ANGULAR MOMENTUM EFFECTS IN HIGHLY EXCITED NUCLEI

BY W. AUGUSTYNIAK AND A. MARCINKOWSKI

Department of Nuclear Reactions, Institute of Nuclear Research, Warsaw*

(Received July 27, 1978; final version received September 26, 1978)

Rotational properties of nearly spherical nuclei are discussed in terms of the statistical model including pairing interaction. The possibility of appearance of back-bending in the moment of inertia within the shell model approach is shown. The yrast states are calculated and discussed.

1. Introduction

The rotations of paired nuclei from the spherical region, with finite angular momenta have been discussed in terms of the statistical model including the BCS Hamiltonian [1]. These rotations are not connected with definite rotational quantum states and are considered to be "classical" in nature in so far as the rotational energy is shared among all states with angular momentum. The model parameters like the angular velocity and the moment of inertia were calculated in Ref. [1] with the use of three different single-particle level schemes. It has been stated that the equidistant single-particle levels with constant spin projections lead to a strong back-bending in the plot of the moment of inertia versus the square of the angular velocity, whereas more realistic spin projection distributions like the rectangular distribution as well as the shell model orbital spin projections wash out the effects of a sudden transition between the superfluid and the normal phase. This conclusion drawn by Moretto in Ref. [1] does not seem to be generally valid. We have shown that the back-bending of the moment of inertia may appear also within the shell model level scheme. In addition the yrast levels which are of primary interest in determining the angular momentum effects at higher excitation energies have been calculated and discussed.

* Address: Instytut Badań Jądrowych, Hoża 69, 00-681 Warszawa, Poland.

2. *Calculations and results*

In the calculations we have used the statistical functions describing the number of nucleons N and the angular momentum projection M, derived in Refs. [2, 3]:

$$N = \sum_{k,s} \left[\tfrac{1}{2} - \frac{\varepsilon_k - \lambda}{2E_k} (\mathrm{tgh}\, \tfrac{1}{2} \beta E_{ks}) \right], \tag{1}$$

$$M = -\sum_{k,s} m_k \left(\frac{s}{1+\exp \beta E_{ks}} \right), \tag{2}$$

where $E_{ks} = E_k + s\omega m_k$ with $E_k = [(\varepsilon_k - \lambda)^2 + \Delta^2]^{\frac{1}{2}}$ being the quasiparticle excitation energy and $s = \pm 1$ defining the sign of the spin projection m_k of the single-particle state ε_k. The energy gap parameter Δ, the chemical potential λ, the collective angular velocity ω and β, the inverse of the nuclear temperature T, are related by the gap equation:

$$\frac{2}{G} = \sum_{k,s} \frac{1}{2E_k} (\mathrm{tgh}\, \tfrac{1}{2} \beta E_{ks}). \tag{3}$$

In order to obtain the quantities describing the whole nucleus it was assumed that the proton and neutron gas are in termal and rotational equilibrium, e.g. $T_p = T_n$ and $\omega_p = \omega_n$, and the total angular momentum I was substituted with the sum of the angular momentum projection of protons M_p and neutrons M_n. The identification $I \equiv M$ limits the consideration to spherical nuclei only [3].

Equations (1), (2) and (3) were solved at fixed T, N_p, N_n, I, providing Δ_p, Δ_n, λ_p, λ_n and ω. The fact that equation (3) is no longer valid for $\omega(T)$ surpassing the critical angular velocity $\omega_{cr}(T)$, obtained from equations (1) and (3) with $\Delta = 0$, was taken into account.

The moment of inertia $\mathscr{I}$ was calculated from the relation:

$$I = \mathscr{I}\omega. \tag{4}$$

It depends not only on the temperature of the excited nucleus but also on the angular momentum, which influences the intrinsic state of the superfluid phase through the dependence of the gap parameter Δ upon the angular velocity.

In the calculations performed use was made of the Nilsson single-particle levels at zero deformation. The number of shells taken into consideration and the corresponding pairing constants G_p and G_n were taken the same as in the paper of Decowski et al. [4].

The calculated dependence of the angular velocity on the angular momentum at different temperatures is shown in figure 1. In the case of ^{76}Se this dependence follows the isotherms obtained by Moretto [1]. For ^{56}Fe and ^{108}Pd the display of ω as a function of angular momentum shows that in the region close to ω_{cr} the angular velocity decreases with angular momentum. This implies a back-bending in the display of the moment of inertia as a function of square of the angular velocity as shown in figure 2. These results seem to indicate that in the realistic shell model scheme the effective single-particle spins

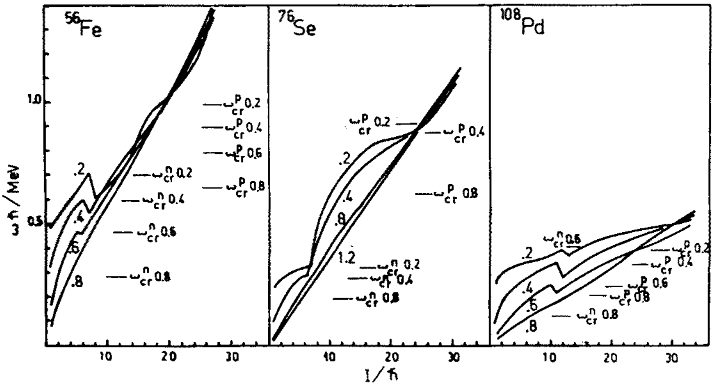

Fig. 1. Angular velocity as a function of angular momentum for ^{56}Fe, ^{76}Se and ^{108}Pd nuclei. Each curve corresponds to different nuclear temperature given in MeV. The positions of the critical angular velocities, for neutrons and protons, for various tempertures are also shown

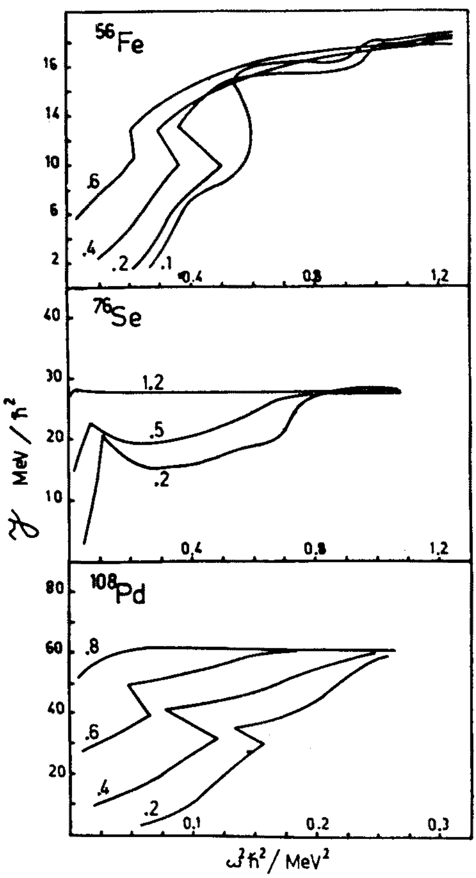

Fig. 2. Moment of inertia as a function of the square of angular velocity for ^{56}Fe, ^{76}Se and ^{108}Pd nuclei at various temperatures. The curves are labelled with the temperature given in MeV

distribution shows an intermediate shape between the two extreme models, the constant spin and the rectangular distribution, considered in Ref. [1].

The dramatic changes of the superfluid phase produced by the shell model demonstrate themselves in the dependence of the energy gap parameters for protons and neutrons on the temperature and angular momentum. This dependence is shown in figure 3. It is visible that these are mainly protons, which contribute to the formation of the total angular momentum of the ^{108}Pd nucleus in the range 5—30 $\hbar$. Only at higher angular

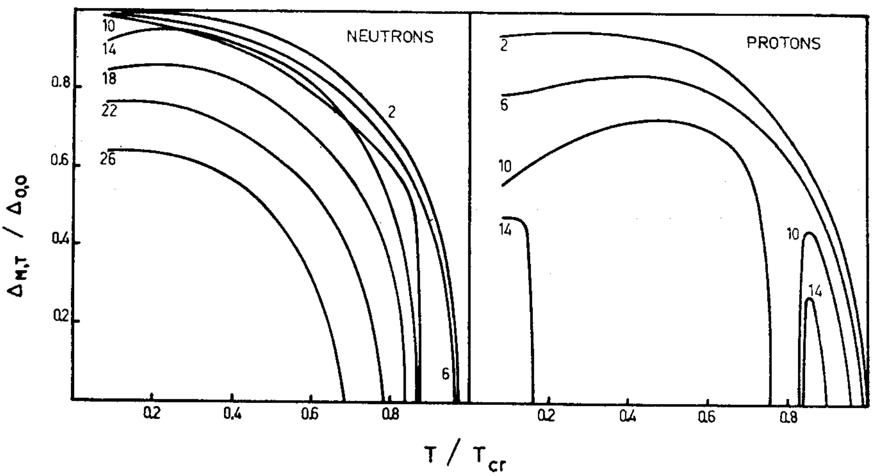

Fig. 3. Energy gap for neutrons and protons in the ^{108}Pd nucleus, as a function of temperature, for various total angular momenta. The angular momenta are given in $\hbar$, the temperature is given as a fraction of its critical value. The subscripts M and T labelling the energy gap parameter Δ denote the angular momentum projection and the temperature, respectively

momenta the reduction of the neutron pairing correlation is noticeable. This is connected with the proximity of the proton chemical potential λ_p to the $1g_{9/2}$ shell. The effects shown should be present along the yrast line and higher isotherms of nearly spherical nuclei.

The behaviour of the moment of inertia along the yrast line is of particular interest in application to the determination of the angular momentum dependence of the level density. In order to define the yrast line equations (1)—(3) have to be solved in the zero temperature limit, where they undergo the following modifications

$$\frac{2}{G} = \sum_{E_k > \omega m_k} \frac{1}{E_k}, \tag{5}$$

$$N = \sum_{E_k < \omega m_k} 1 + \sum_{E_k > \omega m_k} \left[1 + \frac{\varepsilon_k - \lambda_0}{E_k} \right]. \tag{6}$$

Again equation (5) is not valid for $\omega > \omega_{cr}$, where ω_{cr} corresponds to $\Delta_0 = 0$ or $E_k = |\varepsilon_k - \lambda_0|$. By solving equations (5) and (6) at given ω values, Δ_0 and λ_0 were extracted and the effective rotational energy was calculated from the expressions originally given by Kammuri [2],

$$E = \sum_k (\varepsilon_k - \lambda_0) + \lambda_0 N + \frac{\Delta_0^2}{G} - \sum_{E_k > \omega m_k} E_k, \tag{8}$$

$$E_{\text{rot}} = E - E_{\omega=0}, \tag{9}$$

$$M = \sum_{E_k < \omega m_k} m_k, \tag{10}$$

as a function of the total angular momentum $I \equiv M$ (for the whole nucleus both E and M are composed of the contributions due to protons and neutrons).

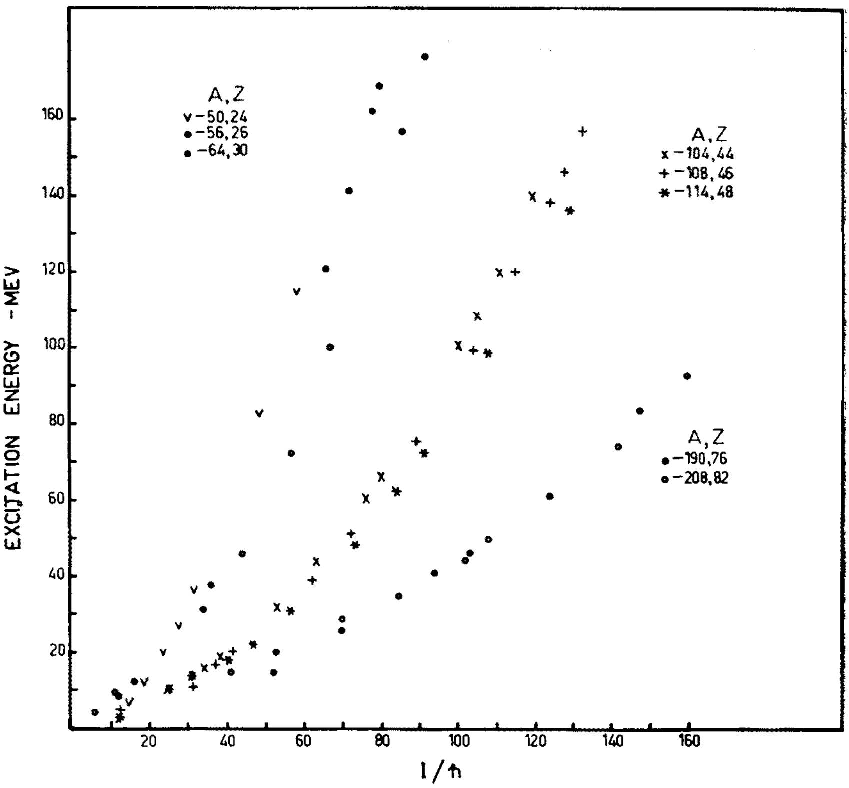

Fig. 4. Yrast states calculated for different nuclei

Such calculations when omitting some spurious solutions provide the yrast states shown in figure 4. For practical purposes the calculated yrast lines may be approximated by the following expression

$$E_{\text{yrast}} = aI_{\text{yrast}}^2 + bI_{\text{yrast}}. \tag{11}$$

Here a is a simple function of the mass number A of the nucleus, $a = 0.072 \exp[-0.02039\, A]$, and b is constant $b = 0.26$. This formula has proved satisfactory in reproducing the results of microscopic calculations performed by Grover and Gilat [5–7], in the nuclear mass range $40 < A < 210$.

Discussions with Dr P. Decowski and Professor J. Turkiewicz are greatly appreciated by the authors.

REFERENCES

[1] L. G. Morretto, *Nucl. Phys.* **A226**, 9 (1974).
[2] T. Kammuri, *Progr. Theor. Phys.* **31**, 595 (1964).
[3] L. G. Moretto, *Nucl. Phys.* **A185**, 145 (1972).
[4] P. Decowski, W. Grochulski, A. Marcinkowski, *Nucl. Phys.* **A194**, 380 (1972).
[5] J. R. Grover, *Phys. Rev.* **157**, 832 (1967).
[6] J. Gilat, *Phys. Rev.* **C1**, 1432 (1970).
[7] J. Gilat, J. R. Grover, *Phys. Rev.* **C3**, 734 (1971).