THE EQUALITY OF RADII CHARACTERIZING ρ AND ψ TRAJECTORIES

By N. A. Kobylinsky

Institute for Theoretical Physics, Academy of Sciences of the Ukrainian SSR, Kiev*

(Received July 17, 1978)

An analysis of the data on ρ and ψ trajectories is carried out. In the framework of a model with square-root asymptotics both trajectories have asymptotically the same slope. This slope is determined by the π-meson radius. The leading thresholds for ρ and ψ trajectories are disposed at $\sqrt{s} \simeq 2.4$ and 4.9 GeV, respectively.

The opinion that the slopes are equal for all Regge trajectories has been widely accepted until recently (we shall treat here the meson trajectories only; a left-hand cut must be taken into account when baryon trajectories are analyzed). No significance has been attached to the dispersion observed in the slopes. The requirement that the dual narrow-resonance models should be polynomially bounded serves as a theoretical argument for the slope equality in the case of the linear approximation for the trajectories.

The recent discovery of the ψ-meson family [1] has shown that the trajectories for these mesons have much smaller slopes. This fact was discussed in detail in Refs. [2–4] where a factorization of Regge slopes was suggested. This factorization can be derived from dual narrow-resonance model, but this model loses its main properties, as was already mentioned, when the slopes are different in asymptotics.

As regards to the dual analytic model [5], they require that asymptotically $\alpha(s) \sim \sqrt{s}$. Various arguments in favour of these trajectories are given in Refs. [6, 7]. The models of trajectories which satisfy this condition were elaborated in Refs. [7, 8].

We analyze the data on ϱ nad ψ trajectories in the framework of a model with square-root asymptotics. Let

$$\alpha_R(s) = \bar{\alpha}_R(s) + \sigma_R(s), \tag{1}$$

where R = ϱ or ψ, $\bar{\alpha}(s)$ is the contribution of leading singularities — heavy thresholds, and $\sigma(s)$ is the light threshold contribution providing the instability of resonances.

* Address: Institute for Theoretical Physics, Academy of Sciences of the Ukrainian SSR, Kiev-130, USSR.

For the ϱ trajectory, taking into account resonance widths, we can use (all parameters given below are in units of GeV)

$$\sigma_\varrho(s) = -0.145\sqrt{4m_\pi^2 - s}\,. \tag{2}$$

The analysis [9] of all the data on πN scattering yields

$$\alpha_\varrho(s) \simeq 0.49+0.82\,s \quad \text{at} \quad -1.5 \lesssim s \leqslant 0. \tag{3}$$

Taking into account (1)–(3), we have in the scattering region

$$\bar{\alpha}_\varrho(s) \simeq 0.53+0.73\,s. \qquad d$$

This expression and the data of the ϱ and g meson masses show the decrease of slope when s slows down, that is, a mean slope for different segments is

$$\bar{\alpha}'_\varrho = \begin{cases} 0.886 & \text{at} \quad s = 0.6-2.86, \\ 0.786 & \text{at} \quad s = 0-0.6, \\ 0.727 & \text{at} \quad s = -1.5-0. \end{cases}$$

This results in

$$\bar{\alpha}''_\varrho(1.0) \simeq 0.070, \qquad \bar{\alpha}''_\varrho(-0.2) \simeq 0.056.$$

If away from a leading threshold at $s = s_R$

$$\bar{\alpha}_R(s) \simeq \lambda_R - \gamma_R\sqrt{s_R - s}\,, \tag{4}$$

then

$$\bar{\alpha}''_R(s)/\bar{\alpha}'_R(s) \simeq 1/2(s_R - s).$$

Using the values of $\bar{\alpha}'_\varrho$ and $\bar{\alpha}''_\varrho$ determined above, we obtain $\sqrt{s_\varrho} \simeq 2.64$ from the behaviour of α_ϱ at $s = 0-2.86$; $\sqrt{s_\varrho} \simeq 2.56$ from the behaviour of α_ϱ at $s = -1.5-0.6$. These estimates agree well with each other as well as with the results of Ref. [8] where the value $\sqrt{s_\varrho} = 2.3$ has been obtained from the first to the $\pi^-p \to \pi^0n$ cross-section and to the difference of the π^+p and π^-p total cross-sections.

We emphasize that the smallness of the second derivative of the ϱ trajectory ($\alpha''_\varrho \simeq \frac{1}{13}\alpha'_\varrho$) does not mean that the approximate linearity of the trajectory will hold in the region $s \gg 5$ too. Indeed, $\alpha''_\varrho \simeq \alpha'_\varrho/2s_\varrho$ implies that the required relation between α'_ϱ and α''_ϱ will already hold when $s_\varrho = 6-7$.

Let us consider now the ψ resonance family [1]. The parameters of the J/ψ(3.098), ψ'(3.684) and ψ'''(4.414) mesons have been measured with high precision. There is also a number of indications on the existence of the ψ''($\sim$4.100) resonance (because of the error in ψ'' mass, the final parameters of the model are calculated from the data on the J/ψ, ψ' and ψ''' only). Suppose that ψ', ψ'' and ψ''' are located on even daughter trajectories which are parallel to the parent trajectory. Then we get for the mean slope of the ψ trajectory

$$\bar{\alpha}'_\psi = \begin{cases} 0.500 & \text{at} \quad s = 9.6-13.6, \\ 0.625 & \text{at} \quad s = 13.6-16.8, \\ 0.740 & \text{at} \quad s = 16.8-19.5. \end{cases}$$

Similarly to the ϱ trajectory, the ψ trajectory slope also increases with s. The change of $\bar{\alpha}'_\psi$ implies that $\bar{\alpha}''_\psi(15) \simeq 0.035$. Eq. (5) gives $\sqrt{s_\psi} \simeq 4.86$. If Eq. (4) approximates the ϱ and ψ trajectories rather well, then the ratio $\bar{\alpha}'_\varrho(ys_\varrho)/\bar{\alpha}'_\psi(ys_\psi)$ must be the same for any y, i. e. a scale for each of the trajectories is judged by the mass of its leading threshold. When this ratio is expressed through the parameters γ_R and $\sqrt{s_R}$, we obtain

$$\frac{\gamma_\varrho}{\gamma_\psi} = \frac{\sqrt{s_\varrho}}{\sqrt{s_\psi}} \, \frac{\bar{\alpha}'_\varrho(ys_\varrho)}{\bar{\alpha}'_\psi(ys_\psi)} . \tag{6}$$

Taking into account the above calculations and the results of Ref. [8], we write

$$\sqrt{s_\varrho} = 2.50, \qquad \sqrt{s_\psi} = 4.86, \qquad ys_\varrho = (m_\varrho^2 + m_g^2)/2 = 1.73,$$

$$\bar{\alpha}'_\varrho(1.73) = 0.886, \qquad ys_\psi = 6.53.$$

To determine $\bar{\alpha}'_\psi\,(6.53)$, we use

$$\bar{\alpha}'_\psi(s_2) \simeq \bar{\alpha}'_\psi(s_1) + \bar{\alpha}''_\psi(s_1)\,(s_2 - s_1), \qquad \bar{\alpha}''_\psi(s_1) \simeq \bar{\alpha}'_\psi(s_1)/2s_\psi .$$

Then

$$\bar{\alpha}'_\psi(s_1) \simeq \frac{\bar{\alpha}'_\psi(s_2)}{1 + (s_2 - s_1)/2s_\psi} ,$$

and with $s_1 = ys_\psi = 6.53$, $s_2 = (m_{J/\psi}^2 + m_{\psi'}^2)/2 = 11.6$, $\bar{\alpha}'_\psi(11.6) = 0.500$, we get

$$\bar{\alpha}'_\psi(6.53) = 0.452.$$

When substituting the values of $\sqrt{s_R}$ and $\bar{\alpha}'_R(ys_R)$ in Eq. (6), we obtain

$$\gamma_\varrho/\gamma_\psi \simeq 1.01,$$

i.e. the both trajectories have practically equal slopes in asymptotics (the idea that nonlinear trajectories should be parallel asymptotically has been surmized and substantiated in Ref. [10]).

As regards to the values of the parameters γ_ϱ and γ_ψ, we obtain by means of the parametrization (4) and the data drawn above $\gamma_\varrho \simeq 3.6$, $\gamma_\psi \simeq 3.4$.

Notice that asymptotically

$$\bar{\alpha}_R(s) \simeq il(s),$$

where $l(s) = 2\gamma k$ is the orbital momentum when two particles with momentum $k \simeq \sqrt{s}/2$ revolve on a circle with radius γ. It is natural to expect that for hadron trajectories $\gamma \leqslant r_\pi \equiv 1/2m_\pi$. Referring to the ϱ and ψ trajectories, the above analysis has revealed that

$$\gamma_\varrho \simeq \gamma_\psi \simeq r_\pi = 3.57.$$

Hence, the main terms of the ϱ and ψ trajectories are characterized by the π meson radius and they may be chosen in the form

$$\bar{\alpha}_R(s) = \lambda_R - \frac{1}{2m_\pi} \sqrt{s_R - s} ,$$

where $\sqrt{s_\varrho} = 2.43$, $\sqrt{s_\psi} = 4.94$, $\lambda_\varrho = 9.21$, $\lambda_\psi = 14.75$.

In conclusion we stress the fact that the parameters obtained satisfy well the equality

$$\tfrac{1}{2}(\sqrt{s_\psi}-\sqrt{s_\varrho}) = m_{D^*}-m_\varrho,$$

where both parts can be interpreted as a difference of the charmed and up-down quark masses.

REFERENCES

[1] Particle Data Group, *Rev. Mod. Phys.* **48**, No.2, part II (1976); *Phys. Lett.* **68B**, 1 (1977).
[2] K. Igi, *Phys. Lett.* **66B**, 276 (1977); *Phys. Rev.* **D16**, 196 (1977).
[3] J. Pasupathy, *Phys. Rev. Lett.* **37**, 1336 (1976).
[4] M. Kuroda, B.-L. Young, *Phys. Rev.* **D16**, 204 (1977).
[5] A. I. Bugrij, G. Cohen-Tannoudji, L. L. Jenkovszky, N. A. Kobylinsky, *Fortschr. Phys* **21**, 427 (1973); L. Gonzalez Mestres, R. Hong Tuan, Preprint LPTHE-72/20, Orsay 1972.
[6] A. A. Trushevsky, *Ukr. Fiz. Zh.* **22**, 353 (1977).
[7] A. I. Bugrij, N. A. Kobylinsky, *Ann. Phys.* (Germany) **32**, 297 (1975); Preprint ITP-75-50E Kiev 1975.
[8] N. A. Kobylinsky, A. B. Prognimak, *Acta Phys. Pol.* **B9**, 149 (1978).
[9] V. Barger, R. J. N. Phillips, *Phys. Lett.* **53B**, 195 (1974); D. Joynson, E. Leader, B. Nicolescu, C. Lopez, *Nuovo Cimento* **30A**, 345 (1975); N. A. Kobylinsky, V. V. Timokhin, *Ukr. Fiz. Zh.* **22**, 1871 (1977); Preprint ITP-78-55E, Kiev (1978); H. Navelet, P. R. Stevens, *Nucl. Phys.* **B118**, 475 (1977).
[10] D. Crewther, G. C. Joshi, *Phys. Rev.* **D9**, 1446 (1974).