perp^{-4}$ parton transverse momenta are not sufficient to obtain the experimental behaviour $\sim p_\perp^{-8}$. This picture seems to change if the QCD predictions are used as a starting point as recently found by Monte Carlo calculations [9, 10]. In the present paper we use an approximate analytic method [7] to study the influence of quark gluon transverse momenta on the large $p_\perp$ distributions according to QCD which we compute as in Ref. [1]. The approximation of our treatment consists in the following:

i) We use factorized quark and gluon distributions in the form

$$F(x, Q^2, k_\perp^2) = F(x, Q^2)g(k_\perp^2), \tag{2}$$

with $F(x, Q^2)$ according to predictions from asymptotic freedom [4] and

$$g(k_\perp^2) = \frac{1}{\pi\langle k_\perp^2\rangle} \exp\left(\frac{-k^2}{\langle k_\perp^2\rangle}\right). \tag{3}$$

We work out the predictions for transverse momenta of quarks and gluons equal to $\langle k_\perp^2\rangle^{1/2} \simeq 0.7$ and 1 GeV/c. Such rather large values of $\langle k_\perp^2\rangle$ are suggested by experiment [11].

ii) We call the system of the 2 jets produced a "di-jet". It has the variables Y, $\vec{P}_\perp$ and the mass $\hat{s}$. We calculate first the di-jet production cross section. In terms of the parton distributions $q(x, \vec{k}_\perp) = (1/x)\ f(x, \vec{k}_\perp)$ for "di-jet" rapidity $Y \approx 0$ and resulting jets observed at $\theta = 90°$ in the c. m. s., the "di-jet" production cross section behaves like

$$\frac{d\sigma}{d\hat{s}dYdP_\perp^2} \approx \frac{\text{const}}{\hat{s}^{n/2}} \sum_{i,j} \int d^2k_\perp f_i(x_1, \vec{k}_\perp) f_j(x_2, \vec{P}_\perp - \vec{k}_\perp), \tag{4}$$

where we have already used conservation of the transverse momentum. We limit our discussion to the kinematic region where $\hat{s} \gg 1\ (\text{GeV})^2$ and $\hat{s} \gg P_\perp^2$.

To study the influence of parton transverse momenta on single-jet production, the $P_\perp$ dependence of Eq. (4) is more essential than the Y or $P_\parallel$ dependence. Modifying the $P_\parallel$ dependence, we would still expect to find the essential effects caused by the parton $k_\perp$. The longitudinal momentum distribution of Eq. (4) is determined by the parton x distribution and a transverse momentum distribution determined by the parton $k_\perp$ distribution. We replace the exact distribution by an isotropic one where the di-jet longitudinal distribution is the same as the transverse one $\left(E\dfrac{d^4\sigma}{d\hat{s}\ dP^3}\right)_{\text{isotropic}}$ which at $\theta = 90°$ ($Y = 0$) agrees with (4).

The energy distribution $d\sigma/(d\hat{s}\ dE)$ of the "di-jet" is easily obtained from an isotropic "di-jet" distribution. If the "di-jet" decays isotropically in its rest frame into two jets i and j with momenta $|\vec{p}_i^{\,*}| = |\vec{p}_j^{\,*}| = \sqrt{\hat{s}}/2$ and polar angles θ_i and $\theta_j = \theta_i + \pi$, the energy distribution of the single jet i in the collision c. m. s. is given by [7]

$$\frac{d\sigma}{d\varepsilon^*} = \int_0^1 dE d\hat{s} \frac{d\sigma}{d\hat{s}dE} \frac{1}{P}, \quad E_0 = \varepsilon + \frac{\sqrt{\hat{s}}}{4\varepsilon}, \quad E_1 = \frac{\sqrt{s}}{2}. \tag{5}$$

We use Eq. (5) only for jets produced at $\theta = 90°$ in the collision c. m. s. From Eq. (5) the invariant single-jet distribution at $\theta = 90°$ is

$$\varepsilon \frac{d\sigma}{dp_i^3}\bigg|_{\substack{\theta = 90° \\ p_i = p_{i\perp}}} = \frac{1}{2\pi p_i} \frac{d\sigma}{d\varepsilon_i}. \tag{6}$$

We perform calculations with the factorized distributions (2) and Gaussian $k_\perp$ distributions (3).

In this case all integrals but one can be performed analytically and we obtain for the single-jet spectrum at $\theta = 90°$

$$\varepsilon_i \left.\frac{d^3\sigma}{dp_i^3}\right|_{\theta=90°} = \frac{1}{2\pi p_{i\perp}} \frac{\text{const}}{2\pi} \int \frac{d\hat{s}}{\hat{s}^{n/2}} \exp\left[-\frac{1}{2\langle k_\perp^2\rangle}\left(p_{i\perp} - \frac{\hat{s}}{4p_{i\perp}}\right)^2\right]$$
$$\times \frac{1}{\sqrt{2\pi\langle k_\perp^2\rangle}} F_1\left(Q^2, \sqrt{\frac{\hat{s}}{s}}\right) F_2\left(Q^2, \sqrt{\frac{\hat{s}}{s}}\right). \tag{7}$$

An asymptotic estimate for this integral using $F_i(\sqrt{\hat{s}/s}) = \text{const}$ leads to $\varepsilon d^3\sigma/dp^3 \sim 1/p_\perp^n$. Thus, asymptotically no change of the power n can be expected, with F_i independent of Q^2, i. e. no scale breaking. In our calculation we use Eq. (7) in the form

$$E \left.\frac{d^3\sigma}{d^3p}\right|_{\text{jet}} = H(p_\perp, \sqrt{s}, \langle k_\perp^2\rangle) = \frac{1}{4p_\perp\sqrt{2\pi\langle k_\perp^2\rangle}} \int d\hat{s} H(p_\perp, \sqrt{s}, \langle k_\perp^2\rangle = 0)$$
$$\times \exp\left[-\frac{\left(p_\perp - \frac{\hat{s}}{4p_\perp}\right)^2}{2\langle k_\perp^2\rangle}\right], \tag{8}$$

where $H(p_\perp = \sqrt{\hat{s}}/2, \sqrt{s}, \langle k_\perp^2\rangle = 0)$ is the single jet distribution without parton transverse momenta. Contrary to the claim in Ref. [8], we use in this approximation *not* only symmetric parton parton collisions with $x_1 = x_2$. Considering the well known parton momentum distributions in hadrons we find that our approximation (which uses the average parton longitudinal momenta *equal* to the parton transverse momenta) is expected to be a rather good one at the energies presently of interest, $\sqrt{s} \simeq 20$ to 30 GeV and leads to results which agree with Monte Carlo calculations of other authors.

3. *Numerical calculation and comparison with data at ISR and FNAL-energies*

We define

$$R(p_\perp, \sqrt{s}, \langle k_\perp^2\rangle) = \frac{H(p_\perp, \sqrt{s}, \langle k_\perp^2\rangle)}{H(p_\perp, \sqrt{s}, \langle k_\perp^2\rangle = 0)}. \tag{9}$$

In figures 2a and b we plot the ratio R as obtained by inserting the QCD single jet cross sections into Eq. (8). R depends strongly on $p_\perp$ and $\sqrt{s}$; it is largest where the large $p_\perp$ single jet distribution is steep like at small $\sqrt{s}$ or for the gluon gluon contribution. At Fermi-lab energies R depends dramatically on the average parton transverse momentum, as shown in Fig. 2a.

In Fig. 2c we plot the effective power $n_{\text{eff}}(\sqrt{s}, x_\perp)$ for the total jet distribution (quark--quark, quark-gluon, and gluon-gluon contributions) for $\sqrt{s} = 20$, 30 and 50 GeV and $\langle k_\perp^2\rangle^{1/2} = 1$ GeV/c. In the $p_\perp$ region around 5 GeV/c we find indeed an effective power near to 8, as found experimentally; but it is to be noted that n_{eff} depends strongly on $x_\perp$ decreasing with rising $x_\perp$.

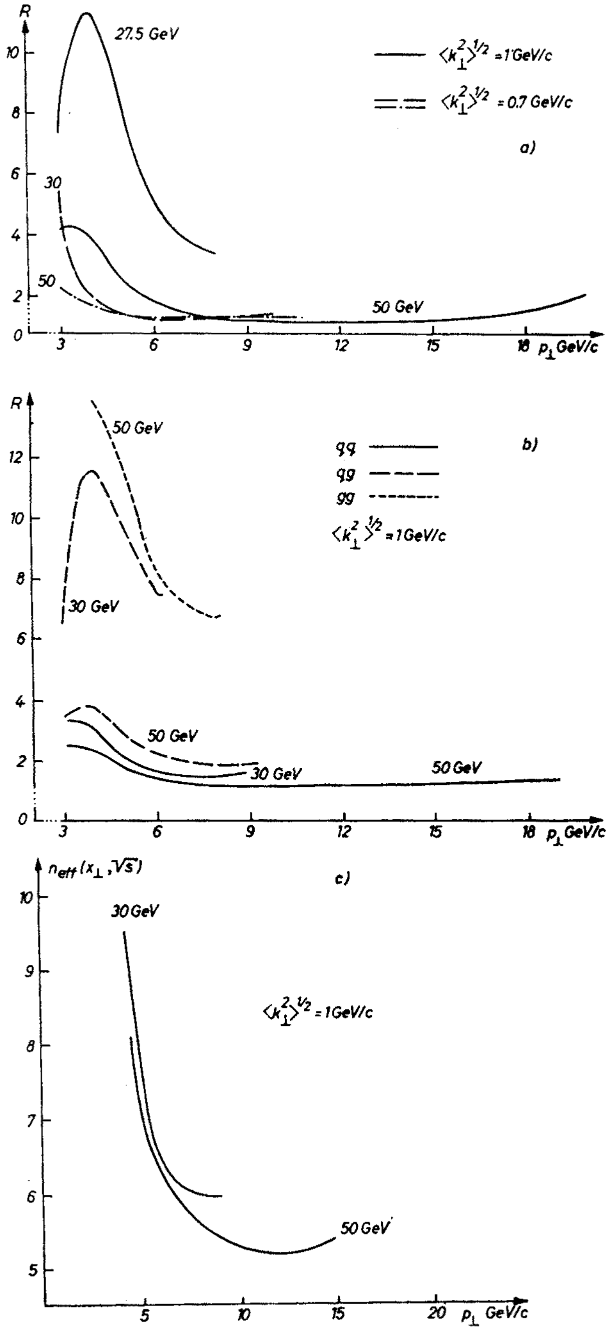

Fig. 2 (a, b). The influence of parton $k_\perp$ distributions on single jet distributions. The ratio R between the large $p_\perp$ jet cross section calculated with and without parton transverse momenta as defined in Eq. (9) plotted as function of $p_\perp$ for different values of $\sqrt{s}$ and $\langle k_\perp^2\rangle^{1/2}$. (a) The ratio R for the total jet cross section for two different $\langle k_\perp^2\rangle^{1/2}$; (b) The ratio R calculated separately for the three contributing mechanisms at $\langle k_\perp^2\rangle^{1/2} = 1$ GeV/c; (c) The effective power $n_{\text{eff}}(\sqrt{s}, x_\perp)$ for the sum of all three mechanisms, at $\langle k_\perp^2\rangle^{1/2} = 1$ GeV/c and $\sqrt{s} = 30$ and 50 GeV

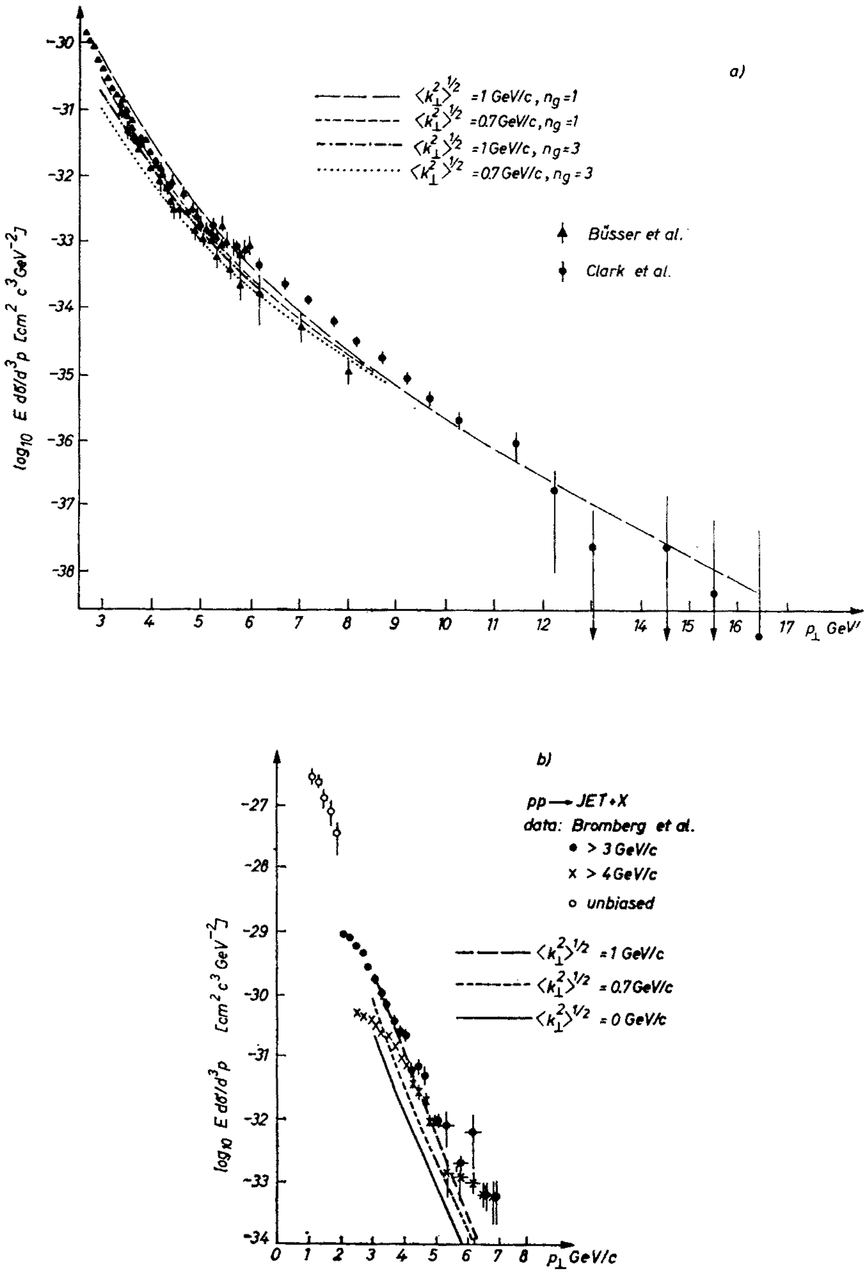

Fig. 3. (a) Comparison of the QCD prediction for π^0 production at large $p_\perp$ at $\sqrt{s} = 62.4$ GeV and $\theta = 90°$. The data are due to Büsser et al. [12] and Clark et al. [13]. The four curves are for average parton transverse momenta $\langle k_\perp^2\rangle^{1/2} = 0.7$ and 1 GeV/c and gluon fragmentation functions $G_g^h(z) \sim (1-z)^{n_g}$ with $n_g = 3$ and 1; (b) Comparison of the QCD prediction for the large $p_\perp$ single jet cross section in pp collisions at $\sqrt{s} = 19.4$ GeV and $\theta = 90°$. The data is due to Bromberg et al., Ref. [14]. The three curves are the QCD predictions for $\langle k_\perp^2\rangle^{1/2} = 0$, 0.7 and 1 GeV/$c$

In figure 3 we compare the predicted cross sections with experimental data [12, 13, 14] at ISR and Fermi-lab energies. We find at ISR energies: The good agreement with data found already in Refs. [1] and [2] is confirmed also for the new data [13] with $p_\perp$ up to 16 GeV/c. At low $p_\perp$ the agreement is improved by taking the parton transverse momentum into account, especially when using the more likely gluon distribution function of the form $(1-z)^3$.

At Fermi-lab energies we compare with the jet production cross section [14]. This has the advantage that the comparison is not influenced by the choice of the parton fragmentation function. The predictions without parton transverse momenta are approximately a factor 10 below the data. This was already found in Ref. [1] comparing with large $p_\perp$ particle cross sections. Using average parton transverse momenta as large as 1 GeV/c the predictions agree also at this energy much better with the data. We stress, however, that the rise in the single jet cross section with growing $\langle k_\perp^2 \rangle^{1/2}$ is very steep at these energies, as shown in figure 2. Parton transverse momenta of similar size were necessary in the Monte Carlo calculation in Ref. [9].

We conclude: The predictions for large $p_\perp$ particle production obtained by using lowest order QCD perturbation theory and taking parton transverse momenta into account agree rather well with presently known single particle and single jet production cross sections at large $p_\perp$.

To confirm the relevance of QCD predictions for large $p_\perp$ production in purely hadronic collisions further tests are needed, e. g. concerning the spin dependence of large $p_\perp$ particle production [15] and correlations between two particles or jets in large $p_\perp$.

We acknowledge discussions with B. R. Combridge, A. P. Contogouris, P. Darriulat, J. Ellis, J. Kripfganz and R. G. Roberts. We thank Mr. Discher for assistance with numerical calculations.

REFERENCES

[1] B. R. Combridge, J. Kripfganz, J. Ranft, *Phys. Lett.* **70B**, 234 (1977).
[2] R. Cutler, D. Sivers, *Phys. Rev.* **D17**, 196 (1978).
[3] H. D. Politzer, *Phys. Reports* **14C**, 129 (1974).
[4] A. J. Buras, K. J. F. Gaemers, *Nucl. Phys.* **B132**, 249 (1978).
[5] C. T. Sachrajda, *Phys. Lett.* **76B**, 100 (1978); W. Furmański, *Phys. Lett.* **77B**, 312 (1978).
[6] M. Della Negra et al., *Nucl. Phys.* **B** (1977); R. D. Field, R. P. Feynman, G. C. Fox, *Nucl. Phys.* **B128**, (1977); R. Baier, B. Petersson, Bielefeld preprint BI-TP 77/08 (1977); M. Fontannaz, D. Schiff, Orsay preprint (1977).
[7] G. Ranft, J. Ranft, *Nuovo Cimento Lett.* **20**, 669 (1977).
[8] F. Halzen, G. A. Ringland, R. G. Roberts, Rutherford preprint, RL-77-115/A (1977).
[9] R. D. Field, *Phys. Rev. Lett.* **40**, 997 1978).
[10] A. P. Contogouris, R. Gaskall, A. Nicolaidis, *Phys. Rev.* **D17**, 839 (1978).
[11] S. W. Herb et al., *Phys. Rev. Lett.* **39**, 252 (1977).
[12] F. W. Büsser et al., *Nucl. Phys.* **B106**, 1 (1976).
[13] A. G. Clark et al., *Phys. Lett.* **74B**, 267 1978.
[14] C. Bromberg et al., Proceedings VIII Int. Symp. on Multiparticle Dynamics, Kayserberg, France, June 1977, p. B-89.
[15] J. Ranft, G. Ranft, *Phys. Lett.* **77B**, 309 (1978).