CHIRAL SYMMETRY BREAKING IN THE BAG MODEL

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(Received October 9, 1978)

The question of compatibility of the MIT bag model with standard chiral symmetry breaking is investigated in the $(3,\bar{3})\oplus(\bar{3},3)$ model. We find that the chiral symmetry breaking parameter c is given by $c = -0.79$ and compute the meson and baryon σ terms in the bag model. For example, in case of the πN scattering σ term, we obtain 104 MeV.

1. Introduction

Considerable interest has been devoted in the literature to the srudy of possible applications of the MIT bag model [1-11]. The main advantage of this model consists in providing a relativistic framework in which the quark wave functions may be explicitly computed at least in the cavity approximation. In particular, the several applications of the model to the study of the hadronic mass spectrum, static parameters of hadrons and a variety of other strong, electromagnetic and weak processes are in reasonably good agreement with experiment.

On the other hand, ever since the original suggestion that strong interactions are approximately invariant under SU(3)$\otimes$SU(3), a great deal of attention has been centered around the study of chiral symmetry breaking. In particular, once the transformation properties of the symmetry breaking part of the Hamiltonian are assumed as, for example, $(3,\bar{3})\oplus(\bar{3},3)$ one can then proceed to compute the matrix elements of the scalar densities, the symmetry breaking parameter and the σ terms by making use of the soft pion limit and SU(3) symmetry assumptions. In view of the successes of the MIT bag model it is thus of interest to see to what extent chiral symmetry breaking considerations are compatible with the bag model assumptions. In this paper we proceed to discuss the previous questions and compute the meson and baryon matrix elements of the scalar densities by writting them in terms of quarks and carrying out the computation in the bag model. Furthermore, we then calculate the symmetry breaking parameter and the σ terms in this model by making use, additionally, of PCAC.

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2. Notation

In the bag model, the ground state wave functions are given by

$$\psi(\vec{r}, t) = \frac{N(x)}{\sqrt{4\pi}} \begin{pmatrix} i\alpha u \\ -\beta\vec{\sigma}\frac{\vec{r}}{|\vec{r}|}u \end{pmatrix} e^{-i\omega t} \tag{2.1}$$

in the case of quarks, and

$$\varphi(\vec{r}, t) = \frac{N(x)}{\sqrt{4\pi}} \begin{pmatrix} -i\beta\vec{\sigma}\frac{\vec{r}}{|\vec{r}|}u \\ \alpha u \end{pmatrix} e^{i\omega t} \tag{2.2}$$

in the case of antiquarks. In the above equation we have defined

$$\alpha = \left(\frac{\omega+m}{\omega}\right)^{\frac{1}{2}} j_0\left(\frac{xr}{R}\right) \tag{2.3}$$

and

$$\beta = \left(\frac{\omega-m}{\omega}\right)^{\frac{1}{2}} j_1\left(\frac{xr}{R}\right). \tag{2.4}$$

Furthermore, $N(x)$ is a normalization factor which is given by

$$N^{-2}(x) = R^3 j_0^2(x) \frac{2\omega\left(\omega-\frac{1}{R}\right)+\frac{m}{R}}{\omega(\omega-m)} \tag{2.5}$$

with ω defined as

$$\omega = \frac{1}{R}\,[x^2+(mR)^2]^{\frac{1}{2}}, \tag{2.6}$$

where R is the radius of the spherical cavity, m is the quark or antiquark mass and $x = x(m,R)$ is the smallest positive root of the equation.

$$\tan x = \frac{x}{1-mR-[x^2+(mR)^2]^{\frac{1}{2}}}. \tag{2.7}$$

We start by writing down the chiral symmetry breaking Hamiltonian $\mathscr{H}'$ in the $(3,\bar{3})\oplus(\bar{3},3)$ representation in the form

$$\mathscr{H}' = \varepsilon_0 u_0 + \varepsilon_8 u_8, \tag{2.8}$$

where the scalar and pseudoscalar densities u_i and v_i may be expressed in terms of quark fields as $u_i = \bar{q}\lambda_i q$ and $v_i = \bar{q}\lambda_i\gamma_5 q$, respectively. In particular, u_0 and u_8 may then be expressed as

$$u_0 = \sqrt{\tfrac{2}{3}}(\bar{u}u+\bar{d}d+\bar{s}s) \tag{2.9}$$

and

$$u_8 = \sqrt{\tfrac{1}{3}}(\bar{u}u+\bar{d}d-2\bar{s}s). \tag{2.10}$$

In the above u, d and s denote, as is customary, the up, down and strange quarks, respectively. In the following we shall only make use of the operator properties of u_i but make no further assumptions as to the structure of its matrix elements. Furthermore, one also obtains, as customary

$$m_\pi^2 = \langle\pi|\mathscr{H}'|\pi\rangle \tag{2.11}$$

and

$$m_K^2 = \langle K|\mathscr{H}'|K\rangle \tag{2.12}$$

upon making use of PCAC for mesons, or

$$m_B = \langle B|\mathscr{H}'|B\rangle + m_0 \tag{2.13}$$

for baryons, where m_0 is the chiral invariant contribution to the baryon mass.

To calculate the matrix elements in Eqs (2.11) and (2.12), one then makes use of the bag model. For this purpose, we need the relation

$$\langle P_\alpha|u_i|P_\alpha\rangle = 2m_\alpha\, {}_B\langle P_\alpha| \int_{\mathrm{Bag}} u_i(x)d^3x|P_\alpha\rangle_B \tag{2.14}$$

where $|P_\alpha\rangle$ denotes a covariantly normalized pseudoscalar meson state, $|P_\alpha\rangle_B$ the corresponding pseudoscalar meson state in the bag model (normalized to ${}_B\langle P_\alpha|P_\alpha\rangle_B = 1$), m_α its mass and u_i a scalar density.

3. Calculation

We now proceed to compute the matrix elements in Eqs (2.11), (2.12) and (2.13) in the bag model. We thus obtain

$$ {}_B\langle\pi| \int_{\mathrm{Bag}} u_0(x)d^3x|\pi\rangle_B = 2a_u\sqrt{\tfrac{2}{3}}, \tag{3.1}$$

$$ {}_B\langle\pi| \int_{\mathrm{Bag}} u_8(x)d^3x|\pi\rangle_B = 2a_u\sqrt{\tfrac{1}{3}}, \tag{3.2}$$

$$ {}_B\langle K| \int_{\mathrm{Bag}} u_0(x)d^3x|K\rangle_B = \sqrt{\tfrac{2}{3}}\,(a_u+a_s(R)), \tag{3.3}$$

and

$$ {}_B\langle K| \int_{\mathrm{Bag}} u_8(x)d^3x|K\rangle_B = \sqrt{\tfrac{1}{3}}\,(a_u-2a_s(R)) \tag{3.4}$$

for mesons. Similarly, we also obtain

$$ {}_B\langle P| \int_{\mathrm{Bag}} u_0(x)d^3x|P\rangle_B = 3a_u\sqrt{\tfrac{2}{3}}, \tag{3.5}$$

$$ {}_B\langle P| \int_{\mathrm{Bag}} u_8(x)d^3x|P\rangle_B = 3a_u\sqrt{\tfrac{1}{3}}, \tag{3.6}$$

$$_{\mathrm{B}}\langle\Sigma| \int_{\mathrm{Bag}} u_0(x)d^3x|\Sigma\rangle_{\mathrm{B}} = (2a_{\mathrm{u}}+a_{\mathrm{s}}(R))\sqrt{\tfrac{2}{3}}\,, \tag{3.7}$$

$$_{\mathrm{B}}\langle\Sigma| \int_{\mathrm{Bag}} u_8(x)d^3x|\Sigma\rangle_{\mathrm{B}} = (2a_{\mathrm{u}}-2a_{\mathrm{s}}(R))\sqrt{\tfrac{1}{3}}\,, \tag{3.8}$$

$$_{\mathrm{B}}\langle\Xi| \int_{\mathrm{Bag}} u_0(x)d^3x|\Xi\rangle_{\mathrm{B}} = \sqrt{\tfrac{2}{3}}\,(a_{\mathrm{u}}+2a_{\mathrm{s}}(R)), \tag{3.9}$$

and

$$_{\mathrm{B}}\langle\Xi| \int_{\mathrm{Bag}} u_8(x)d^3x|\Xi\rangle_{\mathrm{B}} = \sqrt{\tfrac{1}{3}}\,(a_{\mathrm{u}}-4a_{\mathrm{s}}(R)). \tag{3.10}$$

In the preceding a_{u} and $a_{\mathrm{s}}(R)$ denote the contributions to these equations from the nonstrange and strange quarks and antiquarks respectively and may be expressed in the form

$$a_{\mathrm{u}} = \frac{1}{2(x-1)} \tag{3.11}$$

and

$$a_{\mathrm{s}}(R) = \frac{\omega+2m_{\mathrm{s}}R\omega-2m_{\mathrm{s}}}{2\omega(\omega R-1)+m_{\mathrm{s}}}\,, \tag{3.12}$$

with a nonstrange quark mass equal to zero, a strange quark mass m_{s} equal to 0.279 GeV, and where R and ω have the numerical values given by the fit of Ref. [3]. For the matrix elements in Eqs (3.1) to (3.10) we thus obtain the following numerical values:

$$_{\mathrm{B}}\langle\pi| \int_{\mathrm{Bag}} u_0(x)d^3x|\pi\rangle_{\mathrm{B}} = 0.78, \tag{3.13}$$

$$_{\mathrm{B}}\langle\pi| \int_{\mathrm{Bag}} u_8(x)d^3x|\pi\rangle_{\mathrm{B}} = 0.55, \tag{3.14}$$

$$_{\mathrm{B}}\langle\mathrm{K}| \int_{\mathrm{Bag}} u_0(x)d^3x|\mathrm{K}\rangle_{\mathrm{B}} = 0.89, \tag{3.15}$$

$$_{\mathrm{B}}\langle\mathrm{K}| \int_{\mathrm{Bag}} u_8(x)d^3x|\mathrm{K}\rangle_{\mathrm{B}} = -0.43 \tag{3.16}$$

for mesons, and

$$_{\mathrm{B}}\langle\mathrm{P}| \int_{\mathrm{Bag}} u_0(x)d^3x|\mathrm{P}\rangle_{\mathrm{B}} = 1.17, \tag{3.17}$$

$$_{\mathrm{B}}\langle\mathrm{P}| \int_{\mathrm{Bag}} u_8(x)d^3x|\mathrm{P}\rangle_{\mathrm{B}} = 0.83, \tag{3.18}$$

$$_{\mathrm{B}}\langle\Sigma| \int_{\mathrm{Bag}} u_0(x)d^3x|\Sigma\rangle_{\mathrm{B}} = 1.33, \tag{3.19}$$

$$_{\mathrm{B}}\langle\Sigma| \int_{\mathrm{Bag}} u_8(x)d^3x|\Sigma\rangle_{\mathrm{B}} = -0.22, \tag{3.20}$$

$$_{\mathrm{B}}\langle\Xi| \int_{\mathrm{Bag}} u_0(x)d^3x|\Xi\rangle_{\mathrm{B}} = 1.48, \tag{3.21}$$

and

$$_{\mathrm{B}}\langle\Xi| \int_{\mathrm{Bag}} u_8(x)d^3x|\Xi\rangle_{\mathrm{B}} = -1.26 \tag{3.22}$$

for baryons. Comparing, for example, Eqs (3.13) with (3.15) and (3.17), (3.19) and (3.21), we see to what extent these matrix elements are SU(3) symmetric. Note once again that these equations involve the matrix elements with respect to bag model wave functions. Of course, if the matrix elements with respect to covariantly normalized states are used, we would then obtain a large amount of SU(3) breaking.

Making use of Eqs (2.8), (2.11), (2.12), (2.14) and (3.13) to (3.16), we are then in a position to calculate ε_0 and ε_8. We obtain

$$\varepsilon_0 = 202 \text{ MeV} \tag{3.23}$$

and

$$\varepsilon_8 = -160 \text{ MeV}. \tag{3.24}$$

From the above equations we also obtain

$$c = \frac{\varepsilon_8}{\varepsilon_0} = -0.79. \tag{3.25}$$

In the above computations we have used the physical value for the pion mass.

The value $c = -0.79$, calculated in Eqs (3.25), is approximately equal to the value $c = -0.8$ calculated in Ref. [13], making use of different techniques.

In order to check the consistency of the values for ε_0 and ε_8 calculated in Eqs (3.23) and (3.24) making use of the meson matrix elements, we employ Eqs (2.13) for a proton state to calculate m_0 and then proceed to calculate m_Σ and m_Ξ by means of Eqs (3.19) to (3.22). We obtain:

$$m_0 = 834 \text{ MeV}, \tag{3.26}$$

$$m_\Sigma = 1138 \text{ MeV} \tag{3.27}$$

and

$$m_\Xi = 1334 \text{ MeV}. \tag{3.28}$$

From Eqs (3.27) and (3.28) we thus see consistency of the bag value results for mesons and baryons.

In order to compare further the bag model predictions with standard chiral symmetry breaking considerations, we then proceed to compute the σ terms for πN and KN scattering. We obtain:

$$\sigma_{\rm NN}^{\pi\pi} = \frac{\varepsilon_0}{3}(\sqrt{2}+c)\langle {\rm P}|\sqrt{2}\,u_0+u_8|{\rm P}\rangle = 104 \text{ MeV} \tag{3.29}$$

and

$$\sigma_{\rm NN}^{\rm KK} = \frac{\varepsilon_0}{3}(\sqrt{2}-\tfrac{1}{2}c)\langle {\rm P}|\sqrt{2}\,u_0+\frac{\sqrt{3}}{2}u_3-\tfrac{1}{2}u_8|{\rm P}\rangle = 151 \text{ MeV}. \tag{3.30}$$

The above is to be compared with the experimental values

$$\sigma_{\rm NN}^{\pi\pi} = 65 \pm 5 \text{ MeV} \tag{3.31}$$

and

$$\sigma_{NN}^{KK} = 180 \pm 55 \text{ MeV}. \tag{3.32}$$

Furthermore, the remaining σ terms for meson-meson and meson-baryon scattering may, of course, be computed in an analogous fashion. In particular, in the case of $\langle\pi|\sigma^{\pi\pi}|\pi\rangle$ the result is the same as in Weinberg's low energy $\pi\pi$ analysis and in $(3,\bar{3})\oplus(\bar{3},3)$, namely, m_π^2. Before concluding this section, we also wish to note that matrix elements of the type $\langle\Omega|v_\pi|\pi\rangle$, where v_π denotes the pseudoscalar density for the pion in the $(3,\bar{3})\oplus(\bar{3},3)$, are not reliably computable in the bag model. Furthermore, from our results for c in Eq. (3.25) and the standard $(3,\bar{3})\oplus(\bar{3},3)$ result

$$\frac{m_\pi^2}{m_K^2} = \frac{\sqrt{2}+c}{\sqrt{2}-\frac{1}{2}c}\,\frac{\langle\Omega|v_\pi|\pi\rangle}{\langle\Omega|v_K|K\rangle}, \tag{3.33}$$

we see that the bag model predicts a strong symmetry breaking for $\langle\Omega|v_\alpha|P_\alpha\rangle$.

4. *Conclusions*

In this paper, we have computed the meson and baryon matrix elements of the scalar densities in the $(3,\bar{3})\oplus(\bar{3},3)$ representation by explicitly making use of their expression in terms of quark fields and the bag model.

In particular, we have computed the chiral symmetry breaking parameter c in the bag model as $c = -0.79$. Furthermore, we have also calculated the meson and baryon matrix elements of the σ terms and, for example, obtained for the πN σ terms the result $\sigma_{NN}^{\pi\pi} = 104$ MeV, which is in reasonable agreement with the experimental value.

Note, of course, that in spite of the fact that the nonstrange quark masses are also here taken to be zero as in the standard Gell-Mann, Oakes and Renner model, we nevertheless do not obtain $\sigma_{NN}^{\pi\pi} = 25$ MeV as in that scheme, in view of the SU(3) symmetry breaking of the bag model wave functions.

The author wishes to thank J. Katz for stimulating discussions.

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