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THE COULOMB DISINTEGRATION OF RELATIVISTIC CARBON IONS INTO THREE ALPHA PARTICLES

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The Coulomb disintegration of relativistic ^{12}C ions into three α-particles has been considered. The basic assumption of the dissociation mechanism is that a preliminary excitation of the carbon ion is followed by its decay into three α-particles. The total cross section of such reactions have been calculated as well as the energy distribution of secondary α-particles.

1. In recent years, peripheral collisions of relativistic heavy ions, the so-called fragmentation reactions attract more and more attention of physicists. One of main results is the evidence for the Q_{gg}-systematics [1], i. e. the exponential cross-section dependence on the rearrangement energy of the initial and final states of fragmenting nuclei proposed in Refs. [2, 3]. Another important result is the understanding of the essential role of the Coulomb mechanism of nuclear disintegration with a preliminary excitation of high-lying resonance states [4, 5]. The investigation of the Coulomb mechanism itself is a very important problem. Really, since the electromagnetic potential of the ion-nuclear interaction can be assumed to be known, it appears possible to study the structure of high-lying nuclear states, in addition to the analysis of a specific relativistic Coulomb excitation of nuclei. Here it is necessary, of course, to choose among many processes of fragmentation only those in which the Coulomb mechanism dominates. One of those processes is the dissociation of carbon ions in collision with a heavy nucleus.

Other mechanisms, in which the nucleus is excited through the quasi-elastic scattering of target nucleons on one or several ion nucleons, will result in a radical rearrangement of the ion (e. g. in stripping of several nucleons) and in a decrease in the number of α-particles detected simultaneously.

In this paper we study the cross section of the dissociation of carbon ions into three α-particles and the energy distribution of the α-particles. In so doing we restrict ourselves to spherical target nuclei. (The restriction is not fundamental but essentially simplifies our formulae).

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2. The main contribution to the transition amplitude from the state $|i\rangle$ to the state $|f\rangle$ at a relativistic initial energy and a small momentum transfer $-t^2 = q_0^2 - \vec{q}^2 \ll m^2$ comes from the one-photon exchange. Because of the diffractive nature of the process, the transition amplitude can be represented in the form

$$f_{if} = \frac{ip}{2\pi} \int d\vec{b} e^{i\vec{q}_\perp \vec{b}} \theta(b - b_{\min}) \Gamma_{if}(\vec{b}, q_\parallel), \tag{1}$$

where p is the initial momentum, $\theta(b - b_{\min})$ represents the strong nuclear absorption in the region $b < b_{\min}$, ($b_{\min} = R_i + R_t$, R_i, R_t are radii of the ion and target nucleus $R = 1.2\, A^{1/3}$ fm, and the profile function Γ_{if} in the one-photon exchange has the form [6]

$$\Gamma_{if} = \frac{1}{2\pi ip} \int d\vec{q}_\perp e^{-i\vec{q}_\perp \vec{b}} \frac{2Z_t m_i e^2 F_e(t) J^\mu_{if}(p+p')_\mu}{t}, \tag{2}$$

where m_i is the ion mass, Z_t and $F_e(t)$ are the charge and electric form factor of the target, respectively, p^μ and p'^μ are the four-momenta of the target before and after collision, $\hat{J}$ is the electromagnetic transition operator; the scalar product being defined as follows: $a_\mu b^\mu = a_0 b_0 - \vec{a}\vec{b}$. Further consideration is carried out in the antilab system.

We transform the product $J_\mu(p+p')^\mu$ in the integrand (2) so that the electromagnetic transition amplitudes of the photonuclear reaction are given in the explicit form. Because of the gauge invariance $J_\mu q^\mu = 0$, the electromagnetic transition vector $\vec{J}$ in the spherical basis $\vec{e}_\lambda = \dfrac{\vec{e}_1 + i\lambda\vec{e}_2}{\sqrt{2}}$, $\vec{e}_3 = \vec{q}/|\vec{q}|$ can be represented as follows

$$\vec{J} = J_1\vec{e}_1 + J_2\vec{e}_2 + \frac{J_0' q_0}{q}\vec{e}_3 \equiv \vec{J}_\perp + \frac{J_0 q_0}{q}\vec{e}_3. \tag{3}$$

Then the scalar product reads

$$\frac{J_\mu(p+p')^\mu}{t} = -\frac{J_0(p+p')_0}{q^2} - \frac{J_\perp(p+p')_\perp}{t} \simeq \frac{2J_0 p_0}{q^2} - \frac{\sqrt{2}q_\perp}{qt}\sum_{\lambda=\pm 1} \lambda J_\lambda p e^{i\lambda\varphi}, \tag{4}$$

where φ is the azimuthal angle of $\vec{q}$.

The matrix elements J_{0if}, $J_{\lambda if}$ in the antilab system are expressed in terms of the electromagnetic transition amplitudes $\hat{T}_J$ [6].

$$\hat{J}_0 = 4\pi \sum_{JM} (-i)^J Y^*_{JM}(\hat{q}) \hat{T}^{\text{coul}}_{JM}(q), \tag{5}$$

$$\hat{J}_\lambda = -\sum_{JM} (-i)^J \sqrt{2\pi(2J+1)}\, \mathscr{D}^J_{M\lambda}(\varphi_q, -\vartheta_q, \varphi_q)\, [T^{\text{el}}_{JM} + \lambda T^{\text{mag}}_{JM}]. \tag{6}$$

Formulae (1), (2), (4)–(6) define completely the amplitude and consequently the cross section of the ion Coulomb excitation in the state $|f\rangle$ with the excitation energy ω

$$\sigma_1(\omega) = \frac{1}{(2pm_i)^2} \int d\vec{q}_\perp |f_{if}/4\pi|^2. \tag{7}$$

If $|f\rangle$ is one of the resonance states, the cross section of the resonance excitation in the interval ω, $\omega+d\omega$ is

$$d\sigma = \sigma_1(\omega)\,\varrho(\omega)\,d\omega, \tag{8}$$

where $\varrho(\omega)$ is the density of states in this interval. The cross section of excitation of the carbon nucleus with a subsequent decay into three α-particles has the form

$$\frac{d\sigma}{dE_\alpha} = \int d\omega\sigma_1(\omega)\varrho(\omega)W(\omega, E_\alpha), \tag{9}$$

where $W(\omega, E_\alpha)$ is the probability of decay of one α-particle with kinetic energy E_α in the interval E_α, $E_\alpha+dE_\alpha$ normalized by the condition

$$\int W(\omega, E_\alpha)dE_\alpha = \frac{\Gamma_f(\omega)}{\Gamma(\omega)}, \tag{10}$$

where $\Gamma_f(\omega)$ and $\Gamma(\omega)$ are the partial and total decay widths, respectively.

3. To calculate the amplitudes of the Coulomb excitation (1), it is necessary to define explicitly the amplitudes $\hat{T}_J$ of photonuclear reactions in (5) and (6). Due to the smallness of the transferred momentum

$$qR_i \ll 1 \tag{11}$$

it is acceptable for this purpose to use the long-wave approximation [6]

$$T_J^{\text{coul}} = q^J C_J, \qquad T_J^{\text{el}} = q^{J-1} E_J, \qquad T_J^{\text{mag}} = q^J M_J, \tag{12}$$

$$T_J^{\text{coul}} = \sqrt{\frac{J}{J+1}}\,\frac{q}{\omega}\,T_J^{\text{el}}. \tag{12'}$$

In our case this representation can only be applied to calculate the amplitudes of dipole and quadrupole electric T_2^{el} resonances. The analysis of data on $\gamma^{12}\text{C} \to 3\alpha$ reactions shows that the main contribution to the cross section comes from E1, M1, E2, transitions. The contribution to the amplitude from the term T_2^{coul} calculated by formula (12) results in a divergent integral. Therefore, further calculations require another, more realistic at large q, representation for T_2^{coul}. For our purpose it is sufficient to use the simplest liquid--drop formula

$$T_2^{\text{coul}} = \frac{3Z_i}{4\pi}\sqrt{\frac{5}{2\sqrt{B_2C_2}}}\,j_2(qR_i), \tag{13}$$

where B_2 and C_2 are, respectively, the stiffness and mass parameters of the model.

The multipole expansion of $\hat{J}$ provides the following form for the profile function

$$F_{if} = 4Z_t e^2 m_t (-1)^{J_f-M_f} \sum_J (\Gamma_J^{\text{el}}+\Gamma_J^{\text{coul}}+\Gamma_J^{\text{mag}}), \tag{14}$$

where the amplitudes Γ_J are completely determined by the above formulae. The excitation cross section then has the form

$$\sigma_1(\omega) = (4\pi\alpha Z_t)^2 \sum \int bdb\{|\Gamma_J^{\mathrm{el}}|^2+|\Gamma_J^{\mathrm{coul}}|^2+|\Gamma_J^{\mathrm{mag}}|^2\}. \tag{15}$$

Condition (11) allows one more approximation, i. e. the following representation of the target form factor [7]:

$$F_e(t) = \frac{1}{1-a^2 t}, \tag{16}$$

where the parameter a is close in magnitude to the parameter of diffuseness of the nuclear density, δ, which characterizes the decrease of the form factor with increasing $F(t) \sim \exp(t\delta^2)$, [8] and equals $a \simeq 1$ fm.

Then, by using (12)–(16) we arrive at the following form of the resonance excitation cross section

$$\sigma_1(\omega) = (4\pi\alpha Z_t)^2 \left[\frac{|E1|^2}{\omega^2} I_{\mathrm{E1}}(\omega)+|M1|^2 I_{\mathrm{M1}}(\omega)+|E2|^2 I_{\mathrm{E2}}(\omega)\right], \tag{17}$$

where the relativistic radial integrals $I(\omega)$ depend upon the minimal impact parameter b_{min}, initial energy E and level excitation energy and decrease with increasing ω. The factorized form of (17)

$$\sigma_1 = \sigma_{\gamma\mathrm{E1}} \frac{dn_1}{d\omega} + \sigma_{\gamma\mathrm{E2}} \frac{dn_2}{d\omega} + \sigma_{\gamma\mathrm{M1}} \frac{dn_3}{d\omega} \tag{18}$$

is more convenient for us. Here σ_γ is the cross section of the nuclear excitation by γ-quanta, $dn/d\omega$ is the number of photons, for instance:

$$\frac{dn_1}{d\omega} = \frac{2\alpha Z^2}{\pi\omega} I_{\mathrm{E1}}(\omega), \qquad \sigma_{\gamma\mathrm{E1}} = \frac{4\pi^2\alpha}{\omega} \frac{\Gamma|E1|^2}{(\omega-\omega_r)^2+\Gamma^2/4}. \tag{19}$$

To calculate the α-spectrum, it is necessary to define, in (9), the probability W of the level decay into a registered channel. This probability can be found within concrete nuclear models. However, to establish the main qualitative regularities, one can use directly experimental data on the reaction $\gamma^{12}\mathrm{C} \to 3\alpha$. Thus, the product $\sigma_\gamma W$ is the differential cross section of the photonuclear reaction $d\sigma_\gamma/dE_\alpha$. Finally, for the total and differential cross section of the relativistic Coulomb excitation we obtain

$$\sigma = \int d\omega\sigma_\gamma \frac{\Gamma_f(\omega)}{\Gamma(\omega)} \left\{\frac{dn_1}{d\omega}\alpha_1 + \frac{dn_2}{d\omega}\alpha_2 + \frac{dn_3}{d\omega}\alpha_3\right\}, \tag{20}$$

$$\frac{d\sigma}{dE_\alpha} = \int d\omega \frac{d\sigma_\gamma}{dE_\alpha} \left\{\frac{dn_1}{d\omega}\alpha_1 + \frac{dn_2}{d\omega}\alpha_2 + \frac{dn_3}{d\omega}\alpha_3\right\}, \tag{21}$$

where $\sigma_\gamma \Gamma_f/\Gamma$ and $d\sigma_\gamma/dE_\alpha$ are the total and differential cross sections, respectively, for photodisintegration of the carbon into three α-particles, α_i is the weight of each multipolarity defined experimentally.

4. Figure 1 shows the cross section for the Coulomb disintegration of carbon relativistic ions into three α-particles calculated as a function of the target charge and initial energy E_0. σ_γ and α_i are taken from Ref. [9]. As is seen from the calculation, the cross section increases rapidly with growing Z_t (approximately as Z_t^2) and E_0. In Fig. 2 the differential

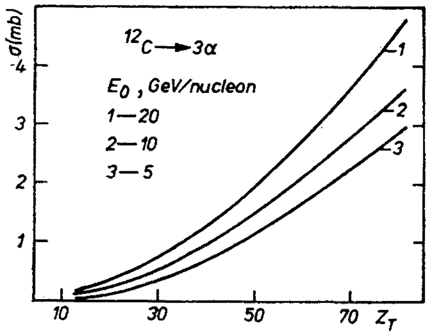

Fig. 1. The Coulomb disintegration cross section of the $^{12}C + Z_t \to 3\alpha + Z_t$ reaction

cross section of α-particles is plotted: a) for the reaction $\gamma + {}^{12}C \to 3\alpha$, and b) for $^{208}Pb + {}^{12}C \to {}^{208}Pb + 3\alpha$.

It is evident that the cross section of the relativistic Coulomb disintegration is somewhat deformed as compared with the cross section in Fig. 2a. The reason is that the number of effective photons in (21) decreases rapidly with growing ω, approximately as $1/\omega \ln$

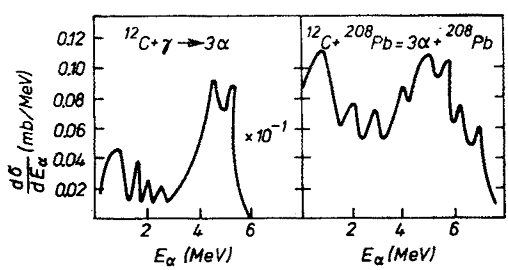

Fig. 2. The differential cross section of a) $\gamma + {}^{12}C \to 3\alpha$ and b) $^{12}C + {}^{208}Pb \to 3\alpha + {}^{208}Pb$ reaction

$\times (1/\omega b_{min})$, for $E_0 \gg m_i$. However, in magnitude, the cross section in Fig. 2b exceeds considerably the photodisintegration cross section because of the great number of effective photons in a single nuclear collision.

In conclusion we note that the present analysis is very much like a prediction and indicates the possibility of experimentally studying the relativistic Coulomb of nuclei.

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APPENDIX

We write the relativistic radial integrals in the explicit form

$$I_{\mathrm{E1}}(\omega) = \frac{b_{\min}^2}{2\omega^2}\left\{q_l^2 q_L^2\left[\mathscr{L}_l(0,2,1) - 2\mathscr{K}_{L0}\,\frac{\mathscr{K}_{l2}-\mathscr{K}_{l0}}{1-a^2\omega^2}\right] + q_l^4\mathscr{M}_l(1,0)\right.$$

$$+[q_L^4 + a^4\omega^4 c^4]\,\frac{\mathscr{M}_c(1,0)}{(1-a^2\omega^2)^2} + q_L^2 c^2[1+a^4\omega^4]\,\frac{\mathscr{L}_c(0,2,1)}{(1-a^2\omega^2)^2} + \frac{4q_L^3\mathscr{K}_{L1}}{b_{\min}(1-a^2\omega^2)^2}\,(\mathscr{K}_{L0}-\mathscr{K}_{c0})$$

$$\left. + \frac{4a^2c\omega^2}{b_{\min}}\left[c q_l\omega^2 a^2\,\frac{\mathscr{K}_{c0}\mathscr{K}_{l1}}{1-a^2\omega^2} - q_L^2\,\frac{\mathscr{K}_{L0}\mathscr{K}_{c1}}{(1-a^2\omega^2)^2} + q_l^2\mathscr{K}_{l0}\mathscr{K}_{c1}\right]\right\}, \tag{A.1}$$

$$I_{\mathrm{M1}}(\omega) = \frac{b_{\min}^2}{2}\left[q_l^2\mathscr{L}_l(0,2,1) + c^2\mathscr{L}_c(0,2,1) - \frac{4cq_l a^2}{b_{\min}}\,\mathscr{N}_{cl}(0)\right],$$

$$I_{\mathrm{E2}}(\omega) = I_{\mathrm{E2}}^{\mathrm{el}}(\omega) + I_{\mathrm{E2}}^{\mathrm{coul}}(\omega), \tag{A.2}$$

$$I_{\mathrm{E2}}^{\mathrm{el}}(\omega) = \frac{b_{\min}^2}{\omega^4}\left\{\tfrac{3}{2}\,q_L^2 q_l^4\mathscr{M}_l(1,0) + \tfrac{3}{2}\,q_L^6\mathscr{M}_L(1,0) - \frac{6q_L^4 q_l^2}{\omega^2 b_{\min}}\,\mathscr{N}_{Ll}(-1)\right.$$

$$+\frac{q_L^6}{2}\mathscr{L}_L(1,3,2) + \frac{q_L^2 q_l^4}{2}\mathscr{L}_l(1,3,2) - \frac{2q_L^4 q_l^2}{\omega^2 b_{\min}}\,\mathscr{N}_{Ll}(1) + 2\left(q_L^2 q_l - \frac{\omega^2}{2}\,q_l\right)\mathscr{L}_l(0,2,1)$$

$$\left.+2q_L^6\mathscr{L}_L(0,2,1) - \frac{4q_L^3 q_l}{\omega^2 b_{\min}}\,(2q_L^2-\omega^2)\mathscr{N}_{Ll}(0)\right\}, \tag{A.3}$$

$$I_{\mathrm{E2}}^{\mathrm{coul}}(\omega) = \frac{b_{\min}^2 q_L^2}{2\omega^4(1-a^2\omega^2)^2}\left[\frac{2q_L^4}{75}\,\mathscr{L}_L(0,2,1) + 6q_L^2c^2G^2\mathscr{L}_c(0,2,1)\right.$$

$$+\frac{8q_L^3ca^2G}{5b_{\min}(1-a^2\omega^2)}\,\mathscr{N}_{cL}(0) + \tfrac{3}{2}\,c^4G^2\mathscr{L}_c(1,3,2) + \frac{q_L^4}{450}\,\mathscr{M}_L(1,0) + \tfrac{1}{2}\,(c^2-2q_L^2)^2G^2\mathscr{M}_c(1,0)$$

$$\left.-\frac{2a^2q_L^2(c^2-2q_L^2)G}{75b_{\min}(1-a^2\omega^2)}\,\mathscr{N}_{cL}(-1) + \frac{q_L^4}{150}\,\mathscr{L}_L(1,3,2) + \frac{2q_L^2c^2a^2G}{5b_{\min}(1-a^2\omega^2)}\,\mathscr{N}_{cL}(1)\right]. \tag{A.4}$$

Here we use the notations:

$$q_l = \frac{\omega}{\sqrt{\gamma^2-1}}, \quad q_L = \frac{\gamma\omega}{\sqrt{\gamma^2-1}}, \quad \gamma = \frac{E_0}{m_{\mathrm{i}}}, \quad c^2 = a^{-2}+q_l^2,$$

$$G = -\frac{j_2(y)}{y^2}, \quad y = \frac{R_{\mathrm{i}}}{a}\sqrt{a^2\omega^2+1},$$

$$\mathscr{L}_s(m,n,k) \equiv \mathscr{K}_{sm}\mathscr{K}_{sn} - \mathscr{K}_{sk}^2, \tag{A.5}$$

$$\mathscr{M}_s(m,n) \equiv \mathscr{K}_{sm}^2 - \mathscr{K}_{sn}^2, \tag{A.6}$$

$$\mathscr{N}_{st}(m) \equiv q_s\mathscr{K}_{sm}\mathscr{K}_{tm+1} - q_t\mathscr{K}_{sm+1}\mathscr{K}_{tm}, \tag{A.7}$$

$\mathscr{K}_{sm} \equiv \mathscr{K}_m(x_s)$ are the Mac Donald functions with the arguments

$$x_L = q_L b_{\min}, \quad x_l = q_l b_{\min}, \quad x_c = cb_{\min}. \tag{A.8}$$

Considering the fact, that for $x \to 0$ the functions $\mathscr{K}_n(x)$ will be

$$\mathscr{K}_n^{(x)} \simeq \frac{(n-1)!2^{n-1}}{x^n}, \quad u \neq 0, \quad \mathscr{K}_0(x) \simeq \ln\left(\frac{2}{x}\right) - 0.5772, \tag{A.9}$$

it is possible to show that the radial integrals go over into the well known Weizsacker--Williams formulae in the limit of large initial energies $E_0 \to \infty$ and small excitation energies $\omega \to 0$:

$$I(\omega) \approx \ln\left(\frac{2\gamma}{\omega b_{\min}}\right) - 1.077.$$

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