LETTERS TO THE EDITOR

THE OZI RULE, PSEUDO-DIMENSION RULE AND $\psi''(3770)$-DECAY

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It is well known that the OZI rule is not an exact rule and, in fact, it is subjected to violations (sometimes very serious) in varying degrees in all cases without a single exception. This fact does not enable one to explain the complete absence of the OZI rule forbidden decay modes $\psi(3100)\pi\pi$, $\psi(3100)\eta$, $\psi(3100)\gamma\gamma$, $\psi(3100)\pi$, 5π, $\omega\pi\pi$ in $\psi''(3770)$-decay. In this note it has been pointed out that the complete absence of the modes mentioned above in ψ''-decay is due to the fact that they are forbidden by the pseudo-dimensionality based selection rule (abbreviated as pseudo-dimension rule).

It may be recalled that the quark duality diagram constraint i.e. the OZI rule [1] derives its importance from the fact that it is considered as a valuable aid for understanding some phenomena related to ψ-particles (which are treated as $c\bar{c}$-bound states with hidden charm). From the point of view of OZI rule $\psi''(3770)$-decay is specially interesting as for it the OZI rule favoured decay channel $D\bar{D}$ is also energetically allowed (as ψ'' is produced above charm threshold) and, as expected, the DD mode has been observed [2] to be dominant. However, the most striking feature in $\psi''(3770)$-decay is the complete absence [3, 4] of the hadronic modes which are forbidden according to the OZI rule. This fact, needless to mention, does not imply by any means that $\psi''(37770)$-decay is an ideal example of the OZI rule. This is so because the OZI rule is *not* an exact rule [1] as it is subjected to violations in varying degrees in *all* cases without a single exception. The validity of this statement becomes immediately transparent by considering the decays [2] of the $\phi(1020)$, $\psi(3100)$, $\psi'(3685)$, $\chi(3415)$, $\chi(3510)$ and $\chi(3555)$ for all of which the OZI rule forbidden modes are observed. There is hardly any need of emphasizing that the OZI rule, not being an exact rule [1], can only demand that the OZI rule favoured decay channel(s) will be *relatively* dominant over the decay channels which are forbidden by the rule concerned. It is well known that an unstable particle, if it is a resonance, can have (in principle at

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least) electromagnetic and weak decay channels apart from the usual strong ones. Clearly, then, in $\psi''(3770)$-decay theoretically allowed decays include isospin-invariant decays $\psi''(3770) \to \psi(3100)\pi\pi, \psi(3100)\eta, 5\pi, \omega\pi\pi$ as well as isospin non-conserving decays $\psi''(3770) \to \psi(3100)\pi$, $\psi(3100)\gamma\gamma$ if we assume $I^G = 0^-$ for the $\psi''(3770)$. We repeat to emphasize that the OZI rule, if applied to the decays of the $\psi''(3770)$ which is considered to be $c\bar{c}$-bound state (with hidden charm), only demands that the OZI rule favoured decay $\psi''(3770) \to D\bar{D}$ will *relatively* dominate over the decays $\psi''(3770) \to \psi(3100))\,\pi\pi$, $\psi(3100)\gamma\gamma$, $\psi(3100)\eta$, $\psi(3100)\pi$, 5π, $\omega\pi\pi$ which are forbidden according to the OZI rule.

It may be recalled that the phase space available for each one of the modes $\psi\pi\pi$, $\psi\eta$, $\psi\gamma\gamma$, 5π, $\psi\pi$, $\omega\pi\pi$ is larger in ψ''-decay compared to the same in ψ-decay. Inspite of this the modes concerned have not been seen [2–4] in ψ''-decay although, interestingly enough, they have been observed [2] in ψ'-decay. It may not be out of place if we mention that the existence of an OZI rule allowed decay channel for an unstable meson does not imply that the decay of the particle concerned will proceed via that channel only. It may be recalled that apart from the OZI rule allowed mode $K\bar{K}$ the OZI rule forbidden mode 3π is also observed [2] in $\phi(1020)$-decay. Clearly, then, one cannot merely throw away by hands the theoretically expected modes $\psi\eta$, $\psi\pi\pi$, $\psi\gamma\gamma$, 5π, $\psi\pi$, $\omega\pi\pi$ in ψ''-decay simply because they are forbidden by the OZI rule which is, by no means, an exact rule [1]. Our motivation in this note is to point out that the complete absence of the modes mentioned above in ψ''-decay is due to the fact that the modes under considerations are forbidden by the pseudo-dimensionality based selection rule [5, 6] (to be abbreviated hereafter as pseudo-dimension rule).

The pseudo-dimension, denoted by d, of a free field carrying the actual spin S is defined by the following relation [5, 6]:

$$d = -KS,$$

where K is a positive odd integer. The odd integral value of K implies that the pseudo-dimension will be integral for a boson field and odd-half integral for a fermion field (like their respective canonical dimensions). Using the properties of negative integers for assigning a non-zero value of pseudo-dimension to a spin-zero field (since a field cannot be a dimensionless quantity) and utilizing the fact that the photon, unlike the massive gauge fields, has only two states of polarization the following relations are obtained [5, 6]:

$$d(\text{magnitude}) = 3S, \; S \neq 0, \tag{1a}$$

$$d(\text{magnitude}) = 1, \quad S = 0, \tag{1b}$$

$$d(\text{magnitude}) = 2, \text{ for a photon.} \tag{1c}$$

The pseudo-dimension rule [5, 6] is stated as follows: All the allowed decays (*not* occurring through subreactions) of an unstable particle must be governed by *one and only one* of the two constraints

$$d_u \geqslant D, \tag{2a}$$

$$d_u \leqslant D, \tag{2b}$$

where d_u is the magnitude of the pseudo-dimension of the unstable particle and D is the

sum of the magnitudes of the pseudo-dimensions of the fields associated with the particles constituting a decay mode (*not* occurring through a subreaction). For a given unstable particle d_μ is fixed whereas D can take in general a finite spectrum of discrete values by virtue of Eqs. (1a)—(1c) corresponding to a finite number of allowed decay modes. As Eqs. (1a)—(1c) refer to free fields, therefore, the quantity D appearing in relations (2a) and (2b) refers to the decay mode(s) not occurring through subreactions which involve final state interactions which in turn necessitate interacting fields.

To demonstrate how the pseudo-dimension rule for particle decays, dicussed above, reduces the number of theoretically expected decay modes (which are not necessarily strong decay modes for the reasons already discussed) but unobserved modes of unstable particle, we consider as an example the decays of the $\omega(784)$. Many such examples have been discussed in earlier papers [5, 6]. From the observed [2] decays $\omega(784) \to 2\pi$, 3π, $\pi^0\gamma$, e^+e^- we have for the pseudo-dimension d_u of the field associated with the $\omega(784)$, $d_u = 3$ which follows from Eq. (1a) as the particle concerned is a spin-one boson and for the 2π mode $D = d_\pi + d_\pi = 1+1 = 2$ which is evident from Eq. (1b). By taking advantage of Eqs. (1a)—(1c) one can easily find out the D values corresponding to the observed modes 3π $(D = d_\pi + d_\pi + d_\pi = 3)$, $\pi^0\gamma$ $(D_\pi = d_\pi + d_\gamma = 1+2 = 3)$, e^+e^- $(D = d_{e^+} + d_{e^-} = 3/2+3/2 = 3)$. Clearly, then, the observed decays $\omega(784,\ d_u = 3) \to 2\pi$ $(D = 2)$, 3π $(D = 3)$, $\pi^0\gamma$ $(D = 3)$, e^+e^- $(D = 3)$ indicate that the appropriate constraint in $\omega(784)$-decay is $d_u \geqslant D$ which must have to be satisfied by all other theoretically expected modes since according to the pseudo-dimension selection rule the decays of a given unstable particle is governed by one and the same constraint. It is interesting to note that the constraint $d_u \geqslant D$, valid in $\omega(784)$-decay, is *not* satisfied by the theoretically expected modes $\pi^+\pi^-\gamma$ $(D = 4)$, $\pi^0\pi^0\gamma$ $(D = 4)$, $\pi^0\mu^+\mu^-$ $(D = 4)$, 3γ $(D = 6)$ since $d_u = 3$ for the $\omega(784)$ and, as expected, have not been seen [2] so far. From the observed [2] decays $\psi''(3770,\ d_u = 3) \to e^+e^-$ $(D = 3)$, $D(1865)$ $\bar{D}(1865)$ $(D = 2)$ it is evident that $\psi''(3770)$-decay is governed by the constraint $d_u \geqslant D$ which, however, is not satisfied by the otherwise expected modes like $\psi(3100)$ $\pi\pi$ $(D = 5)$, $\psi(3100)\eta$ $(D = 4)$, $\psi(3100)\gamma\gamma$ $(D = 7)$, 5π $(D = 5)$, $\omega\pi\pi$ $(D = 5)$. $\psi(3100)\pi$ $(D = 4)$, which are, therefore, forbidden according to the pseudo-dimension rule. It is highly interesting to note that the modes $\psi\eta$, $\omega\pi\pi$, 5π, $\psi\pi$, $\psi\pi\pi$ for ψ''-decay are forbidden both by the OZI rule and the pseudo-dimension rule. In this note we have repeatedly emphasized the well known fact that the OZI rule is not an exact rule [1] and as such the complete absence of the modes mentioned above in ψ''-decay cannot be accounted for by the OZI rule.

From what has been discussed above, we can conclude that the complete absence of the modes concerned in ψ''-decay is due to the fact that they are forbidden by the pseudo-dimension rule. This is so because in the decays of the $\chi(3415)$, $\chi(3510)$, $\chi(3555)$, $\phi(1020)$, $\psi(3100)$ and $\psi'(3685)$ the OZI rule forbidden modes are invariably present. Obviously, then, ψ''-decay phenomena cannot be satisfactorily explained in terms of the OZI rule alone. For this purpose the pseudo-dimension rule is also necessary.

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