SU(4) YANG-MILLS FIELD SOLUTION*

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We find a regular solution of the four-dimensional Euclidean SU(4) Yang-Mills equations. This is the analogue of that of SU(2) found by Belavin, Polyakov, Schwartz and Tyupkin.

The aim of this note is to present a non-singular solution of the SU(4) Euclidean Yang-Mills equations which is the analogue of the BPST solution [1]. Our solution has six gauge fields different from zero.

We shall use the following notation. Greek indices refer to four-dimensional Euclidean space and run from 1 to 4, the Laplacian operator in four dimensions is denoted by $\Box$, i.e. $\Box = \partial_\mu\partial_\mu$, the abbreviation ϕ_μ stands for $\partial\phi/\partial x^\mu \equiv \partial_\mu\phi$, similarly $\phi_{\mu\nu} \equiv \partial_\mu\partial_\nu\phi$, $\phi_\mu^2 \equiv (\phi_\mu)^2$, the isospin indices are denoted by a, b, c and run from 1 to 15.

The Euclidean Yang-Mills field equations for the fifteen gauge fields A_μ^a, $a = 1, 2, \ldots, 15$, are

$$\partial_\mu F^a_{\mu\nu} + f^{abc} A^b_\mu F^c_{\mu\nu} = 0, \tag{1}$$

where the field strengths $F^a_{\mu\nu}$ are defined by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu. \tag{2}$$

In order to simplify Eqs (1) we have chosen an adequate gauge for the fields A_μ^a and we work on a special Euclidean frame. More precisely, we assume that all the gauge fields A_μ^a are derived from a scalar $\phi(x)$ according to the following ansatz[1]:

$$A_\mu^2 = (-\phi_2, \phi_1, 0, 0), \quad A_\mu^5 = (-\phi_3, 0, \phi_1, 0), \quad A_\mu^7 = (0, -\phi_3, \phi_2, 0),$$

$$A_\mu^{10} = (\phi_4, 0, 0, -\phi_1), \quad A_\mu^{12} = (0, \phi_4, 0, -\phi_2), \quad A_\mu^{14} = (0, 0, \phi_4, -\phi_3), \tag{3}$$

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[1] A similar ansatz with complex gauge fields in four-dimensional Minkowski space has been used by Kaku [2], who found a singular solution for the Lienard-Wiechert-type potentials in SU(4).

the gauge fields A_μ^a for $a = 1, 3, 4, 6, 8, 9, 11, 13, 15$ are assumed to be identical to zero. From (3) it follows that the potentials A_μ^a are such that

$$\partial_\mu A_\mu^a = 0 \quad \text{for} \quad a = 1, 2, \ldots, 15. \tag{4}$$

Using the structure constants of SU(4) (see Ref. [3]) and introducing the notation $\chi_{\mu\nu} \equiv \phi_\mu \phi_\nu - \phi_{\mu\nu}$ we find that the field strength components $F_{12}^a, F_{13}^a, F_{14}^a, F_{23}^a, F_{24}^a, F_{34}^a$ (in this order) are given by

$$\begin{aligned}
&\phi_{11}+\phi_{22}+\phi_3^2+\phi_4^2;\ -\chi_{23};\ -\chi_{24};\ \chi_{13};\ \chi_{14};\ 0; && \text{for } a = 2,\\
&-\chi_{23};\ \phi_{11}+\phi_{33}+\phi_2^2+\phi_4^2;\ -\chi_{34};\ -\chi_{12};\ 0;\ \chi_{14}; && \text{for } a = 5,\\
&\chi_{13};\ -\chi_{12};\ 0;\ \phi_{22}+\phi_{33}+\phi_1^2+\phi_4^2;\ -\chi_{34};\ \chi_{24}; && \text{for } a = 7,\\
&\chi_{24};\ \chi_{34};\ -\phi_{11}-\phi_{44}-\phi_2^2-\phi_3^2;\ 0;\ \chi_{12};\ \chi_{13}; && \text{for } a = 10,\\
&-\chi_{14};\ 0;\ \chi_{12};\ \chi_{34};\ -\phi_{22}-\phi_{44}-\phi_1^2-\phi_3^2;\ \chi_{23}; && \text{for } a = 12,\\
&0;\ -\chi_{14};\ \chi_{13};\ -\chi_{24};\ \chi_{23};\ -\phi_{33}-\phi_{44}-\phi_1^2-\phi_2^2 && \text{for } a = 14.
\end{aligned} \tag{5}$$

Due to our special ansatz, it can be shown that Eqs (1) for $a = 1, 3, 4, 6, 8, 9, 11, 13, 15$ are identically satisfied without imposing any restriction on the scalar ϕ. The same property applies for the index ν in (1) for which the corresponding Cartesian components of A_μ^a in (3) are zero. Thus, from the sixty equations (1) only twelve are not identically satisfied. All the restrictions on ϕ are contained in the equation

$$\Box\phi_\beta + 2\{-\phi_\beta \Box\phi + \phi_\alpha \phi_{\alpha\beta} - \phi_\alpha \phi_\alpha \phi_\beta\} = 0. \tag{6}$$

In order to simplify this non-linear equation we introduce a new scalar ψ by means of

$$\phi = \ln \psi, \tag{7}$$

obtaining the following equation for ψ:

$$\Box\psi_\beta - 3\psi^{-1}\psi_\beta \Box\psi = 0. \tag{8}$$

Now it is easy to realize that every solution of the equation

$$\Box\psi = C\psi^3 \tag{9}$$

is also a solution of (8) for any constant C.

The simplest non-singular solution of (9) for $C = -1$ is

$$\psi(x) = \frac{2\sqrt{2}\,|\lambda|}{(x_1^2+x_2^2+x_3^2+x_4^2)+\lambda_1^2}, \tag{10}$$

where λ is an arbitrary number. The solution (10) of (9) gives rise, by means of (7), to the non-singular solution (3) of the Yang–Mills equations (1).

We shall publish a more detailed analysis of our solution in a separate paper.

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