NEW COSMOLOGY

By A. H. Klotz

Department of Applied Mathematics, University of Sydney*

(Received May 22, 1978)

The metric of a cosmology implied by the non-symmetric unified field theory is studied in various coordinate systems. It is shown that a form of it exists leading to a Hubble law of expansion in a first approximation.

1. Introduction

It has been shown previously (Ref. [1]) that the nonsymmetric unified field theory (UFT, Ref. [2]) suggests, if indeed it does not unambiguously prove, that the cosmological world metric is

$$ds^2 = \left(1-\frac{2m}{r}\right)dt^2 - \frac{r_0^4 dr^2}{(r_0^2+r^2)^4\left(1-\frac{2m}{r}\right)} - \frac{r_0^2 r^2}{r_0^2+r^2}(d\theta^2+\sin^2\theta d\phi^2), \tag{1}$$

where m and r_0 are constants. Metric (1) represents a possible cosmology. "Possible" means that it does not lead to immediate conclusions in conflict with observed facts. In particular, a Hubble law of expansion is obtained (approximately) in the range

$$2m \ll r \ll r_0. \tag{2}$$

This, as well as electromagnetic considerations (Ref. [2]) and study of geometry as r tends to infinity published separately (Ref. [3]), imply that r_0 must be regarded as the distance of the Hubble horizon from any observer. The isotropy of the universe described by our metric will be demonstrated below. Although the UFT-universe appears to be radically different from the cosmologies hitherto considered, its geometry bears a considerable resemblance to a de Sitter space-time. Chief differences occur locally where the geometry is Schwarzschild and at infinity. The logic of UFT cosmology is entirely new. The model is independent of any cosmological principle in the recognised sense of the word. It is also independent of the general relativistic field equations. In fact, because of its origin as a consequence of the unified field theory, it is quite incorrect to attempt to describe matter

* Address: Department of Applied Mathematics, University of Sydney, Sydney N. S. W. 2006, Australia.

in terms of the geometry contained in (1). This does not mean that geometrisation of physics is abandoned. Laws of measurement (for example, metric (1) itself) are affected, and indeed, determined by the physical fields present in the world but not through the equations of General Relativity (GR). This leads to an important conclusion.

Unified field theory collapses into GR when skew-symmetry (the electromagnetic field) is removed but its field equations then become the empty space equations of Einstein. Adoption of the UFT cosmology therefore implies rejection of such things as the interior Schwarzschild solution, the Einstein–Maxwell theory and the like. It implies that a universal existence and subsistence of an electromagnetic field is explicitly predicted. A pure gravitational field can only exist locally. The fact that metric (1) gives a local Schwarzschild field can be regarded therefore as demonstrating the logical consistency of the theory although this may throw doubt on the previous, tentative interpretation of $2m$ as the Schwarzschild radius of primeval fire-ball. Actually this is not so and the interpretation may be retained as far as the universe is concerned if it is stressed that Einstein's GR equations ($R_{\mu\nu} = 0 = R_{\nu\mu}$) are strictly locally valid only. And a general relativistic cosmology is, on the basis of the present theory, meaningless. Equations (19) of Ref. [1] must therefore be seen as a purely heuristic, and logically wrong, attempt to see what matter would be like were GR equations with matter ($G_{\mu\nu} = -\kappa T_{\mu\nu}$) correct, and nothing more.

An objection to the cosmological interpretation of (1) was raised by Professor Szekeres (Ref. [4]) that rather than the present speculation, it should be regarded as a prediction of a peculiar structure of the global electromagnetic field. The objection appears to be a matter of taste rather than of science. I prefer to speculate about structure of the universe than about that of electromagnetism. More to the point however, it must be pointed out that the alternative interpretation (a cut-off of the electromagnetic field at some, difficult to specify theoretically, point) is virtually untestable. On the other hand, a cosmological interpretation offers a chance of testing not only particular cosmological predictions but also the unified field theory from which it arises.

The main problem investigated below depends on the interpretation of the coordinate r as the "radial distance" from a local observer. The problem is best discussed by expressing (1) in different coordinate systems and drawing appropriate conclusions. We shall also consider the problem of the red-shift from extragalactic sources.

2. *The light tracks (null geodesics)*

We can easily write down the equations of geodesics of the space-time whose metric is given by (1). If dots denote derivatives with respect to the parameter s, we have

$$\ddot{t} + \frac{2m}{r(r-2m)}\dot{t}\dot{r} = 0,$$

$$\ddot{r} - \left(\frac{2r}{r_0^2+r^2} + \frac{m}{r(r-2m)}\right)\dot{r}^2 - (r-2m)(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2) + \frac{m}{r_0^4 r^3}(r-2m)(r_0^2+r^2)\dot{t}^2 = 0,$$

$$\ddot{\theta} + \left(\frac{2r_0^2}{r(r_0^2+r^2)}\right)\dot{r}\dot{\theta} - \sin\theta\cos\theta\dot{\phi}^2 = 0, \quad \ddot{\phi} + \left(\frac{2r_0^2}{r(r_0^2+r^2)}\right)\dot{r}\dot{\phi} + 2\cot\theta\dot{\theta}\dot{\phi} = 0. \quad (3)$$

As in Schwarzschild case, if initially $\dot{\theta} = 0$, $\theta = \frac{\pi}{2}$, coplanar orbits result and

$$\frac{\dot{\phi} r_0^2 r^2}{r_0^2 + r^2} = h, \quad \text{a constant.} \tag{4}$$

Also

$$\frac{dt}{ds} = \frac{kr}{r-2m}, \tag{5}$$

where k is another constant. Hence, from the identity $1 = g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$, using (4), (5),

$$1 = \frac{k^2 r}{r-2m} - \frac{r_0^4 r \dot{r}^2}{(r_0^2+r^2)^2(r-2m)} - \frac{r_0^2+r^2}{r_0^2 r^2} h^2. \tag{6}$$

Using (6), writing as usual $u = 1/r$, and denoting by dashes derivatives of u with respect to ϕ, we get

$$u'^2 + u^2 = \frac{k^2-1}{h^2} + \frac{2mu}{h^2} + \frac{2mu}{r_0^2} + 2mu^2 - \frac{1}{r_0^2}, \tag{7}$$

whence

$$u'' + u = \frac{m}{h^2} + \frac{m}{r_0^2} + 3mu^2. \tag{8}$$

The term m/r_0^2 is the only difference between (8) and the usual orbit (from which the motion of the perihelion of Mercury, for example, is calculated) of a Schwarzschild space. We get Schwarzschild orbit exactly if $r_0 \to \infty$. However, with the cosmological interpretation of the metric this is of less interest than the corresponding null geodesic obtained by letting

$$ds \to 0 \quad \text{so that} \quad h \to \infty. \tag{9}$$

Then

$$u'' + u = \frac{m}{r_0^2} + 3mu^2. \tag{10}$$

Hence the light tracks are no longer straight lines as in a de Sitter world.

If we suppress the angular coordinates entirely (putting $\dot{\phi} = 0$ as well as $\theta = \frac{\pi}{2}$, $\dot{\theta} = 0$) then for a "radial" null geodesic

$$0 = dt^2 - \frac{r_0^4 dr^2}{(r_0^2+r^2)^2 \left(1 - \frac{2m}{r}\right)^2}, \tag{11}$$

from which, letting $\varrho = r-2m$,

$$t = \pm \frac{mr_0^2}{r_0^2+4m^2}\left[\ln \frac{(r-2m)^2}{(r)^2+r_0^2} + \frac{0}{m}\tan^{-1}\frac{r}{r_0}\right] + \text{constant}. \tag{12}$$

From (12) we can use Eddington's method (Ref. [5]) to get a first estimate for the extragalactic Doppler red shift of a pulse of light emitted from a position ϱ at coordinate time t which reaches an observer at time t'. Thus, if a similar source at the origin where the observer is situated, emits light at the rate $\Delta t_0'$, then

$$\frac{\Delta t'}{\Delta t_0'} = \sqrt{\frac{r}{r-2m}} + \frac{mr_0^2}{r_0^2+4m^2}\left[\frac{2}{r-2m} - \frac{2r-r_0^2-4m+4m^2}{r^2+r_0^2}\right]\sqrt{\frac{r}{r-2m}}\,\frac{dr}{dt}, \tag{13}$$

with the second term corresponding to the required red shift. A more accurate calculation in terms of a different coordinate system will be given in the last section of the work.

3. *Isotropy*

Let us now show that for $r > 2m$, the UFT metric (1) can be put in the isotropic form

$$ds^2 = f^2(\varrho)dt^2 - g^2(\varrho)\,(d\varrho^2 + \varrho^2 d\Omega^2), \tag{14}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. Thus we require

$$g d\varrho = \frac{r_0^2 dr}{(r_0^2+r^2)\sqrt{1-\dfrac{2m}{r}}} \quad \text{and} \quad g\varrho = \frac{rr_0}{\sqrt{r_0^2+\ \ ^2}},$$

or

$$\frac{d\varrho}{\varrho} = \frac{r_0 dr}{\sqrt{(r_0^2+r^2)(r^2-2mr)}}. \tag{15}$$

Let

$$r = \frac{\alpha z+\beta}{\gamma z+\delta}, \quad (\alpha, \beta, \gamma, \delta \text{ constant}).$$

Then

$$dr = \frac{(\alpha\delta-\gamma\beta)dz}{(\gamma z+\delta)^2},$$

and

$$\frac{d\varrho}{\varrho} = \frac{r_0(\alpha\delta-\gamma\beta)dz}{\sqrt{[r_0^2(\gamma z+\delta)^2+(\alpha z+\beta)^2]\,[\alpha z+\beta]\,[(\alpha-2m\gamma)z+\beta-2m\delta]}}$$

$$= \frac{-\sqrt{r_0}\,dz}{\sqrt{(z^2+4m^2)(z+r_0)}},$$

if $\alpha = 2m\gamma$, $r_0^2\delta + 2m\beta = 0$, $\beta = 2mr_0\gamma$. A further substitution $z = \zeta - r_0/3$ reduces this to

$$\frac{d\varrho}{\varrho} = \frac{-\sqrt{r_0}\, d\zeta}{\sqrt{\zeta^3 - (\frac{1}{3} r_0^2 - 4m^2)\zeta + \frac{2}{27} r_0(r_0^2 + 36m^2)}}$$

or

$$\varrho = \exp -2^{2/3} \sqrt{r_0}\, \wp^{-1}(\zeta, g_2, g_3), \tag{16}$$

where $g_2 = 2^{2/3}(\frac{1}{3} r_0^2 - 4m^2)$, $g_3 = -\frac{2}{27} r_0(r_0^2 + 36m^2)$, and $\wp$ is the Weierstrass' elliptic function (Ref. [7]). Hence g and f can be determined easily as functions of ϱ. It follows that the UFT universe is isotropic in the usual coordinate sense.

4. *Kruskal-Szekeres form*

We shall now determine the analogue of the Kruskal–Szekeres coordinates for the UFT metric. As is well known (for example, Ref. [6]) if

$$u = \tfrac{1}{2}\sqrt{r-2m}\left(\exp\frac{t+r}{4m} - \exp\frac{r-t}{4m}\right), \quad v = \tfrac{1}{2}\sqrt{r-2m}\left(\exp\frac{t+r}{4m} + \exp\frac{r-t}{4m}\right),$$

then

$$du^2 - dv^2 = \frac{r\exp\dfrac{r}{2m}}{16m^2}\left[\left(1 - \frac{2m}{r}\right)dt^2 - \frac{dr^2}{1 - \dfrac{2m}{r}}\right],$$

so that the Schwarzschild metric transforms into

$$16m^2 \frac{du^2 - dv^2}{r\exp\dfrac{r}{2m}} - r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

with r defined in terms of u and v by the implicit relation

$$(r - 2m)\exp\frac{r}{2m} = v^2 - u^2.$$

The singularity of the Kruskal-Szekeres space it at the "origin" $r = 0$, but, of course r is not the radial coordinate. We have seen (Ref. [1]) that putting $r = r_0 \tan z/r_0$, $\lambda = 2m/r_0$, transforms (1) into

$$ds^2 = \left(1 - \lambda\cot\frac{z}{r_0}\right)dt^2 - \frac{dz^2}{1 - \lambda\cot\dfrac{z}{r_0}} - r_0^2\sin^2\frac{z}{r_0}(d\theta^2 + \sin^2\theta d\phi^2). \tag{17}$$

Let us now suppose that X and g are functions of z such that

$$1-\lambda\cot\frac{z}{r_0} = \frac{X^2}{g}, \qquad \frac{dz^2}{1-\lambda\cot\frac{z}{r_0}} = \frac{a^2 dX^2}{g}, \tag{18}$$

where a is a constant. Then g is given by

$$\frac{1}{g}\frac{dg}{dz} + \frac{\frac{\lambda}{r_0}\operatorname{cosec}^2\frac{z}{r_0}}{1-\lambda\cot\frac{z}{r_0}} = \pm\frac{2}{a}\frac{1}{1-\lambda\cot\frac{z}{r_0}}. \tag{19}$$

Hence

$$\ln g\left(1-\lambda\cot\frac{z}{r_0}\right) = \pm\frac{2}{a}\int\frac{dz}{1-\lambda\cot\frac{z}{r_0}}. \tag{20}$$

If we take the + sign on the right hand side of equation (20) and let

$$a = \frac{\lambda r_0}{1+\lambda^2}, \tag{21}$$

then

$$g = \frac{1-(1+\lambda^2)\cos^2\frac{z}{r_0}}{1+\lambda\cot\frac{z}{r_0}}\exp\frac{2z}{\lambda r_0}, \tag{22}$$

$$X^2 = \left(1-\lambda\cot\frac{z}{r_0}\right)g(z), \tag{23}$$

and

$$ds^2 = \frac{X^2dt^2-a^2dX^2}{g(z)} - r_0^2\sin^2\frac{z}{r_0}(d\theta^2+\sin^2\theta d\phi^2). \tag{24}$$

The u, v coordinates can be introduced by letting

$$v = aX\cosh\frac{t}{a}, \qquad u = aX\sinh\frac{t}{a}, \tag{25}$$

and then

$$ds^2 = \frac{du^2-dv^2}{g(r)} - \frac{r_0^2r^2}{r_0^2+r^2}(d\theta^2+\sin^2\theta d\phi^2), \tag{26}$$

where

$$v^2 - a^2 = a^2 \left(\frac{r-2m}{r+2m}\right)\left(\frac{r^2-\lambda^2 r_0^2}{r^2+r_0^2}\right) \exp\left(\frac{2}{\lambda} \tan^{-1} \frac{r}{r_0}\right), \tag{27}$$

but $g(r)$ vanishes when $r = 2m$ so that the transformation does not shift the singular surface as in the case of Schwarzschild geometry.

A more convenient coordinate is obtained by simply letting

$$\varrho = \sqrt{1 - \frac{2m}{r}}, \tag{28}$$

when

$$ds^2 = \varrho^2 dt^2 - \frac{16m^2 d\varrho^2}{((1-\varrho^2)^2+\lambda^2)^2} - \frac{4m^2}{(1-\varrho^2)^2+\lambda^2}(d\theta^2 + \sin^2\theta d\phi^2), \tag{29}$$

and apparently ϱ can go from 0 to ∞. But if we adhere to regarding r as a distance measure, then for $r < 2m$, ϱ becomes imaginary and t and ϱ reverse roles. If we then put $t = R$ and $\varrho = iT$, the metric becomes

$$ds^2 = \frac{16m^2 dT^2}{((1+T^2)^2+\lambda^2)^2} - T^2 dR^2 - \frac{4m^2}{(1+T^2)^2+\lambda^2}(d\theta^2 + \sin^2\theta d\phi^2). \tag{30}$$

This leads to a geodesic equation $\frac{d}{ds} T^2 \dot{R} = 0$ or $T^2 \dot{R} = k$, a constant, while for a null geodesic, suppressing angular dependence

$$\frac{dR}{dT} = \frac{4m}{T((1+T^2)^2+\lambda^2)} = \frac{\dot{R}}{\dot{T}} = \frac{k}{T_2^2 \dot{T}}.$$

Hence

$$\frac{4mT\dot{T}}{(1+T^2)^2+\lambda^2} = k, \tag{31}$$

so that

$$1 + T^2 = \lambda \tan \frac{k\lambda}{2m}(s - s_0), \tag{32}$$

and equation (31) gives an approximate measure of the red shift

$$\frac{\Delta t - \Delta s}{\Delta s} \sim \frac{k[(1+T^2)^2+\lambda^2]}{T} - 1. \tag{33}$$

5. *Lemaitre coordinates*

Perhaps the most informative form of the metric (1) is one in which the coefficient of dt^2 is reduced to unity. Let us write our metric as

$$ds^2 = \gamma dt^2 - \frac{w^2}{\gamma} dr^2 - w r^2 (d\theta^2 + \sin^2\theta d\phi^2), \tag{34}$$

where $\gamma = 1-2m/r$, $w = r_0^2/(r_0^2+r^2)$. Let us introduce new coordinates $(\bar{t}, \bar{r}, \theta, \phi)$, by

$$\bar{r} = \exp(kt+h(r)), \quad \bar{t} = t+g(r), \tag{35}$$

where, as indicated, g and h are functions of r only and k is a constant. Then, dashes denoting derivatives with respect to r, $d\bar{r}/\bar{r} = d\bar{s}$ (say) $= k\,dt+h'dr$, $d\bar{t} = dt+g'dr$, and

$$dt = \frac{1}{p}(h'd\bar{t}-g'd\bar{s}), \quad dr = \frac{1}{p}(d\bar{s}-kd\bar{t}), \quad p = h'-kg'.$$

We require

$$\gamma^2 h'^2 - w^2k^2 = \gamma p^2, \quad \text{and} \quad \gamma^2 h'g' - kw^2 = 0, \tag{36}$$

assuming, of course, that $\gamma \neq 0 \neq p$. Eliminating g' between the two equations (36) we get

$$(\gamma-\gamma^2)h'^4 - k^2w^2\left(\frac{2}{\gamma}-1\right)h'^2 + \frac{k^4w^4}{\gamma^3} = 0,$$

whence, $\gamma \neq 1 (r \leqslant \infty)$, $h'^2 = k^2w^2/\gamma$ or $k^2w^2/\gamma^2(1-\gamma)$. The first of these implies $p = 0$, so that

$$h' = \pm\frac{kw}{\gamma\sqrt{1-\gamma}}, \quad g' = \pm\frac{w\sqrt{1-\gamma}}{\gamma}, \tag{37}$$

and we can take the $+ve$ sign without loss of generality. From equation (37)

$$h-kg = \frac{mk}{\lambda\sqrt{2\lambda}}\left[\ln\frac{r-\sqrt{2rr_0}+r_0}{r+\sqrt{2rr_0}+r_0} + 2\tan^{-1}\frac{\sqrt{2rr_0}}{r_0-r}\right], \tag{38}$$

and from the second of the equations (38)

$$g = \frac{\sqrt{mr_0}}{2(1+\lambda^2)}\left[(1+\lambda)\ln\frac{r-\sqrt{2rr_0}+r_0}{r+\sqrt{2rr_0}+r_0} + \sqrt{2\lambda}\ln\left(\frac{\sqrt{r}-\sqrt{m}}{\sqrt{r}+\sqrt{m}}\right)^2 + 2(1-\lambda)\tan^{-1}\frac{\sqrt{2rr_0}}{r_0-r}\right], \tag{39}$$

so that h may be written down from equation (38) at once. We must now calculate the coefficient of $d\bar{r}^2$ in the new coordinates. Using equations (35), (36), (37) and (38) we easily find that

$$\frac{r-\sqrt{2rr_0}+r_0}{r+\sqrt{2rr_0}+r_0}\exp 2\tan^{-1}\frac{\sqrt{2rr_0}}{r_0-r} = \bar{r}^{\nu}\exp\left(-\frac{2\sqrt{2\lambda}}{r_0}\bar{t}\right), \tag{40}$$

where $\gamma = \lambda\sqrt{2\lambda}/mk$, and that the required coefficient is

$$-\frac{2m}{k^2 r\bar{r}^2}, \tag{41}$$

so that

$$ds^2 = d\bar{t}^2 - \frac{2m}{k^2 r\bar{r}^2} d\bar{r}^2 - w^2 r(d\theta^2 + \sin^2\theta d\phi^2), \tag{42}$$

(with r given by (40) in terms of $\bar{r}$ and $\bar{t}$). An approximate explicit expression for r may be obtained from equation (40) when $r \ll r_0$. In fact expanding the left hand side of (40) in terms of r/r_0, and retaining the (lowest order) term $(r/r_0)^{3/2}$ only, we find that

$$r = (\tfrac{9}{32})^{1/3} r_0 \left(1 - \bar{r}^{\nu} \exp\left(-\frac{2\sqrt{2\lambda}}{r_0}\bar{t}\right)\right)^{2/3}, \tag{43}$$

whence

$$\bar{r} \sim \exp k\bar{t} \tag{44}$$

for the approximation to be valid. This condition on the (dimensionless) $\bar{r}$ again has the appearance of a Hubble law at least in an approximate form. If we write $\bar{r}/\bar{r}_0$ instead of $\bar{r}$, so that

$$\bar{r} = \bar{r}_0 \exp(kt + h(r)), \tag{45}$$

the "law" becomes

$$\bar{r} = \bar{r}_0 \exp k\bar{t} + \varepsilon(\bar{t}), \tag{46}$$

where $\varepsilon^2 \ll \varepsilon$, and the "Hubble constant" becomes

$$H \sim \bar{r}_0 k. \tag{47}$$

In the de Sitter case we have, as is well known, $w = 1$, $\gamma = 1 - r^2/r_0^2$, and the transformation

$$\bar{r} = \frac{r}{\sqrt{1 - r^2/r_0^2}} \exp kt, \qquad \bar{t} = t - r_0 \ln\sqrt{1 - r^2/r_0^2}, \qquad kr_0 = 1,$$

maps the metric into

$$ds^2 = d\bar{t}^2 - e^{-2k\bar{t}}(d\bar{r}^2 + \bar{r}^2(d\theta^2 + \sin^2\theta d\phi^2)),$$

but in our case we cannot obtain explicitly the inverse expressions for r and t in terms of $\bar{r}$ and $\bar{t}$.

We may note, however, that in terms of "Lemaitre coordinates we obtain the expansion law when r(not $\bar{r}$) $\ll r_0$ and not as before equation (1) when $2m \ll r$ as well. This is rather curious. Absense of a lower limit for the approximation enables us to avoid having to speculate on the meaning of the constant $2m$ ("Schwarzschild radius of the primeval atom" or the like). Of course, from the point of view of completeness, we have to say what $2m$ (or λ) is meant to be but any such speculation appears to be irrelevant as far as the interpretation of the extragalactic red shift is concerned.

We shall conclude the present study with a note on integration of the radial geodesic equation in the original, $r-t$ coordinates.

6. *The* $r-t$ *equation*

If we suppress θ, ϕ dependence in the original form (1) of the metric, we obtain immediately, as in Schwarzchild geometry

$$\dot{t} = \frac{k}{1-\frac{2m}{r}}, \quad 1 = \left(1-\frac{2m}{r} - \frac{r_0^4}{(r_0^2+r^2)^2\left(1-\frac{2m}{r}\right)}\left(\frac{dr}{dt}\right)^2\right)\dot{t}^2,$$

k, a constant, whence, eliminating $\dot{t}$,

$$\frac{dr}{dt} = \pm\frac{\sqrt{k^2-1}}{k}\left(1+\frac{2\mu}{r}\right)^{1/2}\left(1-\frac{2\mu}{r}\right)\left(1+\frac{r^2}{r_0^2}\right), \tag{48}$$

where $\mu = m/(k^2-1)$, (in Ref. [1] we took $k^2 = 4/3$ but we shall only assume here that $k^2 > 1$, if $k^2 < 1$ then r is confined to a meaningless, finite range and $k^2 = 1$ is clearly a special case; although we can clearly put $k^2 = 1$ by choosing units of time suitably, were we then to require also, say, that $dr/dt = 1$ when $r = r_0$, we would get $r_0 = 4m$. This or any such hypothetical relation would prevent us from assuming freely a convenient interpretation of the constants r_0 and $2m$.) Substitution $\zeta = \sqrt{1+2\mu/r}$ now gives

$$\frac{d\zeta}{dt} = \mp A(\lambda^2-\zeta^2)(\zeta^4-2\zeta^2+v^2), \tag{49}$$

where

$$A = \frac{(k^2-1)^{5/2}}{mk}, \quad \lambda^2 = \frac{k^2}{k^2-1}, \quad v^2 = 1+\frac{4\mu^2}{r_0^2}.$$

Hence

$$\mp A(t-t_0) = \frac{v+2-\lambda^2}{4\gamma v}\ln\frac{\zeta^2-\gamma\zeta+v}{\zeta^2+\gamma\zeta+v} + \frac{1}{2\lambda}\ln\frac{\zeta+\lambda}{\zeta-\lambda}$$
$$+ \frac{\gamma(v+\lambda^2-2^2)}{(v-\lambda^2+2)\sqrt{v-\frac{v^2}{4}}}\tan^{-1}\frac{2\zeta\sqrt{v-\gamma^2 u}}{v-\zeta^2}, \tag{50}$$

where t_0 is a constant and $\gamma^2 = 2(1+v)$.

7. *Conclusions*

We have considered in this work various forms of the cosmological metric (1) according to the choice of the coordinate system. As in most cosmological models the main problem, and indeed one which can only be settled by an hypothesis and eventual comparison with observational data, is to decide what is to represent the radial distance from an observer. It is therefore a problem of measurement, which we have not attempted to solve here. Considerations derived from the unified field theory seem to imply that r in (1) is the appropriate radial coordinate but then the metric recorded here acquire a somewhat unfamiliar form. Of course, the coefficient of $d\theta^2+\sin^2\theta\, d\phi^2$ in both the equations (26) and (40) can be made equal to R^2. In the former case

$$g(r) = g(R) = \frac{(1+\lambda^2)\frac{R^2}{r_0^2}-1}{1+\frac{\lambda}{R}\sqrt{r_0^2-R^2}} \exp + \frac{2}{\lambda}\tan^{-1}\frac{R}{\sqrt{r_0^2-R^2}}$$

and

$$g(r_0) = \lambda^2 \exp\frac{\pi}{\lambda}.$$

If R is regarded as the radial distance from an observer then $g(R)$ vanishes (the coordinate system becomes singular) when $R = r_0/\sqrt{1+\lambda^2}$. An outstanding problem also is a definite identification of the constant $2m$. This cannot be resolved until the problem of matter (that is, definition of an energy-momentum tensor) in unified field theory is settled. Since the latter geometrises both the gravitational and electromagnetic fields in the general relativistic sense though not by the methods of GR, the problem is far from being simple.

REFERENCES

[1] A. H. Klotz, *Acta Phys. Pol.* **B9**, 595 (1978).
[2] A. H. Klotz, *Acta Phys. Pol.* **B9**, 573 (1978).
[3] A. H. Klotz, *Acta Phys. Pol.* **B10**, 3υ7 (1979).
[4] G. Szekeres, private communication.
[5] A. S. Eddington, *Mathematical Theory of Relativity*, Cambridge 1924.
[6] C. W. Misner, K. S. Thorne, J. A. Wheeler, *Gravitation*, San Francisco 1970.
[7] E. T. Copson, *Theory of Functions of a Complex Variable*, Oxford 1935, p. 361.