OCTONIONIC INTERPRETATION OF THE MULTIQUARK STATES IN THE DUAL STRING PICTURE

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Assuming the octonionic quark fields the multiquark string-like states — recently advocated in the dual unitarisation programme — are obtained. An explanation of the n-jet structure in hadron production processes follows naturally from the underlying algebra of octonions. The difference between the schemes of Low–Nussinov and Veneziano can be understood as the difference between standard QCD which allows colour multiplets of higher dimensionality and the octonion algebra where only triplet and antitriplet states are possible.

Construction of dual amplitudes with baryons as external particles has been a difficulty since the beginning of dual models. Recently, within the topological expansion of dual models there have been some proposals of how to define dual baryons [1–3]. In particular the baryon resembling the Y-shaped string was proposed [2] and the new baryonium states were predicted [2, 3]. The topological considerations that have led to dual diagrams for baryon-baryon, baryon-antibaryon scattering predict also a specific n-jet structure of final states in hadron production processes with n depending on the type of the initial colliding particles [2].

These models should be contrasted with those based on standard QCD where the production of particles occurs from a single jet (or bag) [4, 5]. There has been a long lasting misunderstanding between dualists who talk about diagrams and people who believe in QCD and to whom the n-jet dual structure is not understandable. This difficulty can be easily seen by considering this part of meson-meson scattering, the shadow of which gives rise to the Pomeron and which leads to the production of multihadron final states. In the dual picture we have two $q\bar{q}$ jets, whereas from the arguments based on QCD it seems more natural to have one expanding bag with two $q\bar{q}$-pairs in octet representation each one at one end of the bag. The dualists do not consider these interference effects when two quarks act as a single entity in their multijet picture. On the physical grounds with

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experience gained from QED it would seem that such effects should be taken into account and nobody really understands why they are neglected in the dual language. The dualists argue that only some of the diagrams of QCD are allowed. The rule which governs this selection is based on an additional assumption concerning the allowed intermediate states. Let us recall that according to the Veneziano proposal the intermediate multiquark states have the global colour structure of the operators:

$$\varepsilon_{ijk}\varepsilon^{mnp}(\bar{q}A \dots A)^i(\bar{q}A \dots A)^j(A \dots A)^k_p(A \dots Aq)_m(A \dots Aq)_n,$$

$$\varepsilon_{ijk}\varepsilon^{mnp}(\bar{q}A \dots A)^i(AA \dots A)^j_n(AA \dots A)^k_p(A \dots Aq)_m, \tag{1}$$

for M_4^J and M_2^J members of the baryonium family and similarly for other states. It is the aim of this note to sketch briefly how it is possible to obtain such states from the octonion algebra.

First, let us briefly review the concept of the octonionic Hilbert space, the octonion algebra and its connection with $SU(3)_c$. Besides the usual "explanation" of quark confinement as an infrared slavery effect there is a different approach in which unobservability of free quarks emerges from the assumption that Hilbert space has octonionic components. In the approach of Gürsey and Günaydin [6, 7] the usual Hilbert space is enlarged to the octonionic Hilbert space H, which can be divided into two spaces H_L, H_T (longitudinal and transverse). Only one, H_L is observable. The quarks belong to the "fictitious Hilbert space" H_T and, consequently, they are not observable. The octonionic Hilbert space can be understood as an enlargement of the usual complex Hilbert space. One introduces seven different imaginary units $e_1, \dots, e_7$: $e_i^2 = -1$ and defines an octonion algebra which in terms of the so-called split octonions

$$u_k = \tfrac{1}{2}(e_k + ie_{k+3}), \quad u_k^* = \tfrac{1}{2}(e_k - ie_{k+3}),$$

$$u_0 = \tfrac{1}{2}(e_0 + ie_7), \quad u_0^* = \tfrac{1}{2}(e_0 - ie_7), \quad (k = 1, 2, 3;\ e_0 = 1) \tag{2}$$

has the following multiplication rules

$$u_iu_j = \varepsilon_{ijk}u_k^*, \quad u_iu_0 = 0, \quad u_0^2 = u_0, \quad u_0u_i = u_i, \tag{2a–d}$$

$$u_iu_j^* = -\delta_{ij}u_0, \quad u_iu_0^* = u_i, \quad u_0^*u_0 = 0, \quad u_0^*u_i = 0, \tag{2e–h}$$

(plus complex conjugate relations).

The Hilbert space is spanned by states vectors $|\alpha\rangle = \sum_{i/0}^{7} \alpha_i e_i$ with α_i real octonion components [7] which written in terms of the split octonion units are

$$|\alpha) \equiv u_0^*|\alpha\rangle = \sum_{\kappa=0}^{3} |\alpha)_\kappa u_\kappa^*, \tag{3}$$

where $|\alpha)_\kappa$ are complex vectors.

The longitudinal Hilbert space is spanned by vectors with nonzero real components in 1 and/or e_7 direction (u_0^*). The remaining states belong to the transverse Hilbert space, and are not observable. This unobservability of states follows from the propositional calculus developed by Birhkoff and von Neumann [8] and is due to the nonassociativity of the algebra of octonions.

From the unobservable quark fields (we suppress flavour indices)

$$\Psi = \sum_{i=1}^{3} \Phi_i = \sum_{i=1}^{3} q_i u_i, \quad \overline{\Psi} = \sum_{i=1}^{3} \overline{\Phi}_i = \sum_{i=1}^{3} \overline{q}_i u_i^*, \tag{4}$$

we can form states which lie in the longitudinal Hilbert space. The u_i can be interpreted as annihilation and u_i^* as creation operators when acting on the vacuum $u_0|0\rangle$.

It is easy to check that the automorphisms of the octonion algebra (2) which leave unit e_7 invariant form SU(3) group which is identified with usual SU(3) colour.

The operators that create mesons and baryons are

$$\hat{M}(x) = \Psi(x)\overline{\Psi}(x) = -q_i(x)\overline{q}_i(x)u_0,$$

$$\hat{B}(x) = \Psi(x)(\Psi(x)\Psi(x)) = -\varepsilon_{ijk}q_i(x)q_j(x)q_k(x). \tag{5}$$

Let us notice that there is no octet component in the composed (point-like) mesonic field as it would be in the usual SU(3):

$$3\times\overline{3} = 1, \ 3\times 3 = \overline{3}, \ 3\times(3\times 3) = 1. \tag{6}$$

In such a way we get an algebraic suppression of colour excitations. This is crucial in understanding the fundamental difference between the models based on standard QCD and those based on the octonion algebra which, in turn, have suggestive similarly to the dual string picture.

Let us now try to construct nonlocal gauge invariant operators following the ideas of Veneziano and Rossi. We need to know the octonionic counterpart of the gluonic field. The standard QCD equations for the quark fields $q_i(x)$ can be written in terms of the octonionic quark fields $\Psi(x) = \sum_i q_i(x)u_i$ as follows [6]

$$(\gamma_\mu\partial_\mu + m)\Psi(x) - ig\hat{B}_\mu(x)\gamma_\mu\Psi(x) = 0, \tag{7}$$

where

$$\hat{B}_\mu(x) = -R_{u_k^*}B_\mu^{mk}(x)L_{u_m}; \quad B_\mu^{mm}(x) = 0.$$

Here L_{u_m}, $(R_{u_k^*})$ are the operators of multiplications by $u_m(u_k^*)$ from the left (right). We therefore expect the gauge phase to become also an operator

$$\hat{\theta}(x) = -R_{u_k^*}\theta^{mk}(x)L_{u_m}. \tag{8}$$

It is straightforward to check that

$$\hat{\theta}u_i = \theta^{ji}u_j, \quad u_i\hat{\theta} = 0, \quad \hat{\theta}u_i^* = 0, \quad u_i^*\hat{\theta} = \theta^{ij}u_j^*. \tag{9}$$

The bilinear form $\overline{\Psi}\Psi$ is invariant under gauge transformations

$$\Psi \to \Psi'(x) = \exp(-i\hat{\theta}(x))\Psi(x) = U(\hat{\theta}(x))\Psi(x) = [\exp(-i\theta(x))]^{mk}q_k(x)u_m. \tag{10}$$

Let us now consider the field $U_B^C(x_1, x_2)\Psi(x_2)$, where U_B^C is some path — C — dependent string operator. We want the product $U_B^C\ \Psi$ to transform like a Ψ field at the point x_1 undet the gauge transformation θ. We must therefore have

$$U(\hat{\theta}(x_1))U_B^C(x_1, x_2) = U_{B_\theta}^C(x_1, x_2)U(\hat{\theta}(x_2)). \tag{11a}$$

For

$$U_B^C(x_1, x_2) = T \exp\left(-ig \int_{x_1}^{x_2} \hat{B}_\mu(x)dx^\mu\right) \tag{11b}$$

we have

$$\delta\hat{B}_\mu \equiv [\hat{B}_\theta - \hat{B}]_\mu = -\frac{1}{g}\partial_\mu\hat{\theta} - i[\hat{\theta}, \hat{B}_\mu]. \tag{11c}$$

This gives the standard transformation of the gauge fields B^{mk} if we write Eq. (11c) in the matrix representation

$$\delta B_\mu^{km} = -\frac{1}{g}\partial_\mu\theta^{km} - i[[\theta, B_\mu]]^{km}. \tag{12}$$

Now, our mesonic gauge invariant operator has the structure

$$\overline{\Psi}(x_1)U_B^C(x_1, x_2)\Psi(x_2). \tag{13}$$

The gauge invariance of baryonic operators is due to the rule (2a) according to which the product $(\Psi\Psi)$ transform like $\overline{\Psi}$.

We may write down in the octonionic form the gauge invariant operators from ref. (2) for

a) gluonium

$$u_i^* \exp\left(-ig \oint \hat{B}_\mu(x)dx^\mu\right)u_i, \tag{14}$$

b) baryon

$$((U_B(x_1, x_2)\Psi(x_2))\,(U_B(x, x_1)\Psi(x_1)))U_B(x, x_3)\Psi(x_3),$$

c) baryonium M_4^J

$$(((U_B(x, x_1)\Psi(x_1))\,(U_B(x, x_2)\Psi(x_2)))\,(U_B(x, y)\,((\overline{\Psi}(x_3)U_B(x_3, y))\,(\overline{\Psi}(x_4)U_B(x_4, y)))),$$

d) baryonium M_2^J

$$\overline{\Psi}(x_1)U_B(x_1, x)\,((u_m^* U_B^{C_1}(y, x))\,(((U_B(y, x_2)\Psi(x_2))u_m)U_B^{C_2}(y, x))),$$

e) baryonium M_0^J

$$u_i^* U_B^{C_1}(x_1, x_2)\,((u_m^* U_B^{C_2}(x_1, x_2))\,((u_i u_m)U_B^{C_3}(x_1, x_2))).$$

The nontrivial content of this note is better seen when going to the more complicated multiquark objects (c, d, e). We see that the prescription for constructing baryonium operators follows naturally from octonion algebra. There is *no room* for "mock" baryonium states (Ref. [9]) since two quarks can couple only in the colour — antitriplet state.

Our main statement is therefore that: there is a striking similarity between the dual string picture and the models based on the underlying octonion algebra.

Now we turn to the discussion of the differences between the scheme of Veneziano [1, 2] and Low–Nussinov [4, 5] especially concerning the jet structure of multiparticle production processes in both schemes, and the existence or nonexistence of interference

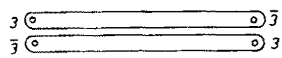

Fig. 1

effects. Recently the universality of quark jets in hadron production reactions was tested carefully and found to be consistent with the data [10]. Many different types of processes involving mesons, baryons, leptons as well as their antiparticles as initial particles were studied [10]. For our purposes it is enough to discuss for instance meson-meson scattering. The picture one has in mind here can be visualized as in Fig. 1. The quark-antiquark pairs after colliding start moving in opposite directions. After some time we have two quark-antiquark pairs. Each pair consists of a quark and an antiquark from different mesons in such a way that it is in a colour singlet state (Fig. 1). The main objection one could pose to such a picture is: why after the collision the initial quark-antiquark pair do

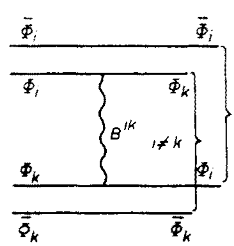

Fig. 2

not act a one entity when this pair is separated from the other pair by a large distance [5]. It seems natural that octet separation should be at work here. On the other side, in the octonionic picture there is no octet at all! Let us discuss it in some detail and imagine soft gluon exchange between two mesons (Fig. 2). In the initial state we have two mesons, each of which can be written as

$$M \sim \sum_{i=1}^{3} \Phi_i \bar{\Phi}_i.$$

After the gluonic exchange has taken place the two initial quark–antiquark pairs have the octonionic structure

$$\sim \sum_{i \neq k} \Phi_k \bar{\Phi}_i.$$

Such a pair *cannot* be treated as one physical object since $u_k u_i^* = 0$ for $k \neq i$.[1] Therefore we must couple quarks in the manner indicated in Fig. 2. This is the only way in which one can couple quarks and antiquarks to obtain physical nonzero amplitude (we neglect at the moment the possible baryonium creation process). In such a way we get highly excited super-clusters which are of the type $qAA...A\bar{q}u_0$. In the case of baryon–antibaryon scattering when different baryonium states can be formed in the direct channel the situation is more complicated (the "junction" appears) but the essence of our reasoning remains unchanged.

We see that two different realisations of the colour confinement dogma lead to different predictions concerning the average multiplicities in hadron production reactions. It is obvious that at present it is not possible to distinguish on the experimental level between 9/4 and 2 (octonions predict $n_{pp}/n_{ee} = 2$ as opposed to the QCD, where octet separation arguments [5] give $n_{pp}/n_{ee} = 9/4$). However, the differences of the two-particle correlations in processes involving different initial particles could help in claryfing what is the underlying physics of colour [11].

The picture favoured by dualists (strings, Y-shaped baryons, different number of jets depending on the type of the initial colliding particles) has an intriguing similarity to the models based on the octonion algebra. It is therefore of high importance to know whether the production of hadron multiparticle states proceeds via formation of universal 3–$\bar{3}$ jets [10, 11]. The confirmation of this pattern would favour the octonionic solution to the quark puzzle and the exceptional groups.

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[1] If $k = i$ then after projecting the result of the action of the gluon operator onto the colour singlet state $\sum_i \Phi_i \bar{\Phi}_i$ of such a pair the only contribution could come from the term $\sum_i B^{ii}(x)$ which is, however, zero.