ELECTROMAGNETIC FIELD OF A ROTATING PERMANENTLY MAGNETIZED SPHERE IN THE INERTIAL AND THE COMOVING FRAMES OF REFERENCE

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The electromagnetic field of a permanently magnetized, rotating sphere without electric charge is calculated up to the first order in the angular velocity with the help of the Maxwell equations in the inertial frame of reference and the previously deduced, generalized Maxwell equations for a non-inertial frame of reference. In this way the field quantities and the charge densities are obtained in both frames of reference directly, i.e. without using transformation formulae for the transition between an inertial frame and a noninertial frame of reference.

1. Some introductory remarks on the problem

The problem of a permanently magnetized, rotating sphere was treated several times in older literature, but with different results. As it seems to us the last detailed treatise was done by Schlomka and Schenkel [1] with critical remarks on a paper by Swann [2] (see also Tate [3], Barnett [4], and Lawrence [5], Cullwick [6]).

In our paper the problem mentioned is treated systematically for the following four alternatives:

a) inertial frame for the observer: resting sphere (I), rotating sphere (II);

b) rotating frame for the observer: resting sphere (rotating in the inertial frame) (III), rotating sphere (resting in the inertial frame) (IV).

The calculations are performed in the inertial frame on the basis of the usual Maxwell equations and in the rotating frame on the basis of generalized Maxwell equations which were previously deduced by us from the general relativistic Maxwell theory [7]. By this direct method we avoid the transformation formulae for the transition to noninertial frames of reference (as is well known, this procedure sometimes gives rise to doubts concerning the correctness of the results). Our field and charge quantities obtained for the four mentioned alternatives are fully consistent. They coincide, as far as the problem was

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treated by them, with the results by Schlomka and Schenkel. Apart from the clarification of the polemic situation our treatment here is an interesting example for the application of the generalized Maxwell equations for non-inertial frames of reference. Furthermore, the results give useful hints for the dynamo problem of generation of the magnetic field of celestial bodies, if treated for simplification in the co-rotating frame of reference.

2. Generalized Maxwell equations, constitutive equations, and boundary conditions for a rotating frame of reference

Let us consider a non-inertial frame of reference, rotating with the constant angular velocity $\boldsymbol{\Omega}_0 = \boldsymbol{k}\Omega_0$ with respect to an inertial frame of reference. According to our results [6] up to the first order in Ω_0 the following generalized Maxwell equations in conventional Gauss units are valid (dash refers to the rotating frame):

$$\mathrm{rot}'\,\boldsymbol{H}' = \frac{1}{c}\left(\frac{\partial'\boldsymbol{D}'}{\partial t} + 4\pi\boldsymbol{j}'\right) + \frac{1}{c^2}\,\frac{\partial'\boldsymbol{H}'}{\partial t} \times (\boldsymbol{\Omega}_0 \times \boldsymbol{r}'), \tag{1}$$

$$\mathrm{div}'\,\boldsymbol{D}' = 4\pi\varrho' + \frac{4\pi\sigma}{c^2}\,\boldsymbol{E}'\boldsymbol{v}' + \frac{2}{c}\,\boldsymbol{\Omega}_0\boldsymbol{H}'$$

$$+ \frac{4\pi}{c^2}\,\boldsymbol{\Omega}_0(\boldsymbol{r}' \times \boldsymbol{j}') - \frac{1}{c}\,(\boldsymbol{\Omega}_0 \times \boldsymbol{r}')\,\mathrm{rot}'\,\boldsymbol{H}', \tag{2}$$

$$\mathrm{rot}'\,\boldsymbol{E}' = -\frac{1}{c}\,\frac{\partial'\boldsymbol{B}'}{\partial t} + \frac{1}{c^2}\,\frac{\partial'\boldsymbol{E}'}{\partial t} \times (\boldsymbol{\Omega}_0 \times \boldsymbol{r}'), \tag{3}$$

$$\mathrm{div}'\,\boldsymbol{B}' = -\frac{2}{c}\,(\boldsymbol{\Omega}_0\boldsymbol{E}') + \frac{1}{c}\,(\boldsymbol{\Omega}_0 \times \boldsymbol{r}')\,\mathrm{rot}'\,\boldsymbol{E}'. \tag{4}$$

The electric current density reads if we use Ohm's law:

$$\boldsymbol{j}' = \varrho'\boldsymbol{v}' + \sigma\left(\boldsymbol{E}' + \frac{1}{c}\,\boldsymbol{v}' \times \boldsymbol{B}'\right), \tag{5}$$

where σ is the electric conductivity. The quantities $\boldsymbol{r}'$ and $\boldsymbol{v}'$ mean the radius vector and velocity of the medium with respect to the rotating frame. Furthermore, ϱ' is the true electric charge density.

From the field equations in the usual way the following generalized continuity equation results:

$$\frac{\partial}{\partial t}\left(\varrho' + \frac{\sigma}{c^2}\,\boldsymbol{E}'\boldsymbol{v}'\right) + \mathrm{div}'\,\boldsymbol{j}' + \frac{1}{c^2}\,\boldsymbol{\Omega}_0\left(\boldsymbol{r}' \times \frac{\partial \boldsymbol{j}'}{\partial t}\right) = 0. \tag{6}$$

The constitutive equations keep the usual form:

$$\text{a) } \boldsymbol{D}' = \boldsymbol{E}' + 4\pi\boldsymbol{P}', \qquad \text{b) } \boldsymbol{B}' = \boldsymbol{H}' + 4\pi\boldsymbol{M}'. \tag{7}$$

The boundary conditions in the rotating frame can be derived in the well known way from the field equations (1) to (4) respectively (6). They keep the usual form for moving media, too:

$$\text{a) } H_t'^2 - H_t'^1 = \frac{4\pi}{c} \lambda' v_f', \quad \text{b) } E_t^2 - E_t^1 = 0, \tag{8}$$

$$\text{b) } D_n'^2 - D_n'^1 = 4\pi\lambda', \quad \text{c) } B_n'^2 - B_n'^1 = 0; \tag{9}$$

$$\frac{\partial \lambda'}{\partial t} = j_n'^2 - j_n'^1. \tag{10}$$

Here 1 and 2 denote medium 1 and medium 2; λ' is the electric surface charge density; v_f' means the velocity of the corresponding charge density in the tangential direction; t respectively n denote the tangential respectively normal direction with respect to the surface.

3. *Alternative I: Resting sphere with respect to the inertial frame (inertial observer)*

Since this case is mathematically rather simple, it can be treated more generally than the others, namely as a spherical shell with inner radius r_1 and outer radius r_0. The other alternatives are only treated for a sphere with the radius r_0.

3.1. Shell ($r_1 < r < r_0$)

Because of the magnetostatic character of this problem we have the simplified situation:

$$\text{a) } \boldsymbol{E} = 0, \quad \text{b) } \boldsymbol{D} = 0, \quad \text{c) } \varrho = 0, \quad \text{d) } \boldsymbol{v} = 0, \quad \text{e) } \boldsymbol{j} = 0. \tag{11}$$

Therefore, the field equations read:

$$\text{a) rot } \boldsymbol{H} = 0, \quad \text{b) div } \boldsymbol{B} = 0, \tag{12}$$

where

$$\boldsymbol{B} = \boldsymbol{H} + 4\pi \boldsymbol{M}. \tag{13}$$

For simplicity the direction of magnetization and rotation will be taken to coincide:

$$\boldsymbol{M} = \boldsymbol{k} M \quad (M = \text{const}). \tag{14}$$

This means axisymmetry of the problem with respect to $\boldsymbol{k}$ (z-axis). Under these circumstances we get from (12a) rot $\boldsymbol{B} = 0$, i.e.

$$\boldsymbol{B} = -\text{grad } \Psi, \tag{15}$$

where Ψ is the magnetic potential, for which from (12b) the potential equation

$$\Delta \Psi = 0 \tag{16}$$

follows. The axisymmetric solution reads

$$\Psi = -\sum_{l=0}^{\infty} (c_l r^l + d_l r^{-(l+1)}) P_l(\cos\theta) \tag{17}$$

(r, θ polar coordinates, c_l and d_l coefficients for the shell). From this last equation for the magnetic fields results:

$$\text{a) } B_r = \sum_{l=0}^{\infty} [l c_l r^{l-1} - (l+1) d_l r^{-(l+2)}] P_l, \quad \text{b) } B_\theta = \sum_{l=1}^{\infty} [c_l r^{l-1} + d_l r^{-(l+2)}] P_l^1; \tag{18}$$

$$\text{a) } H_r = \sum_{l=0}^{\infty} [l c_l r^{l-1} - (l+1) d_l r^{-(l+2)}] P_l - 4\pi M \cos\theta,$$

$$\text{b) } H_\theta = \sum_{l=1}^{\infty} [c_l r^{l-1} + d_l r^{-(l+2)}] P_l^1 + 4\pi M \sin\theta. \tag{19}$$

3.2. Interior ($r \leqslant r_1$)

In a similar way for the interior (vacuum) we find

$$\text{a) } \check{B}_r = \check{H}_r = \sum_{l=1}^{\infty} l \check{c}_l r^{l-1} P_l, \quad \text{b) } \check{B}_\theta = \check{H}_\theta = \sum_{l=1}^{\infty} \check{c}_l r^{l-1} P_l^1 \tag{20}$$

($\check{c}_l$ coefficients for the interior).

3.3. Exterior ($r \geqslant r_0$)

Analogously we get for the exterior (vacuum)

$$\text{a) } \bar{B}_r = \bar{H}_r = -\sum_{l=0}^{\infty} (l+1) \bar{d}_l r^{-(l+2)} P_l, \quad \text{b) } \bar{B}_\theta = \bar{H}_\theta = \sum_{l=1}^{\infty} \bar{d}_l r^{-(l+2)} P_l^1 \tag{21}$$

($\bar{d}_l$ coefficients for the exterior).

3.4. Boundary conditions and final solutions

The boundary conditions for the interface interior/shell

$$\text{a) } \check{B}_r(r_1) = B_r(r_1), \quad \text{b) } \check{B}_\theta(r_1) = H_\theta(r_1) \tag{22}$$

and for the interface shell/exterior

$$\text{a) } B_r(r_0) = \bar{B}_r(r_0), \quad \text{b) } H_\theta(r_0) = \bar{B}_\theta(r_0) \tag{23}$$

lead to the following solution of the problem:

interior:

$$\check{\boldsymbol{B}} = \check{\boldsymbol{H}} = 0, \tag{24}$$

shell:

$$B = \frac{2m}{r_0^3\left[1-\left(\frac{r_1}{r_0}\right)^3\right]} - \frac{\left(\frac{r_1}{r_0}\right)^3}{1-\left(\frac{r_1}{r_0}\right)^3}\frac{1}{r^3}[3(me_r)e_r - m], \tag{25}$$

$$H = -\frac{m}{r_0^3\left[1-\left(\frac{r_1}{r_0}\right)^3\right]} - \frac{\left(\frac{r_1}{r_0}\right)^3}{1-\left(\frac{r_1}{r_0}\right)^3}\frac{1}{r^3}[3(me_r)e_r - m] \tag{26}$$

(superposition of a homogeneous field and a dipole field). The vectors e_r and e_θ are the unit vectors in polar coordinates.

exterior:

$$\bar{B} = \bar{H} = \frac{1}{r^3}[3(me_r)e_r - m]. \tag{27}$$

This means a dipole field with the dipole moment

$$m = k\frac{4\pi}{3}(r_0^3 - r_1^3)M, \tag{28}$$

where

$$k = e_r \cos\theta - e_\theta \sin\theta. \tag{29}$$

3.5. Sphere

For the limiting case of a shell ($r_1 \to 0$) the above results read:

$$\text{a) } B = \frac{2m}{r_0^3}, \quad \text{b) } H = -\frac{m}{r_0^3}, \quad \text{c) } E = 0, \quad \text{d) } D = 0, \quad \text{e) } \varrho = 0,$$

$$\text{f) } \lambda = 0; \tag{30}$$

$$\text{a) } \bar{B} = \bar{H} = \frac{1}{r^3}[3(me_r)e_r - m], \quad \text{b) } \bar{E} = \bar{D} = 0, \tag{31}$$

where

$$m = k\frac{4\pi}{3}r_0^3 M = km. \tag{32}$$

This special case is quoted in some textbooks.

4. *Alternative II: Rotating sphere with respect to the inertial frame (inertial observer)*

4.1. Sphere

The Maxwell equations take the form:

$$\text{a) } \operatorname{rot} \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{j}, \quad \text{b) } \operatorname{div} \boldsymbol{B} = 0, \quad \text{c) } \operatorname{rot} \boldsymbol{E} = 0, \quad \text{d) } \operatorname{div} D = 4\pi\left(\varrho + \frac{\sigma}{c^2}\boldsymbol{v}\boldsymbol{E}\right). \tag{33}$$

The continuity equation, electric current density, and the constitutive equations read:

$$\operatorname{div} \boldsymbol{j} = 0, \tag{34}$$

$$\boldsymbol{j} = \varrho\boldsymbol{v} + \sigma\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}\right), \tag{35}$$

$$\text{a) } \boldsymbol{D} = \boldsymbol{E} + 4\pi\boldsymbol{P}, \quad \text{b) } \boldsymbol{B} = \boldsymbol{H} + 4\pi\boldsymbol{M}. \tag{36}$$

For the velocity of the rotating sphere

$$\boldsymbol{v} = \boldsymbol{\Omega}\times\boldsymbol{r} \quad (\boldsymbol{\Omega} = \boldsymbol{k}\Omega) \tag{37}$$

the relations

$$\text{a) } \operatorname{div} \boldsymbol{v} = 0, \quad \text{b) } \operatorname{rot} \boldsymbol{v} = 2\boldsymbol{\Omega} \tag{38}$$

hold. In our intended approximation the formulae

$$\text{a) } \boldsymbol{P} = \frac{1}{c}\boldsymbol{v}\times\boldsymbol{M}, \quad \text{b) } \boldsymbol{M} = \boldsymbol{k}M \tag{39}$$

can be used. Since the rotating sphere considered must not lead to Joule's heat production, we try to find the solution by the ansatz

$$\boldsymbol{E} = -\frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}. \tag{40}$$

Indeed, in this way we succeed and find the solutions:

$$\text{a) } \boldsymbol{B} = \frac{2\boldsymbol{m}}{r_0^3}, \quad \text{b) } \boldsymbol{H} = -\frac{\boldsymbol{m}}{r_0^3}, \quad \text{c) } \boldsymbol{E} = -\boldsymbol{e}_R\frac{2\Omega mr\sin\theta}{cr_0^3},$$

$$\text{d) } \boldsymbol{D} = \boldsymbol{e}_R\frac{\Omega mr\sin\theta}{cr_0^3}, \quad \text{e) } \varrho = \frac{\Omega m}{2\pi cr_0^3}, \tag{41}$$

where

$$\boldsymbol{e}_R = \boldsymbol{e}_r\sin\theta + \boldsymbol{e}_\theta\cos\theta \tag{42}$$

is the radial unit vector in cylindrical coordinates.

4.2. Exterior (vacuum)

The field equations ($\overline{\boldsymbol{B}} = \overline{\boldsymbol{H}}$, $\overline{\boldsymbol{D}} = \overline{\boldsymbol{E}}$):

$$\text{a) rot}\,\overline{\boldsymbol{B}} = 0, \quad \text{b) div}\,\overline{\boldsymbol{B}} = 0, \quad \text{c) rot}\,\overline{\boldsymbol{E}} = 0, \quad \text{d) div}\,\overline{\boldsymbol{E}} = 0 \tag{43}$$

lead to the solutions

$$\text{a) } \overline{\boldsymbol{B}} = \frac{1}{r^3}\left[3(\boldsymbol{m}\boldsymbol{e}_r)\boldsymbol{e}_r - \boldsymbol{m}\right],$$

$$\text{b) } \overline{\boldsymbol{E}} = -\frac{\Omega m r_0^2}{2cr^4}\left[\boldsymbol{e}_r(1+3\cos 2\theta)+2\boldsymbol{e}_\theta \sin 2\theta\right] \quad \text{(quadrupole field)} \tag{44}$$

and to the electric surface charge density

$$\lambda = -\frac{\Omega m}{4\pi c r_0^2}(1+\cos 2\theta) \tag{45}$$

if we take into account the boundary conditions

$$\text{a) } B_r(r_0) = \overline{B}_r(r_0), \quad \text{b) } H_\theta(r_0) = \overline{B}_\theta(r_0), \quad \text{c) } \overline{E}_r(r_0) - D_r(r_0) = 4\pi\lambda,$$

$$\text{d) } E_\theta(r_0) = \overline{E}_\theta(r_0) \tag{46}$$

which in this approximation result from the general conditions (8) and (9).

4.3. Electric neutrality of the sphere

According to our assumption the sphere should not carry any electric charge Q. Therefore we have to see whether our results (41e) and (45) fulfil this condition:

$$Q = \int \varrho\, dV + \int \lambda\, df = \frac{\Omega m}{2\pi c r_0^3}\cdot\frac{4\pi r_0^3}{3} - \frac{\Omega m}{4\pi c r_0^2}\cdot 2\pi r_0^2 \int_{\theta=0}^{\pi} (1+\cos 2\theta)\sin\theta\, d\theta.$$

Performing the integration we find in fact

$$Q = 0. \tag{47}$$

5. Alternative III: Resting sphere with respect to the rotating frame (noninertial observer)

5.1. Sphere

In this case we have to specialize the generalized Maxwell equations (1) to (4), taking into account that

$$\text{a) } \boldsymbol{\Omega}_0 = \boldsymbol{\Omega}, \quad \text{b) } \boldsymbol{v}' = 0 \tag{48}$$

is valid. We find

$$\text{rot}'\,\boldsymbol{H}' = \frac{4\pi}{c}\boldsymbol{j}', \tag{49}$$

$$\operatorname{div}' \boldsymbol{D}' = 4\pi\varrho' + (\boldsymbol{\Omega H}'), \tag{50}$$

$$\operatorname{rot}' \boldsymbol{E}' = 0, \tag{51}$$

$$\operatorname{div}' \boldsymbol{B}' = -\frac{2}{c}(\boldsymbol{\Omega E}'). \tag{52}$$

The continuity equation is

$$\operatorname{div}' \boldsymbol{j}' = 0. \tag{53}$$

For the electric current density we obtain

$$\boldsymbol{j}' = \sigma \boldsymbol{E}', \tag{54}$$

while the constitutive equations take the form

$$\text{a) } \boldsymbol{D}' = \boldsymbol{E}', \quad \text{b) } \boldsymbol{B}' = \boldsymbol{H}' + k4\pi \boldsymbol{M}. \tag{55}$$

The last two equations are justified, because in the first approximation in Ω we can use the relations

$$\text{a) } \boldsymbol{P}' = 0, \quad \text{b) } \boldsymbol{M}' = k\boldsymbol{M}. \tag{56}$$

Since Joule's heat production does not occur, we conclude

$$\boldsymbol{E}' = \boldsymbol{D}' = 0. \tag{57}$$

Under these circumstances we find from the generalized Maxwell equations (49) to (52)

$$\text{a) } \boldsymbol{B}' = \frac{2\boldsymbol{m}}{r_0^3}, \quad \text{b) } \boldsymbol{H}' = -\frac{\boldsymbol{m}}{r_0^3}, \quad \text{c) } \varrho' = \frac{\Omega m}{2\pi c r_0^3}. \tag{58}$$

5.2. Exterior (vacuum)

In this case the field equations have the form ($\bar{\boldsymbol{B}}' = \bar{\boldsymbol{H}}'$, $\bar{\boldsymbol{D}}' = \bar{\boldsymbol{E}}'$):

$$\text{a) } \operatorname{rot}' \bar{\boldsymbol{B}}' = 0, \quad \text{b) } \operatorname{div}' \bar{\boldsymbol{E}}' = \frac{2}{c}(\boldsymbol{\Omega}\bar{\boldsymbol{B}}'), \quad \text{c) } \operatorname{rot}' \bar{\boldsymbol{E}}' = 0, \quad \operatorname{div}' \bar{\boldsymbol{B}}' = 0. \tag{59}$$

Taking into account the boundary conditions

$$\text{a) } B_r'(r_0) = \bar{B}_r'(r_0), \quad \text{b) } H_\theta'(r_0) = \bar{B}_\theta'(r_0), \quad \text{c) } \bar{E}_r'(r_0) - D_r'(r_0) = 4\pi\lambda',$$
$$\text{d) } E_\theta'(r_0) = \bar{E}_\theta'(r_0), \tag{60}$$

we find the solutions ($r' = r$):

$$\bar{\boldsymbol{B}}' = \frac{1}{r^3}[3(\boldsymbol{m}\boldsymbol{e}_r')\boldsymbol{e}_r' - \boldsymbol{m}], \tag{61}$$

$$\bar{\boldsymbol{E}}' = -\frac{\Omega r_0^2 m}{2cr^4}[\boldsymbol{e}_r'(1+3\cos 2\theta') + 2\boldsymbol{e}_\theta' \sin 2\theta'] - \frac{\Omega m}{2cr^2}[\boldsymbol{e}_r'(1-\cos 2\theta') - 2\boldsymbol{e}_\theta' \sin 2\theta'] \tag{62}$$

and the surface charge density

$$\lambda' = -\frac{\Omega m}{4\pi c r_0^2}(1+\cos 2\theta'). \tag{63}$$

Comparing the results (58c) respectively (63) with (41e) respectively (45), because of $\theta' = \theta$ we obtain the invariance:

$$\text{a) } \varrho' = \varrho, \quad \text{b) } \lambda' = \lambda, \tag{64}$$

which we expected indeed in this approximation.

6. *Alternative IV: Rotating sphere with respect to the rotating frame (noninertial observer)*

6.1. Sphere

This case is realized by the assumptions

$$\text{a) } \boldsymbol{\Omega}_0 = \boldsymbol{\Omega}, \quad \text{b) } \boldsymbol{v}' = -\boldsymbol{\Omega}\times\boldsymbol{r}'. \tag{65}$$

From (1) to (6) the field equations

$$\text{rot}' \boldsymbol{H}' = \frac{4\pi}{c}\boldsymbol{j}', \tag{66}$$

$$\text{div}' \boldsymbol{D}' = 4\pi\varrho' + \frac{2}{c}(\boldsymbol{\Omega}\boldsymbol{H}'), \tag{67}$$

$$\text{rot}' \boldsymbol{E}' = 0, \tag{68}$$

$$\text{div}' \boldsymbol{B}' = -\frac{2}{c}(\boldsymbol{\Omega}\boldsymbol{E}'), \tag{69}$$

the electric current density

$$\boldsymbol{j}' = -\varrho'(\boldsymbol{\Omega}\times\boldsymbol{r}')+\sigma\left[\boldsymbol{E}'-\frac{1}{c}(\boldsymbol{\Omega}\times\boldsymbol{r}')\times\boldsymbol{B}'\right], \tag{70}$$

and the continuity equation

$$\text{div}' \boldsymbol{j}' = 0 \tag{71}$$

result. In this physical configuration the electric polarization and the magnetization have the form

$$\text{a) } \boldsymbol{P}' = -\frac{M}{c}(\boldsymbol{\Omega}\times\boldsymbol{r}')\times\boldsymbol{k}, \quad \text{b) } \boldsymbol{M}' = \boldsymbol{k}M. \tag{72}$$

Hence the constitutive equations (7) read

$$\text{a) } \boldsymbol{D}' = \boldsymbol{E}'-\frac{4\pi M}{c}(\boldsymbol{\Omega}\times\boldsymbol{r}')\times\boldsymbol{k}, \quad \text{b) } \boldsymbol{B}' = \boldsymbol{H}'+\boldsymbol{k}4\pi M. \tag{73}$$

The absence of Joule's heat production leads us to

$$E' = \frac{1}{c}(\boldsymbol{\Omega}\times\boldsymbol{r}')\times\boldsymbol{B}'. \quad (74)$$

Under these circumstances the generalized Maxwell equations (66) to (69) are fulfilled by the solutions

$$\text{a) } \boldsymbol{B}' = \frac{2\boldsymbol{m}}{r_0^3}, \quad \text{b) } \boldsymbol{H}' = -\frac{\boldsymbol{m}}{r_0^3}, \quad (75)$$

$$\text{a) } \boldsymbol{E}' = \boldsymbol{e}'_R \frac{2\Omega m r \sin\theta'}{c r_0^3}, \quad \text{b) } \boldsymbol{D}' = -\boldsymbol{e}'_R \frac{\Omega m r \sin\theta'}{c r_0^3}, \quad \text{c) } \varrho' = 0. \quad (76)$$

6.2. Exterior (vacuum)

The field equations read ($\overline{\boldsymbol{B}}' = \overline{\boldsymbol{H}}'$, $\overline{\boldsymbol{D}}' = \overline{\boldsymbol{E}}'$)

$$\text{a) } \mathrm{rot}'\,\overline{\boldsymbol{B}}' = 0, \quad \text{b) } \mathrm{div}'\,\overline{\boldsymbol{E}}' = \frac{2}{c}(\boldsymbol{\Omega}\overline{\boldsymbol{B}}'), \quad \text{c) } \mathrm{rot}'\,\overline{\boldsymbol{E}}' = 0, \quad \text{d) } \mathrm{div}'\,\overline{\boldsymbol{B}}' = 0. \quad (77)$$

The form of the boundary conditions is in the first order in Ω the same as in (60). Exploiting them, we find the solutions

$$\overline{\boldsymbol{B}}' = \frac{1}{r^3}[3(\boldsymbol{m}\boldsymbol{e}'_r)\boldsymbol{e}'_r - \boldsymbol{m}], \quad (78)$$

$$\overline{\boldsymbol{E}}' = -\frac{\Omega m}{2cr^2}[\boldsymbol{e}'_r(1-\cos 2\theta') - 2\boldsymbol{e}'_\theta \sin 2\theta'], \quad (79)$$

whereas the electric surface charge density reads

$$\lambda' = 0. \quad (80)$$

The results (76c) and (80) are in accordance with (30e) and (30f).

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